CONTENTS

There are no problems and therefore, no problem solutions for the "read-only" Chapters 4, 9, and 13.

1.1

(a) From Table 1.1...
  (i) Si or Ge
  (ii) AlP, AlAs, AlSb, GaN, GaP, GaAs, GaSb, InP, InAs, or InSb

(b) Crystalline material has the same atomic pattern or order throughout the material, while polycrystalline material has crystalline subsections that are misaligned with respect to each other.

(c) A *unit cell* is a small portion of a crystal that could be used to reproduce the crystal. (The preceding is from the first sentence of Subsection 1.2.1.)

(d)

Unit cell	Atoms/unit cell
simple cubic	1
bcc	2
fcc	4
diamond	8

(e) $1\text{Å} = 10^{-8}$ cm
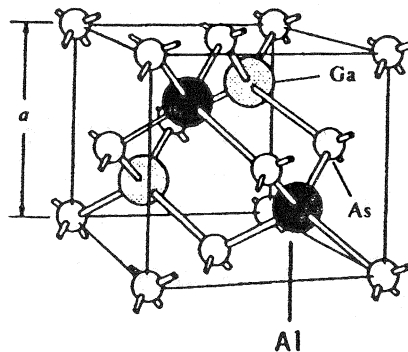
(f) a (one lattice constant)

(g) 4

(h) As summarized in Table 1.3: ( )--crystal plane, { }--equivalent planes, [ ]--crystal direction, and < >--equivalent directions.

(i) See Subsection 1.3.2.

1.2

In the $Al_{0.5}Ga_{0.5}As$ unit cell, pictured below, fcc sublattice sites containing the Column III elements are equally occupied by Al and Ga atoms.

1.3

(a) Ge crystallizes in the diamond lattice where there are 8 atoms per unit cell (see Subsection 1.2.3). Thus

$$\text{DENSITY} = \frac{8}{a^3} = \frac{8}{(5.65 \times 10^{-8})^3} = 4.44 \times 10^{22} \text{ atoms/cm}^3$$

1.4

(a) From Fig. 1.3(c), we conclude nearest-neighbors in the bcc lattice lie along the unit cell body diagonal. Since the body diagonal of a cube is equal to $\sqrt{3}$ times a cube side length (the lattice constant a),

$$\left(\frac{\text{Nearest-Neighbor}}{\text{Distance}}\right) = \frac{\sqrt{3}}{2} a$$

(b) From Fig. 1.3(d), nearest-neighbors in the fcc lattice are concluded to lie along a *cube-face* diagonal. The diagonal of a cube face is equal to $\sqrt{2}$ times a cube side length. Thus

$$\left(\frac{\text{Nearest-Neighbor}}{\text{Distance}}\right) = \frac{\sqrt{2}}{2} a$$

1.5

(a) Looking at Fig. 1.4(a) one concludes
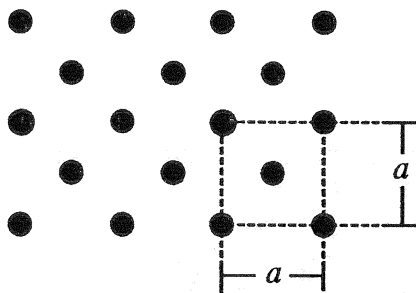
(b) For Si at room temperature $a = 5.43 \times 10^{-8}$ cm. From the above figure one concludes that there are $(1/4 \times 4$ corner atoms$) + 1$ body atom $= 2$ atoms per an area of a^2 on the (100) surface. Thus one has

$$\frac{2}{a^2} = \frac{2}{(5.43 \times 10^{-8})^2} = 6.78 \times 10^{14} \text{ Si atoms/cm}^2$$

(c) For a (110) plane one has the atom placement pictured below

(d) On the (110) plane in the area $a \times \sqrt{2}a$ one has $(1/4 \times 4$ corner atoms$) + (1/2 \times 2$ edge atoms$) + 2$ body atoms $= 4$ atoms. Thus one has

$$\frac{4}{\sqrt{2}a^2} = \frac{2\sqrt{2}}{(5.43 \times 10^{-8})^2} = 9.59 \times 10^{14} \text{ Si atoms/cm}^2$$

(e) MATLAB program script (paralleling Exercise 1.3)...

```
%Solution to Problem 1.5(e)
N=input('input number of atoms on (100) face of unit cell, N = ');
a=input('lattice constant in angstrom, a = ');
surfaceden=N*(1.0e16)/(a^2)      %number of atoms/cm^2
```
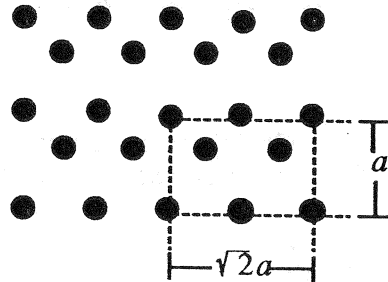
(Note: This and all other problem solutions are available on disk.)

1.6

(a) (i) Following the procedure outlined in the text

1, 3, 1	...intercepts (normalized)
1, 1/3, 1	...[1/intercept]s
3, 1, 3	...reduction to lowest whole-number set
(313)	...Miller index notation for plane

(ii) As noted in the text near the end of Subsection 1.2.4, the normal to a plane in the cubic crystal system has the same Miller indices as the plane.

[313]   ...Miller index notation for normal to plane

(b) (i) Again following the Miller indexing procedure,

1, 1, 1/2	...normalized intercepts
1, 1, 2	...[1/intercept]s
1, 1, 2	...lowest whole-number set
(112)	...Miller indices of plane

(ii) Assume the vector has a length d. Its projections along the x, y, and z axes are then 0, 0, and d, respectively. Reducing to the lowest possible whole-number set and enclosing in square brackets, then yields

$\qquad$ **[001]** $\qquad$ ...Miller indices of direction vector

1.7

For each of the given planes, the Miller indexing procedure must be reversed to determine the intercepts of the given plane on the coordinate axes. Using part (c) as an example, one proceeds as follows:

$\qquad$ (123) $\qquad$ ...Miller indices
$\qquad$ 1, 2, 3 $\qquad$ ...[1/intercept]s
$\qquad$ 1,1/2,1/3 $\qquad$ ...intercepts

The plane in question intercepts the x, y, z coordinate axes at a, $a/2$, and $a/3$, respectively. Note that any multiple of the cited intercept set – such as $3a$, $2a$, a – would also be correct. All such planes are parallel equivalent planes.

