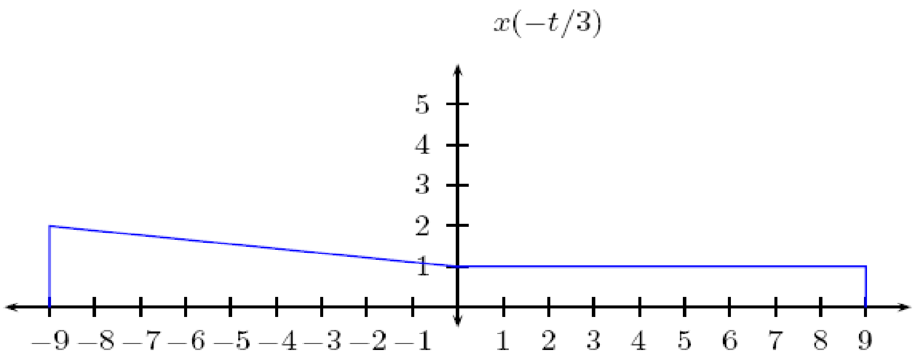
Chapter 2 solutions

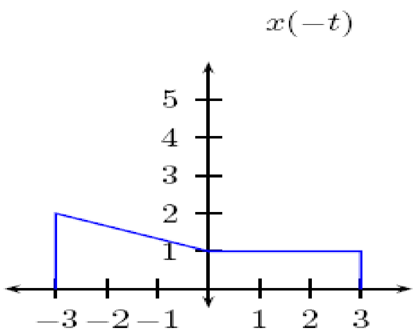
2.1

(a)

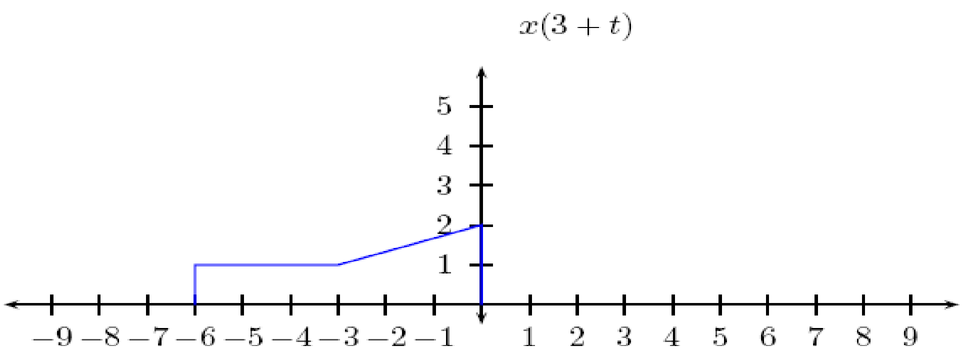
(i)



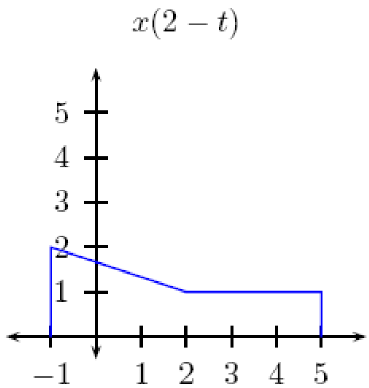
(ii)



(iii)

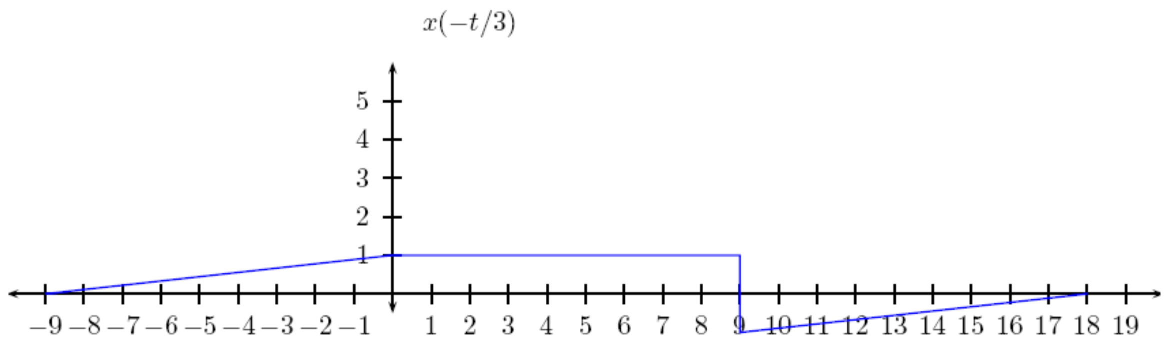


(iv)

