Radiation Sources

■ Problem 1.1. Radiation Energy Spectra: Line vs. Continuous

Line (or discrete energy): a, c, d, e, f, and i.
Continuous energy: b, g, and h.

■ Problem 1.2. Conversion electron energies compared.

Since the electrons in outer shells are bound less tightly than those in closer shells, conversion electrons from outer shells will have greater emerging energies. Thus, the M shell electron will emerge with greater energy than a K or L shell electron.

■ Problem 1.3. Nuclear decay and predicted energies.

We write the conservation of energy and momentum equations and solve them for the energy of the alpha particle. Momentum is given the symbol "p", and energy is "E". For the subscripts, "al" stands for alpha, while "b" denotes the daughter nucleus.

$$p_{al} + p_b = 0 \qquad \frac{p_{al}^2}{2\,m_{al}} = E_{al} \qquad \frac{p_b^2}{2\,m_b} = E_b \qquad E_{al} + E_b = Q \quad \text{and} \quad Q = 5.5\ \text{MeV}$$

Solving our system of equations for E_{al}, E_b, p_{al}, p_b, we get the solutions shown below. Note that we have two possible sets of solutions (this does not effect the final result).

$$E_b = 5.5\left(1 - \frac{m_{al}}{m_{al} + m_b}\right) \qquad E_{al} = \frac{5.5\,m_{al}}{m_{al} + m_b}$$

$$p_{al} = \mp\frac{3.31662\sqrt{m_{al}}\sqrt{m_b}}{\sqrt{m_{al} + m_b}} \qquad p_b = \pm\frac{3.31662\sqrt{m_{al}}\sqrt{m_b}}{\sqrt{m_{al} + m_b}}$$

We are interested in finding the energy of the alpha particle in this problem, and since we know the mass of the alpha particle and the daughter nucleus, the result is easily found. By substituting our known values of $m_{al} = 4$ and $m_b = 206$ into our derived E_{al}equation we get:

$$E_{al} = 5.395\ \text{MeV}$$

Note : We can obtain solutions for all the variables by substituting $m_b = 206$ and $m_{al} = 4$ into the derived equations above :

$$E_{al} = 5.395\ \text{MeV} \qquad E_b = 0.105\ \text{MeV} \qquad p_{al} = \mp 6.570\sqrt{\text{amu} * \text{MeV}} \qquad p_b = \pm 6.570\sqrt{\text{amu} * \text{MeV}}$$

■ Problem 1.4. Calculation of Wavelength from Energy.

Since an x-ray must essentially be created by the de-excitation of a single electron, the maximum energy of an x-ray emitted in a tube operating at a potential of 195 kV must be 195 keV. Therefore, we can use the equation E=hν, which is also E=hc/λ, or λ=hc/E. Plugging in our maximum energy value into this equation gives the minimum x-ray wavelength.

$$\lambda = \frac{h \times c}{E} \qquad \text{where we substitute } h = 6.626 \times 10^{-34}\ J * s,\ c = 299\,792\,458\ m/s \text{ and } E = 195\ \text{keV}$$

$$\lambda = \frac{\mathbf{1.01869\ J\text{–}m}}{\mathbf{KeV}} = \mathbf{0.0636\ Angstroms}$$

■ Problem 1.5. ^{235}U Fission Energy Release.

Using the reaction $^{235}U \rightarrow {}^{117}\text{Sn} + {}^{118}\text{Sn}$, and mass values, we calculate the mass defect of:

$$M\,(^{235}U) - \left[M(^{117}\text{Sn}) + M(^{118}\text{Sn})\right] = \Delta\text{M}$$

and an expected energy release of $\Delta\text{M}c^2$.

$$\boldsymbol{Q} = \mathbf{(235.0439 - (116.9029 + 117.9016))\ AMU} \times \frac{\mathbf{931.5\ MeV}}{\mathbf{AMU}} = \mathbf{223\ MeV}$$

This is one of the most exothermic reactions available to us. This is one reason why, of course, nuclear power from uranium fission is so attractive.