(a) $\qquad$ (b) $\qquad$ (c) $\qquad$ (d)

(e) $\qquad$ (f) $\qquad$ (g) $\qquad$ (h)
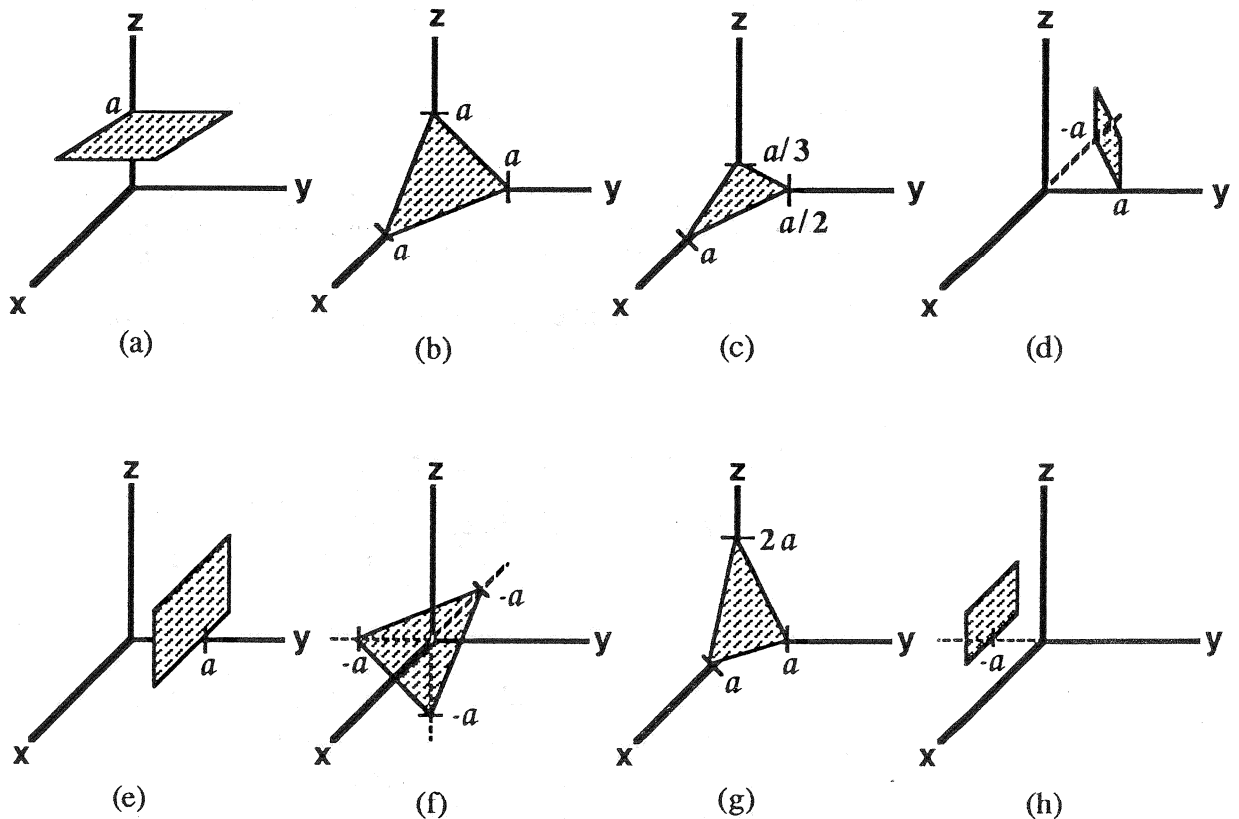
1.8

Miller indices may be viewed as specifying the projection (in arbitrary units) of the to-be-pictured vectors along the coordinate axes. For example, [010] corresponds to a vector with a unit projection along the y-axis and no projection along the x- or z-axes. In other words, [010] is coincident with the $+y$ coordinate axis. The other required direction vectors are deduced in a similar manner and are as pictured below.
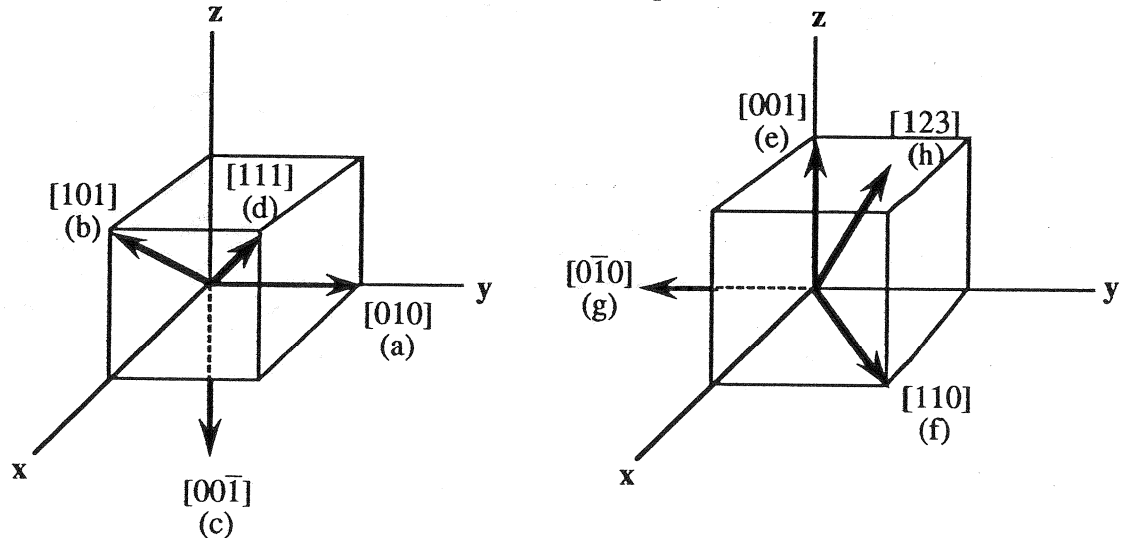
1.9

As noted in the problem statement, two directions $[h_1k_1l_1]$ and $[h_2k_2l_2]$ will be mutually perpendicular if

$$h_1h_2 + k_1k_2 + l_1l_2 = 0$$

(a) Here $[h_1k_1l_1] = [100]$, requiring $h_2 = 0$. *All directions $[0k_2l_2]$ are perpendicular to* [100]. Two specific simple examples are [001] and [011].

(b) Given $[h_1k_1l_1] = [111]$, one requires the Miller indices of the perpendicular direction to be such that $h_2 + k_2 + l_2 = 0$. Two specific examples are [01$\bar{1}$] and [11$\bar{2}$].

1.10

As shown in the following left-hand figure, when the [011] and [010] directions are pictured simultaneously, it becomes obvious that the angle between the two directions is 45°. Alternatively, the angle between the two directions can be computed using the $\cos(\theta)$ relationship in Problem 1.9. Specifically, given $[h_1 k_1 l_1] = [011]$ and $[h_2 k_2 l_2] = [010]$, $\cos(\theta) = 1/\sqrt{2}$ and $\theta = 45°$. The required positioning of the "grooves" on the wafer's surface is pictured in the following right-hand figure.
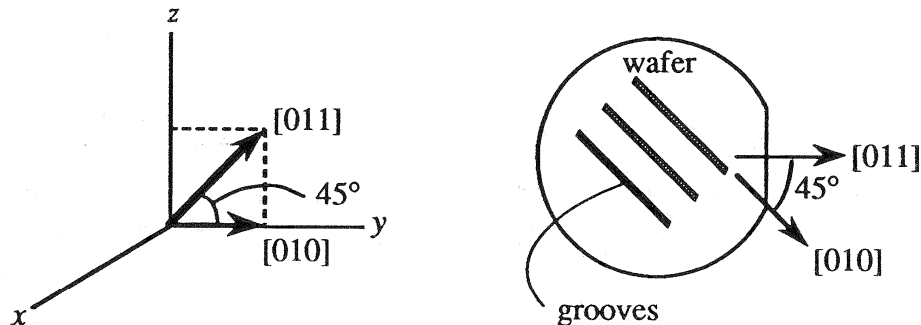
1.11

(a) If the Fig. P1.11 unit cell is conceptually copied and the cells stacked like blocks in a nursery, one concludes the resulting latttice is a $\boxed{\text{simple cubic lattice}}$.

(b) There is one atom inside the unit cell and the unit cell volume is a^3. Thus $\boxed{\text{atoms/unit volume} = 1/a^3}$.