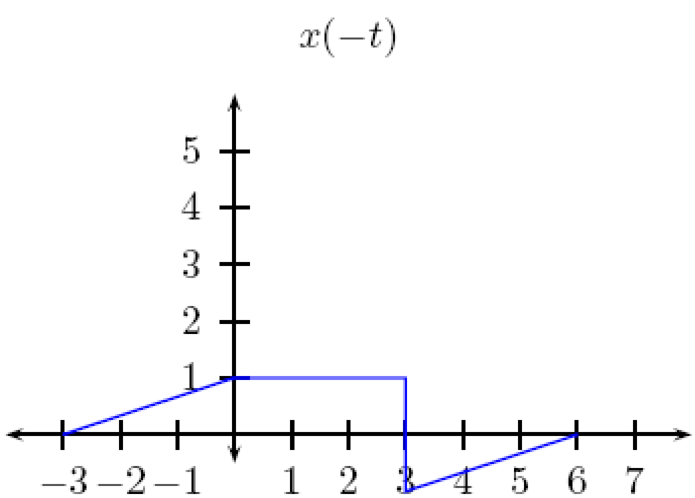


(b)

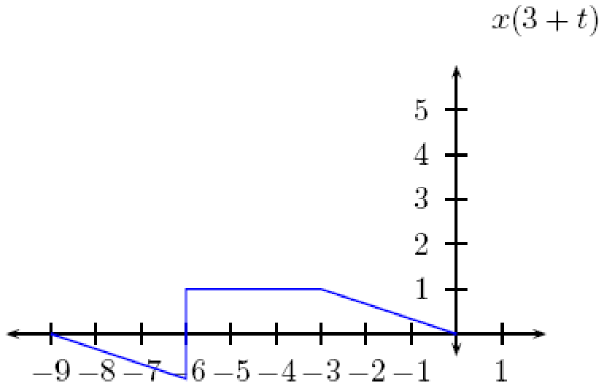
(i)



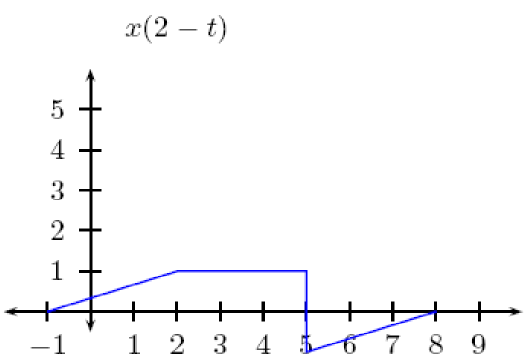
(ii)



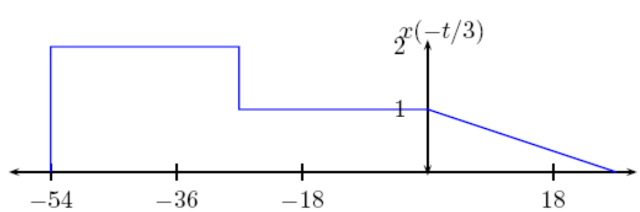
(iii)

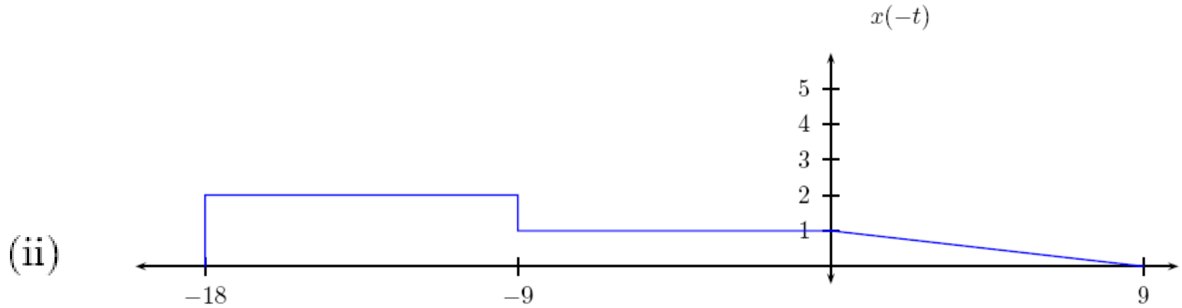


(iv)