■ Problem 1.6. Specific Activity of Tritium.

Here, we use the text equation Specific Activity = (ln(2)*Av)/ ($T_{1/2}$*M), where Av is Avogadro's number, $T_{1/2}$ is the half-life of the isotope, and M is the molecular weight of the sample.

$$\textbf{Specific Activity} = \frac{\mathbf{ln(2) \times Avogadro's\ Constant}}{\boldsymbol{T_{1/2}\,M}}$$

We substitute $T_{1/2}$ = 12.26 years and M=$\frac{3 \text{ grams}}{\text{mole}}$to get the specific activity in disintegrations/(gram–year).

$$\textbf{Specific Activity} = \frac{\mathbf{1.13492 \times 10^{22}}}{\mathbf{gram\text{–}year}}$$

The same result expressed in terms of kCi/g is shown below

$$\textbf{Specific Activity} = \frac{\mathbf{9.73\ kCi}}{\mathbf{gram}}$$

■ Problem 1.7. Accelerated particle energy.

The energy of a particle with charge q falling through a potential ΔV is qΔV. Since ΔV= 3 MV is our maximum potential difference, the maximum energy of an alpha particle here is q*(3 MV), where q is the charge of the alpha particle (+2). The maximum alpha particle energy expressed in MeV is thus:

$$\textbf{Energy} = \mathbf{3\ Mega\ Volts \times 2\ Electron\ Charges = 6.\ MeV}$$

■ Problem 1.8. Photofission of deuterium. ${}^{2}_{1}D + \gamma \to {}^{1}_{0}n + {}^{1}_{1}p$ + Q (-2.226 MeV)

The reaction of interest is ${}^{2}_{1}D + {}^{0}_{0}\gamma \to {}^{1}_{0}n + {}^{1}_{1}p$+ Q (-2.226 MeV). Thus, the γ must bring an energy of at least 2.226 MeV in order for this endothermic reaction to proceed. Interestingly, the opposite reaction will be exothermic, and one can expect to find 2.226 MeV gamma rays in the environment from stray neutrons being absorbed by hydrogen nuclei.

■ Problem 1.9. Neutron energy from D-T reaction by 150 keV deuterons.

We write down the conservation of energy and momentum equations, and solve them for the desired energies by eliminating the momenta. In this solution, "a" represents the alpha particle, "n" represents the neutron, and "d" represents the deuteron (and, as before, "p" represents momentum, "E" represents energy, and "Q" represents the Q-value of the reaction).

$$p_a + p_n = p_d \qquad \frac{p_a^2}{2\,m_a} = E_a \qquad \frac{p_n^2}{2\,m_n} = E_n \qquad \frac{p_d^2}{2\,m_d} = E_d \qquad E_a + E_n = E_d + Q$$

Next we want to solve the above equations for the unknown energies by eliminating the momenta. (Note : Using computer software such as Mathematica is helpful for painlessly solving these equations).

We evaluate the solution by plugging in the values for particle masses (we use approximate values of "m_a," "m_n,"and "m_d" in AMU, which is okay because we are interested in obtaining an energy value at the end). We define all energies in units of MeV, namely the Q-value, and the given energy of the deuteron (both energy values are in MeV). So we substitute $m_a = 4$, $m_n = 1$, $m_d = 2$, Q = 17.6, $E_d = 0.15$ into our momenta independent equations. This yields two possible sets of solutions for the energies (in MeV). One corresponds to the neutron moving in the forward direction, which is of interest.

$$E_n = 13.340\ \text{MeV} \qquad E_a = 4.410\ \text{MeV}$$
$$E_n = 14.988\ \text{MeV} \qquad E_a = 2.762\ \text{MeV}$$