(c) For a (110) surface plane the atom positioning would be as pictured below.

$$\frac{\text{atoms}}{\text{unit area}} = \frac{1 \text{ atom}}{(a)(\sqrt{2}a)} = \frac{1}{\sqrt{2}a^2}$$
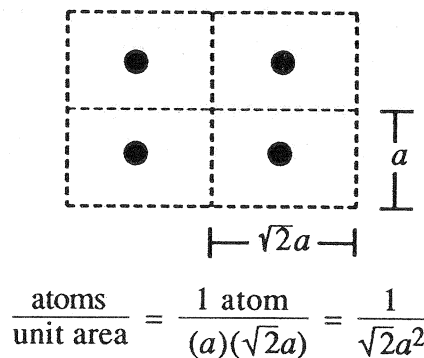
(d) [111] ... The specified vector has equal projections on the three coordinate axes.

1-6

<u>1.12</u>

Equivalent planes: (a) 6, (b) 12, (c) 8.

Equivalent directions: (d) 6, (e) 12, (f) 8.

NOTE: The answers may be deduced from geometrical considerations — or — by noting the total number of possible combinations of the given, and negatives of the given, Miller indices.

<u>1.13</u>

(a) In the simple cubic lattice the nearest-neighbor distance is a, where a is the side length of the cube, and the atomic radius r is therefore $a/2$. Moreover, there is one atom per unit cell. Thus

$$\text{Occupied volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (a/2)^3 = \pi a^3/6$$

$$\text{Total cell volume} = a^3$$

$$\text{Ratio} = \frac{\text{Occupied volume}}{\text{Total volume}} = \frac{\pi}{6}$$

(b) In the body centered cubic lattice the atom in the center and any one of the cube corner atoms are nearest neighbors. Thus 1/2 the nearest neighbor distance is $r = \sqrt{3}\,a/4$. Also, there are two atoms per unit cell.
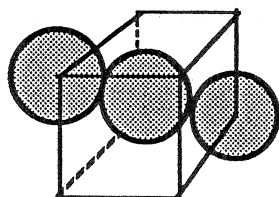
$$\text{diagonal} = 4r = \sqrt{3}\,a$$

$$\text{Occupied volume} = 2(\frac{4}{3}\pi r^3) = \frac{8}{3}\pi (\sqrt{3}\,a/4)^3 = \frac{\sqrt{3}}{8}\pi a^3$$

$$\text{Total cell volume} = a^3$$

$$\text{Ratio} = \frac{\text{Occupied volume}}{\text{Total volume}} = \frac{\sqrt{3}\pi}{8}$$

(c) For a face centered cubic lattice, the closest atoms lie in a cube face. Also, there are four atoms per unit cell in the fcc lattice.

face diagonal $= 4r = \sqrt{2}\,a$; $r = \sqrt{2}\,a/4$

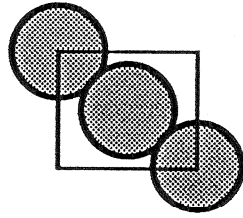Occupied volume $= 4(\frac{4}{3}\pi r^3) = \frac{16}{3}\pi\,(\sqrt{2}\,a/4)^3 = \frac{\sqrt{2}}{6}\pi\,a^3$

Total volume $= a^3$

Ratio $= \dfrac{\sqrt{2}\pi}{6}$

(d) As emphasized in Fig. 1.4(c), the atom in the upper front corner of the unit cell and the atom along the cube diagonal 1/4 of the way down the diagonal are nearest neighbors. Since the diagonal of the cube is equal to $\sqrt{3}$ times a cube side length, the center-to-center distance between nearest-neighbor atoms in the diamond lattice is $\sqrt{3}\,a/4$, and the atomic radius $r = \sqrt{3}\,a/8$. Moreover, there are eight atoms per unit cell in the diamond lattice. Thus

Occupied volume $= 8(\frac{4}{3}\pi r^3) = \frac{32}{3}\pi\,(\sqrt{3}\,a/8)^3 = \frac{\sqrt{3}}{16}\pi\,a^3$

Total volume $= a^3$

Ratio $= \dfrac{\sqrt{3}\pi}{16}$

2.1

(a)/(b) The MATLAB program script yielding both the part (a) and part (b) results is listed below. A combined plot comparing the part (a) result (solid-line) and part (b) result (dashed-line) is included before the program script. At $T = 300$ K the part (a) relationship yields $\boxed{E_G(300K) = 1.1245 \text{ eV}}$.
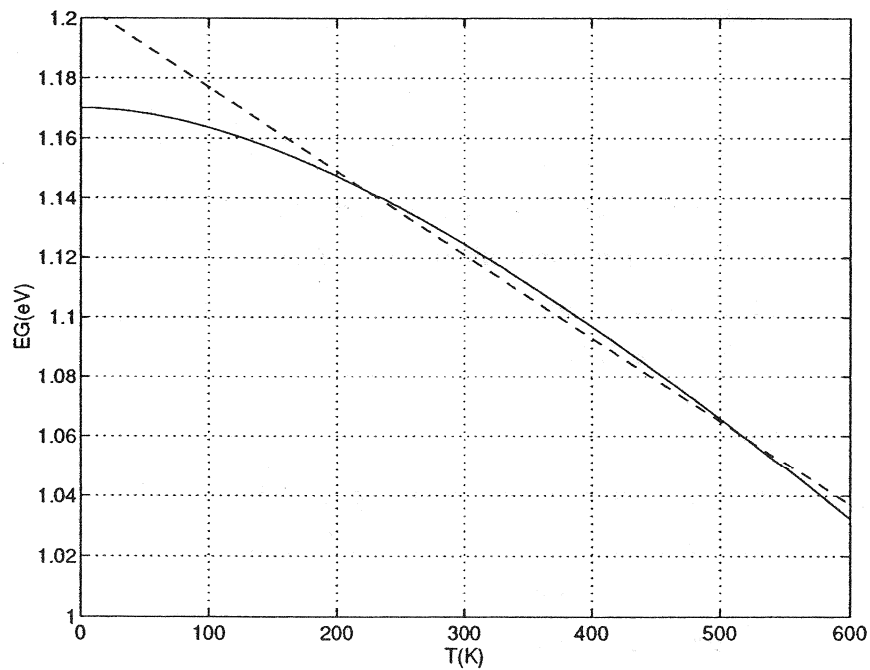
MATLAB program script...

```
%EG Computation (EG versus T)
close
clear

%Parabolic Fit Parameters
EG0=1.170;
a=4.730e-4;
b=636;

%Parabolic computation and plot
T=[0:5:600];
```

```
EG=EG0-a.*(T.^2)./(T+b);
EG300=EG0-a.*(300.^2)./(300+b);
plot(T,EG); axis([0 600 1.0 1.2]); grid;
xlabel('T(K)'); ylabel('EG(eV)');
hold on

%Linear computation and plot
EG0=1.205;
a=2.8e-4;
EG=EG0-a.*T;
plot(T,EG,'b--');
hold off

%T=300K result
EG300
```

2.2

(a) See Fig. 2.4(a).

(b) See Fig. 2.4(b) or the left-hand side of Fig. 2.7(b).

(c) See the left-hand side of Fig. 2.7(c).

(d) See Fig. 2.10(a).

(e) See Fig. 2.10(b).

2.3

(a) See the right-hand side of Fig. 2.7(b).

(b) See the right-hand side of Fig. 2.7(c).

(c) See Fig. 2.13(a).