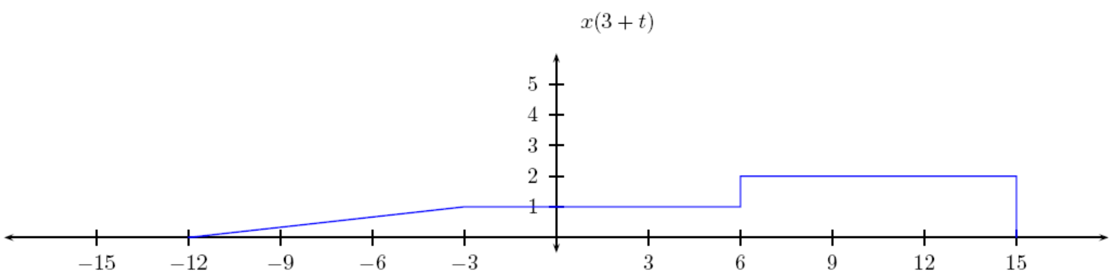
(c) (i)





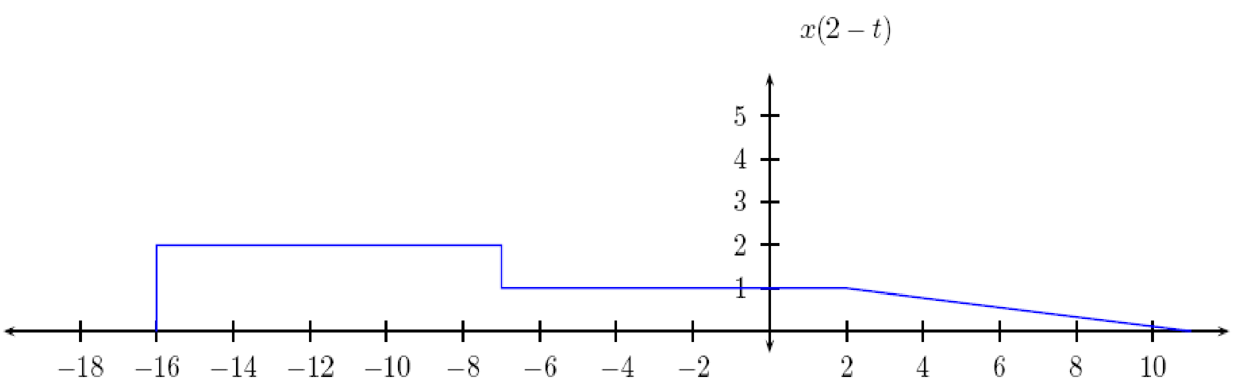
x(−t)
5
4
3
2
1
−18
−9
9

(iii)



x(3 + t)
5
4
3
2
1
−15
−12
−9
−6
−3
3
6
9
12
15

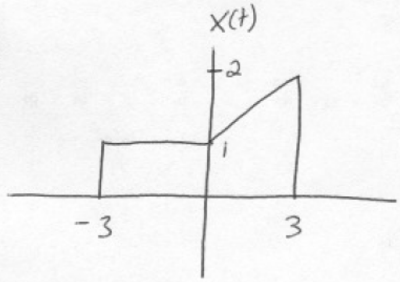
(iv)



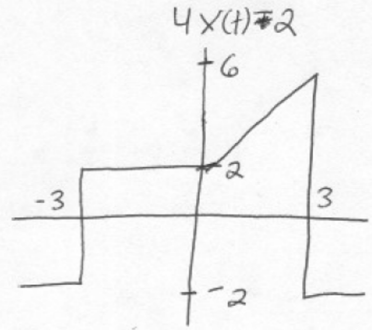
x(2 − t)
5
4
3
2
1
−18
−16
−14
−12
−10
−8
−6
−4
−2
2
4
6
8
10

2.2

(a)



i)



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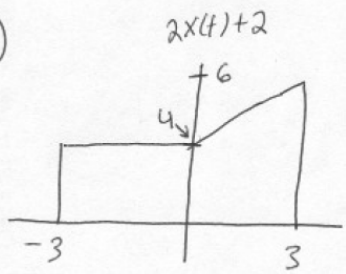

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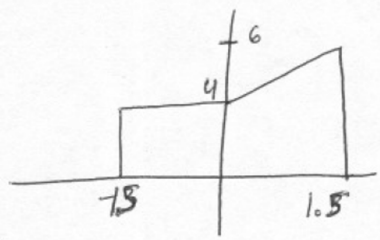