Next we solve for the momenta by eliminating the energies. When we substitute $m_a = 4$, $m_n = 1$, $m_d = 2$, Q = 17.6, $E_d = 0.15$ into these equations we get the following results.

$$p_n = \frac{p_d}{5} \mp \frac{1}{5}\sqrt{2}\,\sqrt{3\,p_d^2 + 352} \qquad p_a = \frac{1}{10}\left(8\,p_d \pm 2\sqrt{2}\,\sqrt{3\,p_d^2 + 352}\right)$$

We do know the initial momentum of the deuteron, however, since we know its energy. We can further evaluate our solutions for p_n and p_a by substituting:

$$p_d = \sqrt{2 \times 2 \times 0.15}$$

The particle momenta (in units of $\sqrt{\text{amu} * \text{MeV}}$) for each set of solutions is thus:

$$p_n = -5.165 \qquad p_a = 5.940$$
$$p_n = 5.475 \qquad p_a = -4.700$$

The largest neutron momentum occurs in the forward (+) direction, so the highest neutron energy of 14.98 MeV corresponds to this direction.

Radiation Interaction Problems

Problem 2.1 Stopping time in silicon and hydrogen.

Here, we apply Equation 2.3 from the text.

$$T_{\text{stop}} = \frac{1.2\ \text{range}\sqrt{\frac{\text{mass}}{\text{energy}}}}{10^7}$$

Now we evaluate our equation for an alpha particle stopped in silicon. We obtained the value for "range" from Figure 2.8 (converting from mass thickness to distance in meters by dividing by the density of Si $\simeq$ 2330 mg/cm^3). The value for "mass" is approximated as 4 AMU for the alpha particle, and the value for "energy" is 5 MeV. We substitute range = $22\ x\ 10^{-6}$, mass = 4 and energy = 5 into Equation 2.3 to get the approximate alpha stopping time (in seconds) in silicon.

$$T_{\text{stop}} = 2.361 \times 10^{-12}\ \text{seconds}$$

Now we do the same for the same alpha particle stopped in hydrogen gas. Again, we obtain the value for "range" (in meters) from Figure 2.8 in the same manner as before (density of H $\simeq$ .08988 mg/cm^3), and, of course, the values for "mass" and "energy" are the same as before (nothing about the alpha particle has changed). We substitute range = 0.1, mass = 4 and energy =5 into Equation 2.3 to get the approximate alpha stopping time (in seconds) in hydrogen gas.

$$T_{\text{stop}} = 1.073 \times 10^{-8}\ \text{seconds}$$

The results from this problem tell us that the stopping times for alphas range from about picoseconds in solids to nanoseconds in a gas.

Problem 2.2. Partial energy lost in silicon for 5 MeV protons.

Clever technique: A 5 MeV proton has a range of 210 microns in silicon according to Figure 2-7. So, after passing through 100 microns, the proton has enough energy left to go another 110 microns. It takes about 3.1 MeV, according to the same figure, to go this 110 microns, so this must be the remaining energy. Thus the proton must have lost 1.9 MeV in the first 100 microns.

Problem 2.3. Energy loss of 1 MeV alpha in 5 microns Au.

From Figure 2.10, we find that $\frac{-1}{\rho}\frac{dE}{dx} \simeq 380\ \frac{\text{MeV}*\text{cm}^2}{g}$. Therefore, $\frac{dE}{dx} \simeq 380\ \frac{\text{MeV}*\text{cm}^2}{g} * \rho$ (ignoring the negative sign will not affect the result of this problem).

$$\text{Energy loss} = \frac{\rho\ (dE/dx)\ \Delta x}{\rho}$$

We substitute $dE/dx = \frac{380\ \text{MeV cm}^2\ \rho}{\text{gram}}$, $\rho = \frac{19.32\ \text{grams}}{\text{cm}^3}$ and $\Delta x = 5$ microns to get the energy loss of the 1 MeV α-particle in 5 μm Au (in non-SI units).