(d) See Fig. 2.13(b).

(e) See the extreme left-hand side of either Fig. 2.13(a) or Fig. 2.22(b).

(f) See the extreme left-hand side of Fig. 2.13(b).

(g) See the extreme right-hand side of Fig. 2.16.

(h) See the extreme left-hand side of Fig. 2.18.

(i) See the middle of Fig. 2.18.

(j) See the extreme right-hand side of Fig. 2.18.

(k) See Fig. 2.19.

(l) See Fig. 2.19.

2.4

(a) The removal of the column III Ga atom with three valence electrons leaves five dangling bonds in the vicinity of the vacancy. The removal of the column V As atom with five valence electrons leaves three dangling bonds in the vicinity of the vacancy.

(b) When a Si atom with four valence electrons is inserted into the missing Ga site, there is one extra electron that does not fit snugly into the bonding pattern. Conversely, when a Si atom is inserted into the missing As site, there are one too few bonds to complete the bonding scheme.--There is a hole in the bonding scheme.
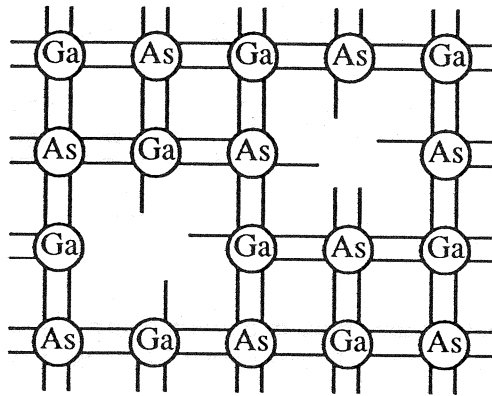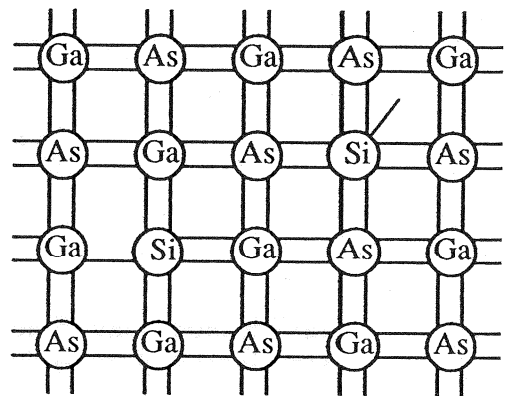
Answer-(a)   Answer-(b)

(c) $\boxed{n\text{-type}}$ ... The extra electron noted in part (b) is readily released yielding an increase in the electron concentration.

(d) $\boxed{p\text{-type}}$ ... The missing bond noted in part (b) is readily filled at room temperature yielding an increase in the hole concentration.

(e)

(i)

E_c
E_F
------ E_i
E_v
n-type

(ii)

E_c
------ E_i
E_F
E_v
p-type

2-3

2.5

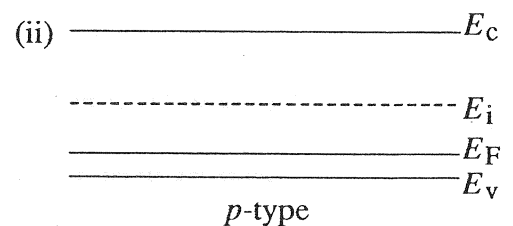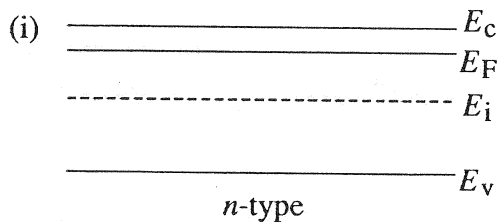As noted in Subsection 2.4.1, $g_c(E)dE$ represents the number of conduction band states/cm^3 lying in the energy range between E and $E + dE$. It follows that the number of states/cm^3 in the conduction band lying between energies E_c and $E_c + \gamma kT$ is simply obtained by integrating $g_c(E)dE$ over the noted range of energies.

$$\text{states/cm}^3 = \int_{E_c}^{E_c+\gamma kT} g_c(E)dE = \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3} \int_{E_c}^{E_c+\gamma kT} \sqrt{E - E_c}\, dE$$

$$= \frac{2}{3} \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3} (E - E_c)^{3/2} \Big|_{E_c}^{E_c+\gamma kT} = \boxed{\frac{2}{3}\left(\frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3}\right)(\gamma kT)^{3/2}}$$

2.6

(a) The probability of electrons occupying states at a given energy under equilibrium conditions is given by the Fermi function. Here we are told the energy of interest is $E = E_F$. Thus

$$f(E_F) = \frac{1}{1 + e^{(E_F - E_F)/kT}} = \frac{1}{2}$$

(b) The desired probability is again given by the Fermi function. Here we are told $E_F = E_c$ and the energy of interest is $E = E_c + kT$. Consequently,

$$f(E_c + kT) = \frac{1}{1 + e^{[(E_c+kT) - E_c]/kT}} = \frac{1}{1 + e^1} = 0.269$$

(c) The problem statement indicates $f(E_c+kT) = 1 - f(E_c+kT)$, or

$$\frac{1}{1 + e^{(E_c+kT-E_F)/kT}} = 1 - \frac{1}{1 + e^{(E_c+kT-E_F)/kT}} = \frac{e^{(E_c+kT-E_F)/kT}}{1 + e^{(E_c+kT-E_F)/kT}} = \frac{1}{1 + e^{(E_F-E_c-kT)/kT}}$$

Thus we must have

$$E_c + kT - E_F = E_F - (E_c + kT)$$

or

$$\boxed{E_F = E_c + kT}$$

<u>2.7</u>

The distribution of electrons in the conduction band is given by $g_c(E)f(E)$; the distribution of holes in the valence band is given by $g_v(E)[1 - f(E)]$. Working with the electron distribution we note,

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}} \cong e^{-(E - E_F)/kT}$$

...for all $E \geq E_c$ if the semi-conductor is nondegenerate

Thus

$$g_c(E)f(E) = \frac{m_n^* \sqrt{2m_n^*(E-E_c)}}{\pi^2 \hbar^3} e^{-(E-E_F)/kT}$$

$$= \kappa (E-E_c)^{1/2} e^{-(E-E_F)/kT} \qquad ...\kappa \equiv \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3}$$

The extrema points of any function are obtained by taking the derivative of the function and setting the derivative equal to zero.

$$\frac{d}{dE}[g_c(E)f(E)] = \frac{\kappa}{2(E-E_c)^{1/2}} e^{-(E-E_F)/kT} - \frac{\kappa}{kT}(E-E_c)^{1/2} e^{-(E-E_F)/kT}$$

$$\text{set} = 0$$

Clearly

$$\frac{1}{2\sqrt{E_{peak}-E_c}} = \frac{\sqrt{E_{peak}-E_c}}{kT}$$

or

$$E_{peak} - E_c = kT/2$$

and

$$\boxed{E_{peak} = E_c + kT/2}$$

...for electrons in the conduction band

The development leading to the peak energy of $E_{peak} = E_v - kT/2$ for holes in the valence band is completely analogous.