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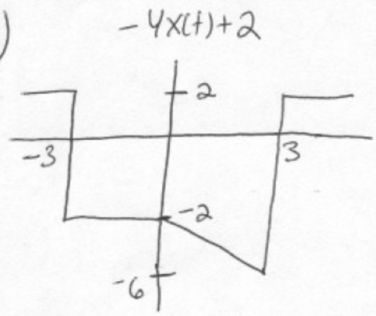
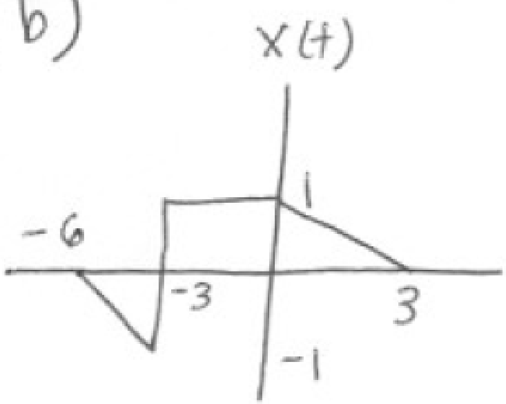
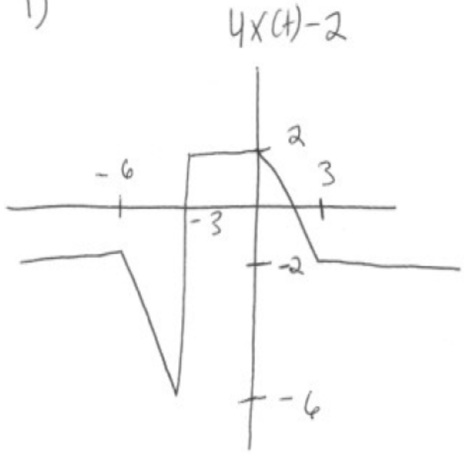


2.2

(b)



i)



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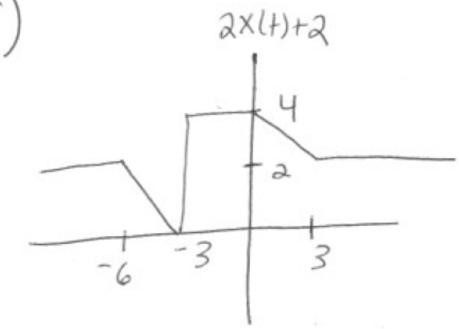

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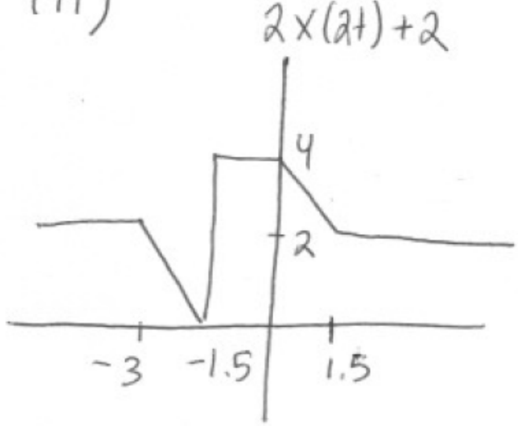