2-5

2.8

The electron population at any energy is given by $g_c(E)f(E)$. Also, since the semiconductor is nondegenerate

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \cong e^{-(E-E_F)kT} \quad \text{...for all } E > E_c$$

The electron population at $E = E_c + 5kT$ normalized to the peak electron population at $E = E_c + kT/2$ is therefore

$$\text{ratio} = \frac{g_c(E_c+5kT)f(E_c+5kT)}{g_c(E_c+kT/2)f(E_c+kT/2)}$$

$$\cong \frac{\sqrt{5kT}}{\sqrt{kT/2}} \frac{e^{-(E_c+5kT-E_F)kT}}{e^{-(E_c+kT/2-E_F)kT}} = \sqrt{10}\, e^{-4.5} = 3.51 \times 10^{-2}$$

2.9

The hole and electron distributions are given respectively by

$$(\text{electron dist.}) = g_c(E)f(E) \cong \frac{m_n^*\sqrt{2m_n^*}}{\pi^2\hbar^3}\sqrt{E-E_c}\, e^{-(E-E_F)/kT}$$

$$= \left(\frac{m_n^*\sqrt{2m_n^*}}{\pi^2\hbar^3} e^{-E_G/4kT}\right)\sqrt{E-E_c}\, e^{-(E-E_c)/kT}$$

and

$$(\text{hole dist.}) = g_v(E)[1-f(E)] \cong \frac{m_p^*\sqrt{2m_p^*}}{\pi^2\hbar^3}\sqrt{E_v-E}\, e^{(E-E_F)/kT}$$

$$= \left(\frac{m_p^*\sqrt{2m_p^*}}{\pi^2\hbar^3} e^{-3E_G/4kT}\right)\sqrt{E_v-E}\, e^{-(E_v-E)/kT}$$

Note that the approximate (non-degenerate) expressions for the Fermi function established in Subsection 2.4.2 were employed in writing down the carrier distributions.

The required MATLAB program script and resultant plots are presented below. Computations were performed employing $E_G = 1.12$ eV, $kT = 0.0259$ eV, $m_n{*}/m_0 = 1.18$ and $m_p{*}/m_0 = 0.81$ from Table 2.1, and $h = 6.63 \times 10^{-34}$ joule-sec and $m_0 = 9.11 \times 10^{-31}$ kg from the table of physical constants (inside back cover).

The plots are clearly consistent with Fig. 2.16 in the text. (Note that the electron distribution scale is multiplied by 10^{15} while the hole distribution scale is multiplied by 10^6.) The distributions peak at $kT/2$ from the band edges graphically reconfirming the peak positions noted in Problem 2.7.

MATLAB program script...

```
%Problem 2.9...Carrier Distributions

%Intialization
close
clear

%Constants
EG=1.12;
kT=0.0259;
m0=9.11e-31;
mnr=1.18;
mpr=0.81;
hbar=6.63e-34/(2*pi);
c1=1.6e-19;    %joules = c1*eV
c2=1.0e-6      %m^3=c2*cm^3

%Computation
deltaE=linspace(0,5*kT);            %deltaE = E-Ec or Ev-E in eV
A=m0*sqrt(2*m0)/(pi^2*hbar^3);
An=mnr^(3/2)*A;   Ap=mpr^(3/2)*A;
e_dist=c1*c2*An*exp(-EG/(4*kT)).*sqrt(c1*deltaE).*exp(-deltaE/kT);
h_dist=c1*c2*Ap*exp(-3*EG/(4*kT)).*sqrt(c1*deltaE).*exp(-deltaE/kT);
%Note use of c1 and c2 to make distribution units number/cm^3-eV

%Plots
subplot(2,1,1), plot(e_dist,deltaE/kT);  grid
xlabel('electron distribution (number/cm^3-eV)');
ylabel('(E-Ec)/kT')
subplot(2,1,2), plot(h_dist,-deltaE/kT);  grid
axis([0,1.4e7,-5,0])
xlabel('hole distribution (number/cm^3-eV)');
ylabel('(E-Ev)/kT')
```
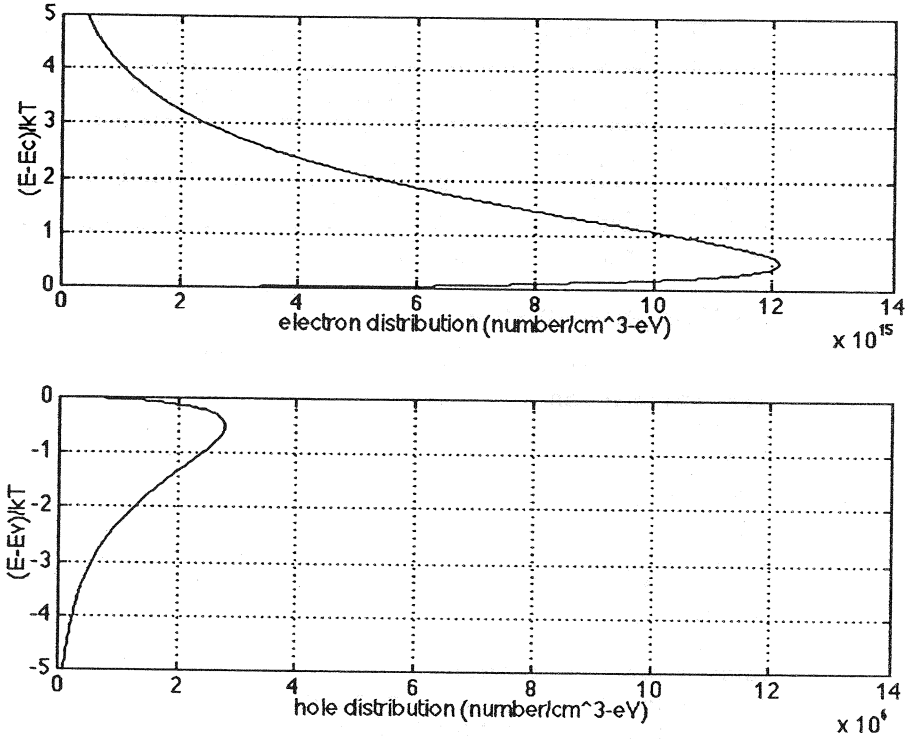
2.10

(a) Utilizing Eq. (2.6a), the approximate (nondegenerate) expression for the Fermi function established in Subsection 2.4.2, Eq. (2.13a), and Eq. (2.16a), one obtains

$$\text{(normalized dist.)} = \frac{g_c(E)f(E)}{n} \cong \frac{\left(m_n^* \sqrt{2m_n^*} / \pi^2 \hbar^3\right) \sqrt{E - E_c} \; e^{-(E - E_F)/kT}}{2\left(m_n^* kT / 2\pi\hbar^2\right)^{3/2} e^{(E_F - E_c)/kT}}$$

$$= \frac{2\sqrt{E - E_c}}{\sqrt{\pi}\,(kT)^{3/2}} \; e^{-(E - E_c)/kT}$$

(b) A plot of the normalized electron distribution verus energy for three different temperatures and the MATLAB program script yielding the plot are given below. From the plot one observes that the peak energy, which occurs at $kT/2$, moves to progressively higher energies with increasing T. More significantly, the distribution becomes less peaked in nature and the height of the peak decreases with increasing temperature.

2-8

MATLAB program script...

```
%Problem 2.10...Normalized Electron Distribution as a function of T
%Intialization
close
clear

%Computation and plot
k=8.617e-5;
T=[300 600 1200];
kT=k.*T;
E_Ec=linspace(0,0.4);
   for i=1:3,
   dist=2*sqrt(E_Ec)/(sqrt(pi)*kT(i)^(3/2)).*exp(-E_Ec/kT(i));
   y(i,:)=dist;
   end
plot(E_Ec,y);   grid
axis([0,0.4,0,20])
xlabel('E-Ec (eV)');   ylabel('normalized distribution (1/eV)');
text(.005,12,'300K','Color','yellow');
text(.01,7,'600K','Color','magenta');
text(.015,3,'1200K','Color','cyan')
```
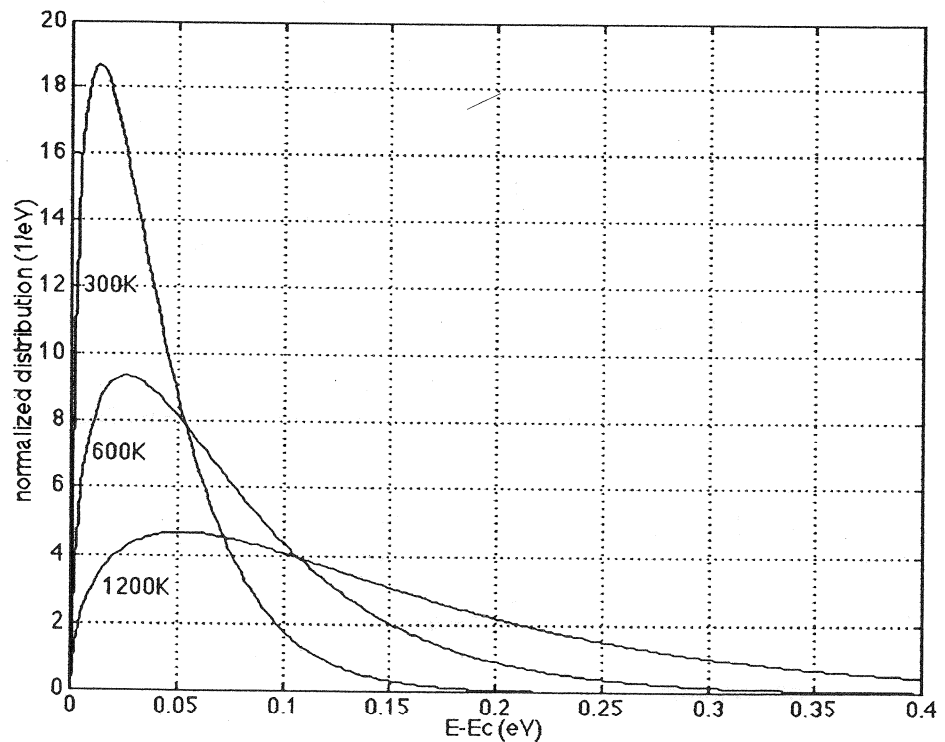
2.11

Substituting the Eq.(2.6b) expression for $g_v(E)$ and the Eq.(2.7) expression for $f(E)$ into Eq.(2.8b), one obtains

$$1 - \frac{1}{1 + e^{(E - E_F)/kT}} = \frac{1}{1 + e^{(E_F - E)/kT}}$$

and

$$p = \frac{m_p^* \sqrt{2m_p^*}}{\pi^2 \hbar^3} \int_{E_{bottom}}^{E_v} \frac{\sqrt{E_v - E}\ dE}{1 + e^{(E_F - E)/kT}} \tag{2.9}'$$

Now letting

$$\eta = \frac{E_v - E}{kT} \tag{2.10a}'$$

$$\eta_v = \frac{E_v - E_F}{kT} \tag{2.10b}'$$

$$E_{bottom} \rightarrow -\infty \tag{2.10c}'$$

yields

$$p = \frac{m_p^* \sqrt{2m_p^*}\ (kT)^{3/2}}{\pi^2 \hbar^3} \int_0^\infty \frac{\eta^{1/2} d\eta}{1 + e^{\eta - \eta_v}} \tag{2.11}'$$

Recognizing

$$F_{1/2}(\eta_v) \equiv \int_0^\infty \frac{\eta^{1/2} d\eta}{1 + e^{\eta - \eta_v}} \qquad \text{...Fermi-Dirac integral of order 1/2} \tag{2.12}'$$

and defining

$$N_v = 2 \left[\frac{m_p^* kT}{2\pi \hbar^2} \right]^{3/2} \tag{2.13b}$$

one obtains

$$p = N_V \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_v)$$

<div align="right">(2.14b)</div>

If the semiconductor is nondegenerate, such that $E_F \geq E_v + 3kT$, then $\eta_v \leq -3$. Since $\eta \geq 0$ in the Fermi-Dirac integral, $\exp(\eta - \eta_v) \geq \exp(3)$ for all η. Thus one obtains

$$F_{1/2}(\eta_v) \cong \int_0^\infty \eta^{1/2} e^{-(\eta - \eta_v)} d\eta = \frac{\sqrt{\pi}}{2} e^{(E_v - E_F)/kT}$$
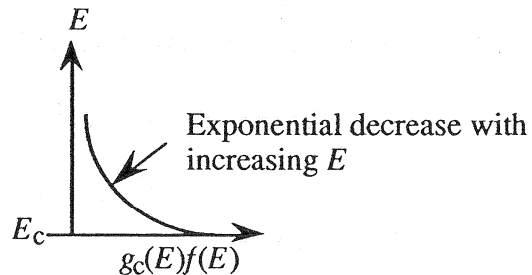
Substituting this approximate relationship into Eq.(2.14b) finally yields

$$p = N_V e^{(E_v - E_F)/kT}$$

<div align="right">(2.16b)</div>

2.12

(a)   electron distribution $= g_c(E)f(E) \cong (N_C/kT) e^{-(E-E_F)/kT}$

where use has been made of the fact that the semiconductor is nondegenerate ($E_F < E_c - 3kT$). Thus

(b) Following the procedure outlined in Subsection 2.5.1 of the text, if

$$g_c(E) = N_C/kT \qquad \dots E \geq E_c$$

then

$$n = \int_{E_c}^{E_{top}} g_c(E)f(E)dE = \frac{N_C}{kT}\int_{E_c}^{E_{top}} \frac{dE}{1 + e^{(E-E_F)/kT}}$$

Let

$$\eta = \frac{E - E_c}{kT} \; ; \qquad \eta_c = \frac{E_F - E_c}{kT} \; ; \qquad E_{top} \to \infty$$

This yields

$$n = N_C \int_0^\infty \frac{d\eta}{1 + e^{\eta-\eta_c}}$$

The integral here can be performed in closed-form.

$$n = N_C\left[\eta - \ln(1+e^{\eta-\eta_c})\right]_0^\infty = N_C\left[\eta_c + \ln(1+e^{-\eta_c})\right]$$

The relationship analogous to Eqs.(2.14a) is therefore

$$\boxed{n = N_C\left[\eta_c + \ln(1+e^{-\eta_c})\right]}$$

If the semiconductor is nondegenerate, $\eta_c \leq -3$. Thus

$$\ln(1+e^{-\eta_c}) = \ln\left[e^{-\eta_c}(1+e^{\eta_c})\right] = -\eta_c + \ln(1+e^{\eta_c})$$

$$\cong -\eta_c + e^{\eta_c} \quad \text{...since } \exp(\eta_c) \ll 1 \text{ and } \ln(1+x) \cong x \text{ if } x \ll 1.$$

We therefore conclude

$$\boxed{n = N_C\, e^{\eta_c} = N_C\, e^{(E_F - E_c)/kT}}$$

The above holds for a nondegenerate semiconductor and is the desired relationship analogous to Eq.(2.16a). Actually, the relationship has turned out to be identical to Eq.(2.16a).