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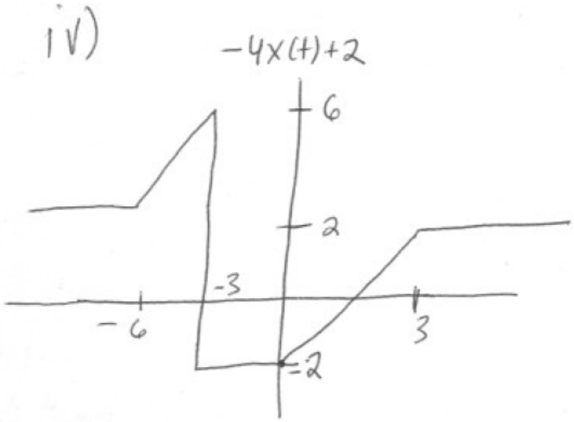
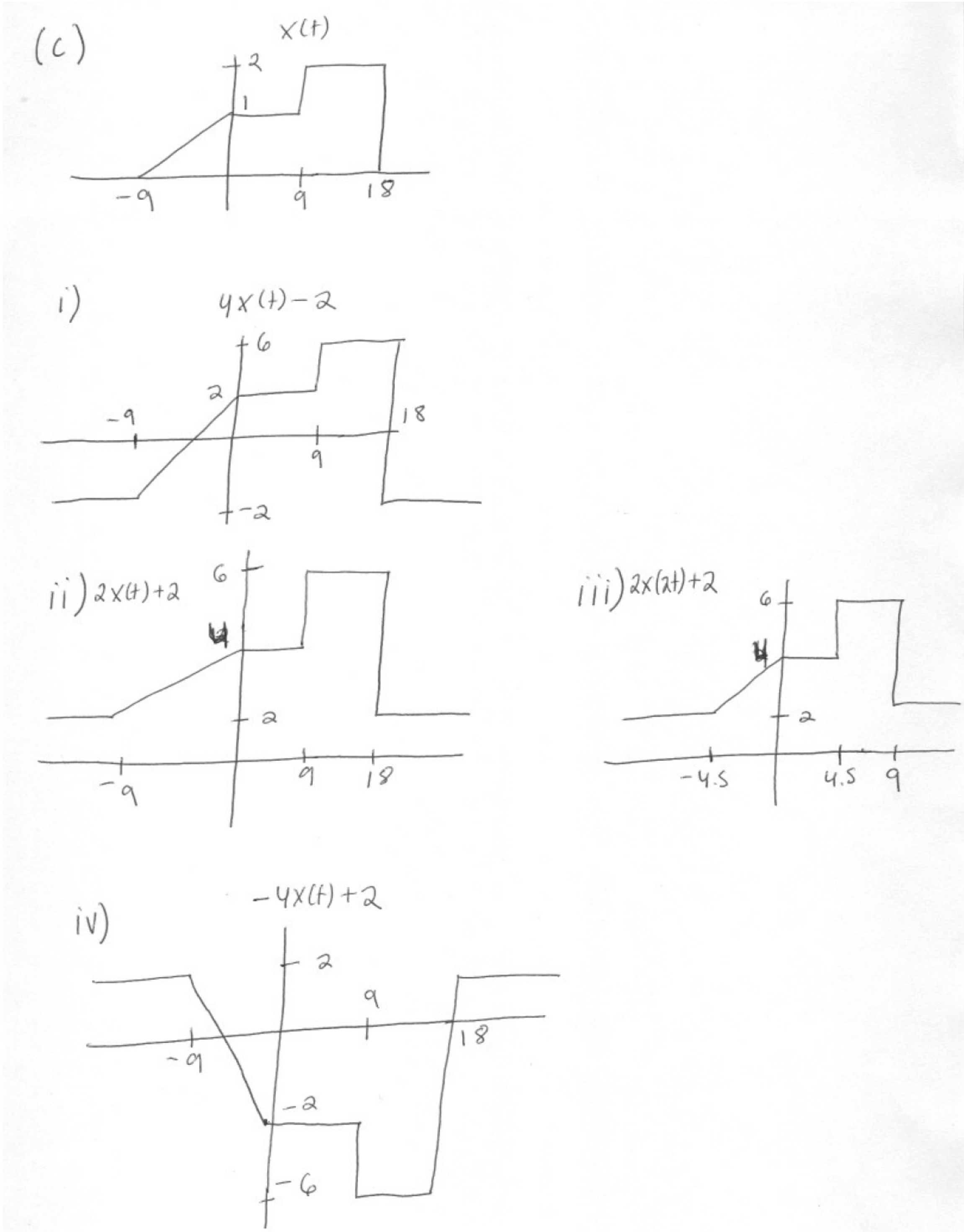
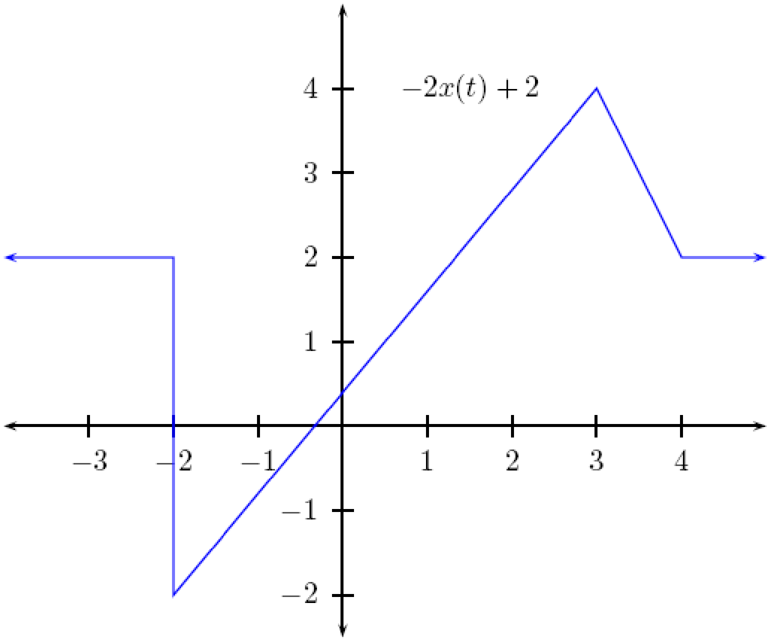
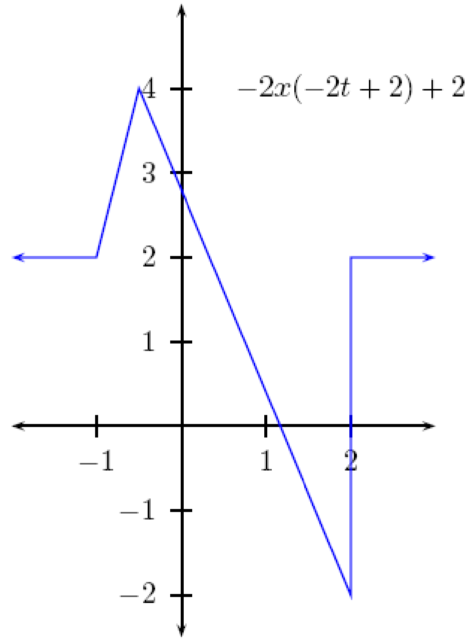


2.2


(c)
x(t)
2
1
-9
9
18
i)
4x(t)-2
6
2
-9
18
9
-2
ii) 2x(t)+2
6
4
2
-9
9
18
iii) 2x(2t)+2
6
4
2
-4.5
4.5
9
iv)
-4x(t)+2
2
9
-9
18
-2
-6

2.3





(a)

$y(t) = -2(x(-2t+2)) + 2$

(b)

t	$y(t)$	$-2t+2$	$-2(x(-2t-1))+2$
-0.5	4	3	4
-1	2	4	2
1	0.4	0	0.4

2.4

(a) $y(t) = -0.5(x(2t-4)) + 1.5$

(b)

t	y(t)	2t-4	-0.5(x(2t-4))+1.5
2	1.5	0	1.5
3	-1	2	-1
4.5	1.5	5	1.5

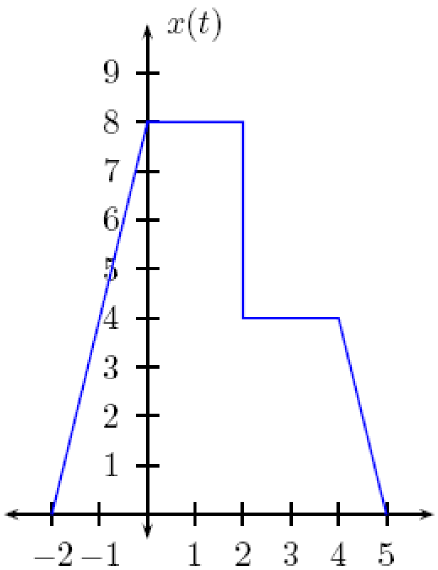
(c) $x(t) = -2y(\frac{t+4}{2}) + 3$

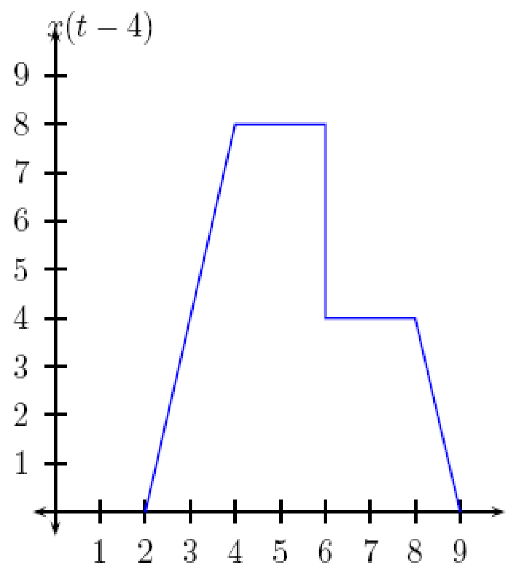
(d)

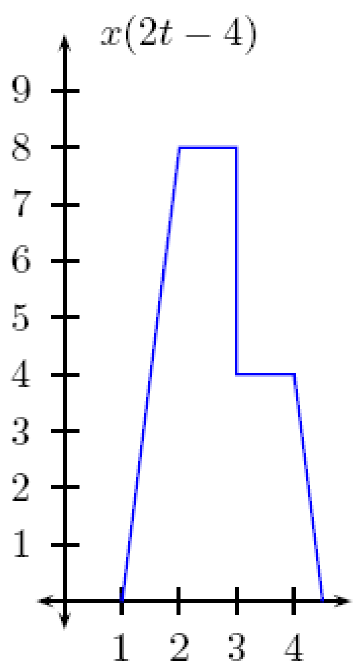
t	x(t)	$\frac{t+4}{2}$	$-2y(\frac{t+4}{2})+3$
0	0	2	0
4	-3	4	-3
5	0	4.5	0