2.13

(a) Rewriting Eqs.(2.13), one obtains

$$N_C = N_0 \left(\frac{m_n^*}{m_0}\right)^{3/2}$$

$$N_V = N_0 \left(\frac{m_p^*}{m_0}\right)^{3/2}$$

where

$$N_0 \equiv 2\left[\frac{m_0 kT}{2\pi\hbar^2}\right]^{3/2}$$

Using the numbers cited in the problem statement, the k-value given on the inside back cover, and remembering to convert from eV to joules, one calculates

$$N_0 = 2\left[\frac{2\pi (9.109 \times 10^{-31})(8.617 \times 10^{-5})(300)(1.602 \times 10^{-19})}{(6.625 \times 10^{-34})^2}\right]^{3/2}$$

$$= 2.510 \times 10^{25}/m^3 = 2.510 \times 10^{19}/cm^3$$

and therefore

$$N_{C,V} = (2.510 \times 10^{19}/cm^3)\ (m^*/m_0)^{3/2}$$

(b)

Semiconductor	N_C (cm^{-3})	N_V (cm^{-3})
Si	3.22×10^{19}	1.83×10^{19}
Ge	1.02×10^{19}	5.42×10^{18}
GaAs	4.26×10^{17}	9.41×10^{18}

2.14

(a) Referring to Fig. 2.20, one concludes:

   (i) $n_i(Si) = n_i(Ge, 300K)$ at $T \cong 430K$.

   (ii) $n_i(GaAs) = n_i(Ge, 300K)$ at $T \cong 600K$.

(b) With the differences in the effective masses neglected,

$$\frac{n_{iA}}{n_{iB}} = \frac{e^{-E_{GA}/2kT}}{e^{-E_{GB}/2kT}} = e^{(E_{GB}-E_{GA})/2kT} = e^{1/0.0518} = \mathbf{2.42 \times 10^8}$$

2.15

The MATLAB program script implementing the requested $n_i(Ge)$ vs. T computation is reproduced below along with sample numerical results. As must be the case since the same computational equation was used in both cases, the numerical results are found to be consistent with the values displayed in Fig. 2.20.

MATLAB program script...

```
%Problem 2.15...ni versus T for Ge
%Initialization
close;  clear

%Computation
k=8.617e-5;
T=[225:25:475];
ni=(1.76e16).*(T.^1.5).*exp(-0.392./(k.*T));

%Display result on screen
j=length(T);
fprintf('\n\nT(K)         ni(Ge)\n');
for ii=1:j,
fprintf('%-10.f%-10.3e\n',T(ii),ni(ii));
end
```

T(K)	ni(Ge)
225	9.841e+10
250	8.705e+11
275	5.251e+12
300	2.375e+13
325	8.597e+13
350	2.611e+14
375	6.888e+14
400	1.620e+15
425	3.463e+15
450	6.838e+15
475	1.263e+16

<u>2.16</u>

(a) As $T \to 0$, $n \to 0$ and $p \to 0$. (See the discussion in Subsection 2.5.7.)

(b) Since $N \gg n_i$, one would have

$$n = N_D \quad \text{and} \quad p = n_i^2/N_D \quad \text{...if a donor}$$

$$p = N_A \quad \text{and} \quad n = n_i^2/N_A \quad \text{...if an acceptor}$$

We are told $n = N$ and $p = n_i^2/N$. Clearly the impurity is a **donor**.

(c) Here we are given the minority carrier concentration, $n = 10^5/cm^3$. As long as the Si is nondegenerate, one can always write

$$np = n_i^2$$

Thus

$$p = n_i^2/n = \frac{(10^{10})^2}{10^5} = 10^{15}/cm^3$$

Note: From previous problems we recognize that the above carrier concentrations do indeed correspond to a nondegenerate semiconductor.

(d) Given $E_F - E_i = 0.259eV$ and $T = 300K$,

$$n = n_i \, e^{(E_F - E_i)/kT} = (10^{10}) \, e^{0.259/0.0259} = 2.20 \times 10^{14}/cm^3$$

$$p = n_i \, e^{(E_i - E_F)/kT} = (10^{10}) \, e^{-0.259/0.0259} = 4.54 \times 10^5/cm^3$$

(e) Employing the np product relationship,

$$np = n^2/2 = n_i^2$$
$$n = \sqrt{2}\,n_i = 1.414 \times 10^{13}/cm^3$$

Next employing the charge neutrality relationship,

$$p - n + N_D - N_A = n/2 - n + N_D = 0$$
$$N_D = n/2 = n_i/\sqrt{2} = 0.707 \times 10^{13}/cm^3$$

2.17

(a) At room temperature in Si, $n_i = 10^{10}/cm^3$. Thus here $N_D \gg N_A$, $N_D \gg n_i$ and

$$n = N_D = 10^{15}/cm^3$$

$$p = n_i^2/N_D = 10^5/cm^3$$

(b) Since $N_A \gg N_D$ and $N_A \gg n_i$,

$$p = N_A = 10^{16}/cm^3$$

$$n = n_i^2/N_A = 10^4/cm^3$$

(c) Here we must retain both N_A and N_D, but $N_D - N_A \gg n_i$.

$$n = N_D - N_A = 10^{15}/cm^3$$

$$p = n_i^2/(N_D - N_A) = 10^5/cm^3$$

(d) We deduce from Fig. 2.20 that, at 450K, $n_i(Si) \approx 5 \times 10^{13}/cm^3$. Clearly, n_i is comparable to N_D and we must use Eq.(2.29a).

$$n = \frac{N_D}{2} + \left[\left(\frac{N_D}{2} \right)^2 + n_i^2 \right]^{1/2} = 1.21 \times 10^{14}/cm^3$$

$$p = \frac{n_i^2}{n} = \frac{(5 \times 10^{13})^2}{1.21 \times 10^{14}} = 2.07 \times 10^{13}/cm^3$$

(e) We conclude from Fig. 2.20 that, at 650K, $n_i \approx 10^{16}/cm^3$. Here $n_i \gg N_D$. Thus

$$n = n_i \approx 10^{16}/cm^3$$

$$p = n_i \approx 10^{16}/cm^3$$

2.18

(i) As established in the text [Eq.(2.36)],

$$E_i = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln(m_p^*/m_n^*)$$

Taking m_p^*/m_n^* to be temperature independent and employing the values listed in Table 2.1, one concludes

part	T(K)	kT (eV)	E_i displacement from midgap (eV)
(a-c)	300	0.0259	−0.0073
(d)	450	0.0388	−0.0109
(e)	650	0.0560	−0.0158

Alternatively, the m_p^*/m_0 and m_n^*/m_0 versus T fit-relationships cited in Exercise 2.4 may be used to compute the m_p^*/m_n^* ratio. One finds

part	T(K)	m_p^*/m_n^*	kT (eV)	E_i displacement from midgap (eV)
(a-c)	300	0.680	0.0259	−0.0075
(d)	450	0.703	0.0388	−0.0103
(e)	650	0.719	0.0560	−0.0139

(ii) $E_F - E_i$ is computed using the appropriate version of Eq.(2.37) or (2.38).