2.5

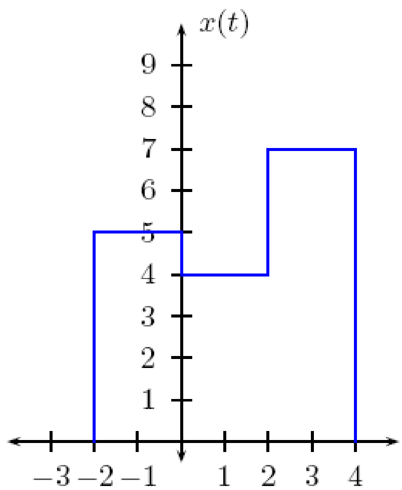
$$x(2t-4) = 4[(2t-2)u(2t-2) - (2t-4)u(2t-4) - u(2t-6) - (2t-8)u(2t-8) - (2t-9)u(2t-9)]$$
$$= 4[(2t-2)u(t-1) - (2t-4)u(t-2) - u(t-3) - (2t-8)u(t-4) - (2t-9)u(t-4.5)]$$

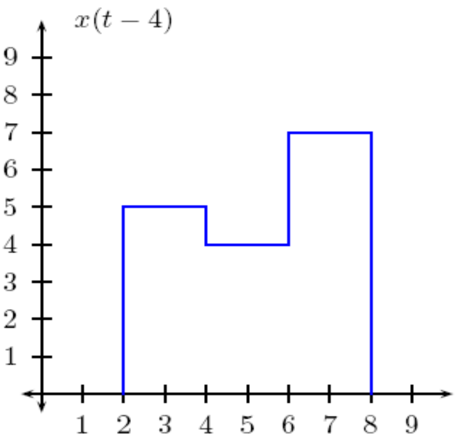


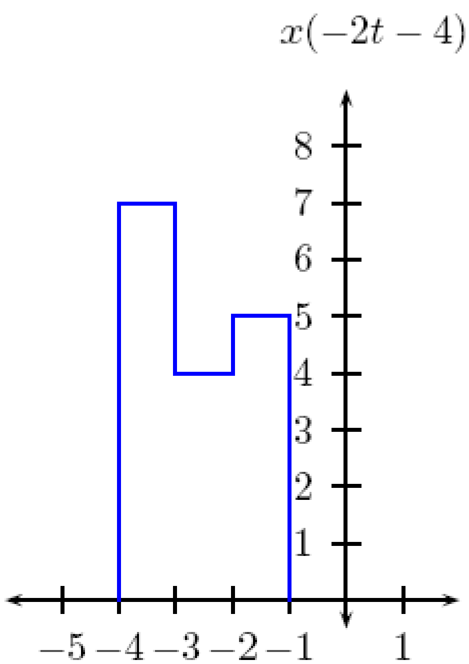




2.6





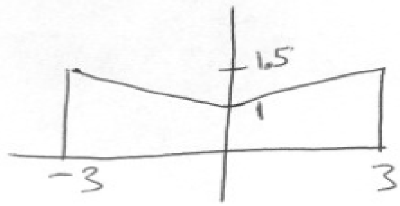


$$
\begin{aligned}
x(t) &= 5u(-2t-2) - u(-2t-4) + 3u(-2t-6) - 7u(-2t-8) \\
&= 5u(-(t+1)) - u(-(t+2)) + 3u(-(t+3)) - 7u(-(t+4))
\end{aligned}
$$

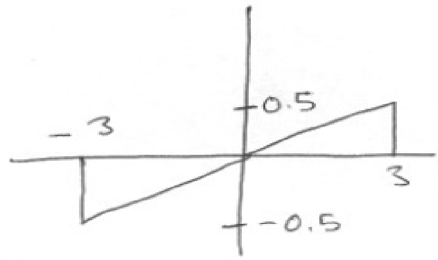
Or $x(t) = 7u(t+4) - 3u(t+3) + u(t+2) - 5u(t+1)$