(a) $E_F - E_i = kT \ln(N_D/n_i) = 0.0259 \ln(10^{15}/10^{10}) = 0.298$ eV

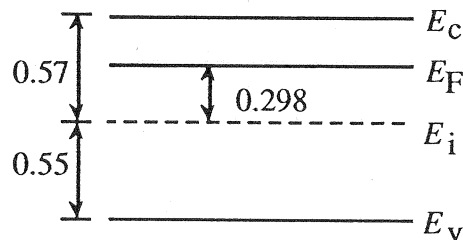(b) $E_i - E_F = kT \ln(N_A/n_i) = 0.0259 \ln(10^{16}/10^{10}) = 0.358$ eV

(c) $E_F - E_i = kT \ln[(N_D-N_A)/n_i] = 0.0259 \ln(10^{15}/10^{10}) = 0.298$ eV
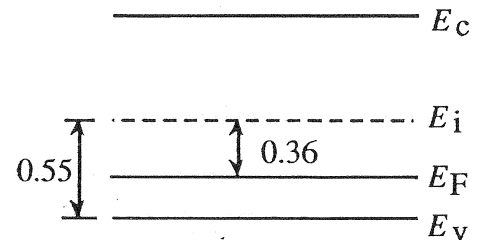
(d) $E_F - E_i = kT \ln(n/n_i) = 0.0388 \ln(1.21 \times 10^{14}/5 \times 10^{13}) = 0.034$ eV

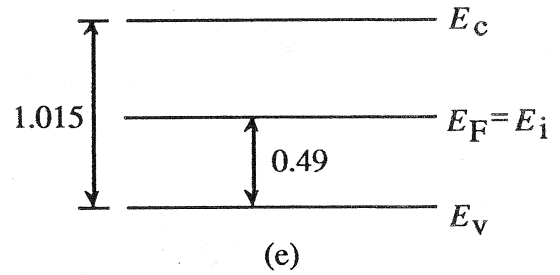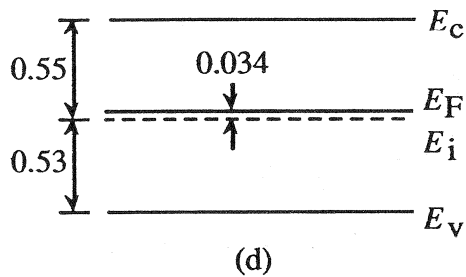(e) $E_F - E_i = kT \ln(n/n_i) \cong 0$ ...$(n \cong n_i)$

(iii)

(a) and (c)   (b)

(d)

(e)

2.19

(a) A sample MATLAB program that computes n, p, and E_F-E_i given T, N_D, and N_A is listed below. The program incorporates the $n_i(T)$ computation given in the solution to Exercise 2.4a.

MATLAB program script...

```
%Calculation of n,p and EF-Ei (nondegenerate, fully ionized)

%Initialization
clear;  close

%Specification of basic parameters
T=input('Please input the temperature, T, in Kelvin...T=');
NA=input('Please input NA(cm-3)...NA=');
ND=input('Please input ND(cm-3)...ND=');
k=8.617e-5;
Nnet=ND-NA;

%ni computation (from Exercise 2.4a solution)
  %Constants and T-range
  A=2.510e19;
  Eex=0.0074;    %Value was adjust to match S&G ni(300K) value
  %
  %Band Gap vs. T
  EG0=1.17;
  a=4.730e-4;
  b=636;
  EG=EG0-a.*(T.^2)./(T+b);
  %
  %Effective mass ratio (mnr=mn*/m0, mpr=mp*/m0)
  mnr=1.028 + (6.11e-4).*T - (3.09e-7).*T.^2;
  mpr=0.610 + (7.83e-4).*T - (4.46e-7).*T.^2;
  %
  %Actual ni calculation
  ni=A.*((T./300).^(1.5)).*((mnr.*mpr).^(0.75)).*exp(-(EG-Eex)./(2
.*k.*T));
```

2-18

```
%Computation of n, p, and EF-Ei
if Nnet==0,
    n=ni;
    p=ni;
    EFi=0;
elseif Nnet>0,
    n=Nnet/2+sqrt((Nnet/2)^2+ni^2);
    p=ni^2/n;
    EFi=k*T*log(n/ni);
else
    p=-Nnet/2+sqrt((Nnet/2)^2+ni^2);
    n=ni^2/p;
    EFi=-k*T*log(p/ni);
end

%Printout of results
format compact;
n
p
EFi
```

(b) Results obtained employing the part (a) program are tabulated below. The Problem 2.17/2.18 part (d) and (e) results are slightly different because of inaccuracies in reading the elevated temperature values of n_i from Fig. 2.20.

Part	T (K)	N_A(cm^{-3})	N_D(cm^{-3})	n_i (cm^{-3})	n (cm^{-3})	p (cm^{-3})	$E_F - E_i$ (eV)
(a)	300	0	10^{15}	1.00×10^{10}	1.00×10^{15}	1.00×10^{5}	0.298
(b)	300	10^{16}	0	1.00×10^{10}	1.00×10^{4}	1.00×10^{16}	-0.357
(c)	300	9×10^{15}	10^{16}	1.00×10^{10}	1.00×10^{15}	1.00×10^{5}	0.298
(d)	450	0	10^{14}	4.71×10^{13}	1.19×10^{14}	1.87×10^{13}	0.0359
(e)	650	0	10^{14}	1.146×10^{16}	1.151×10^{16}	1.141×10^{16}	2.44×10^{-4}

2.20

There is more than one way to work this problem, with alternative approaches likely to yield slightly different answers. The most straightforward approach is recorded here.

At the onset of degeneracy

$$E_F = E_c - 3kT \qquad ...n\text{-type semiconductor}$$

$$E_F = E_v + 3kT \qquad ...p\text{-type semiconductor}$$

and the maximum nondegenerate carrier concentrations are therefore

$$n = N_C \, e^{(E_F - E_c)/kT} = N_C \, e^{-3}$$

$$p = N_V \, e^{(E_v - E_F)/kT} = N_V \, e^{-3}$$

However

$$n \cong N_D \qquad \text{...}n\text{-type Si at room T}$$

$$p \cong N_A \qquad \text{...}p\text{-type Si at room T}$$

Thus

$$N_{D|max} = N_C \, e^{-3} = (3.22 \times 10^{19}) \, e^{-3} = 1.60 \times 10^{18}/cm^3$$

$$N_{A|max} = N_V \, e^{-3} = (1.83 \times 10^{19}) \, e^{-3} = 9.11 \times 10^{17}/cm^3$$

N_C and N_V were computed using the expression $N_{C,V} = (2.51 \times 10^{19}/cm^3)(m_{n,p}^*/m_0)^{3/2}$ and the effective mass values in Table 2.1. The cited computational relationship is given in the text below Eq.(2.14b). Also see Problem 2.13.

2.21

The MATLAB program listed below computes $E_F - E_i$ vs. N_A or N_D up to the nondegenerate limit (N_A or $N_D \cong 10^{18}/cm^3$) and yields results very similar to Fig. 2.21.

MATLAB program script...

```
%EF-Ei versus NA or ND (nondegenerate, fully ionized, 300K)
clear;  close

%Specification of basic parameters
kT=0.0259;
ni=1.0e10;
NB=logspace(13,18);    %NB=ND or NA

%Computation of EF-Ei versus doping
EFiD=kT.*log(NB./ni);
EFiA=-EFiD;

%Plot out Fermi level positioning
semilogx(NB, EFiD, NB, EFiA);
axis([1.0e13,1.0e18,-0.56 0.56]);
grid;  xlabel('ND  or  NA');  ylabel('EF-Ei');
text(1.0e14,0.30,'Donor');  text(1.0e14,-0.30,'Acceptor');
```