2.7

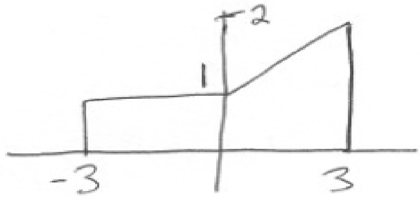
a) even



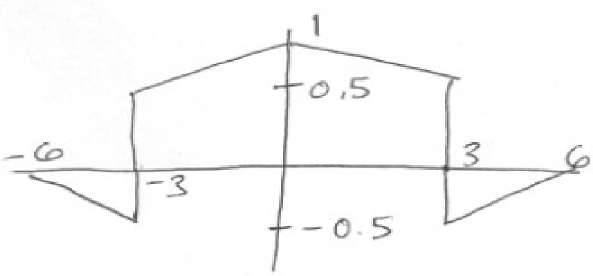
odd



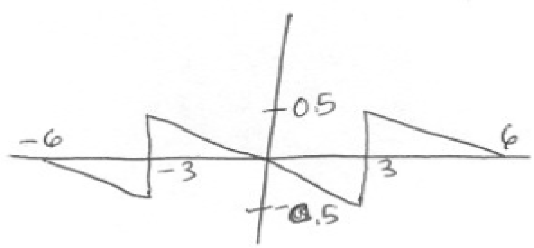
verify: even + odd:



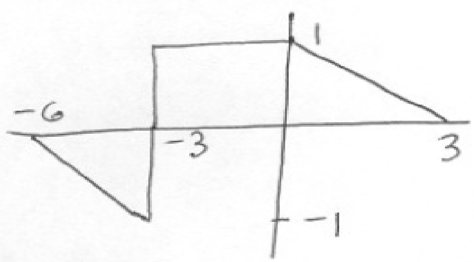
b) even



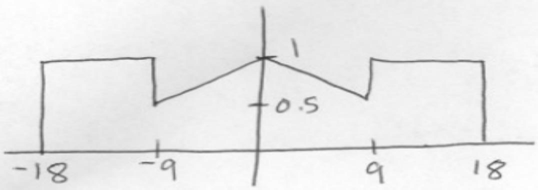
odd



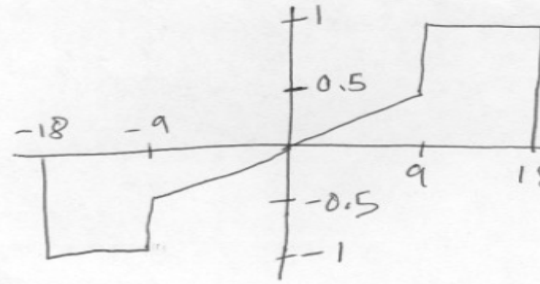
verify: even + odd:



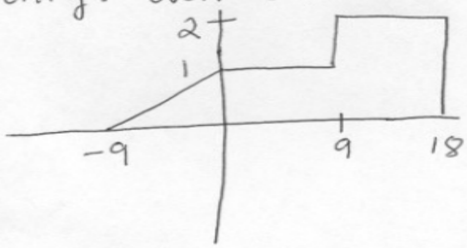
c) even



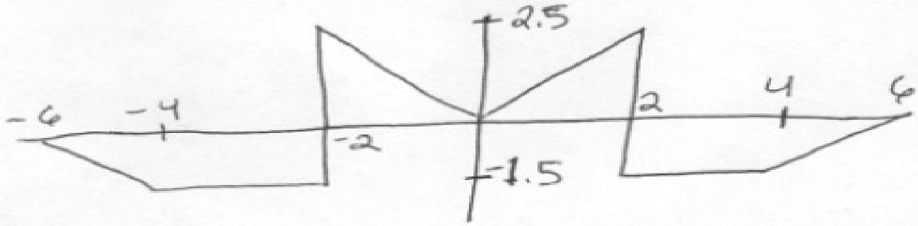
odd



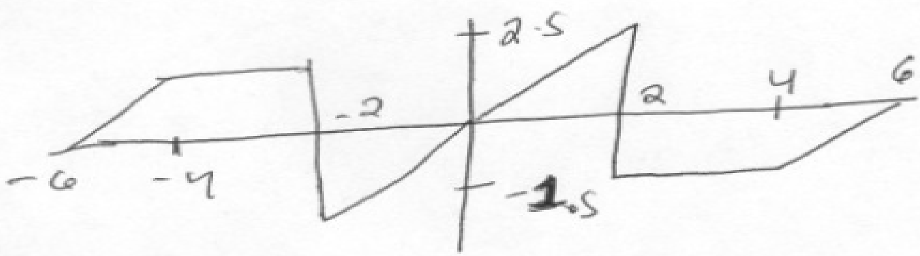
verify: even + odd:



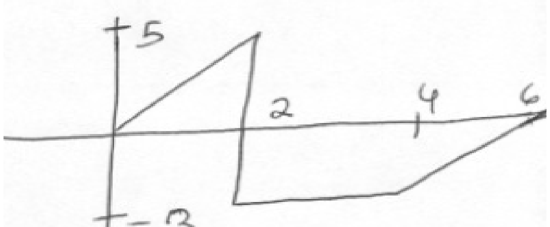
d) even



odd

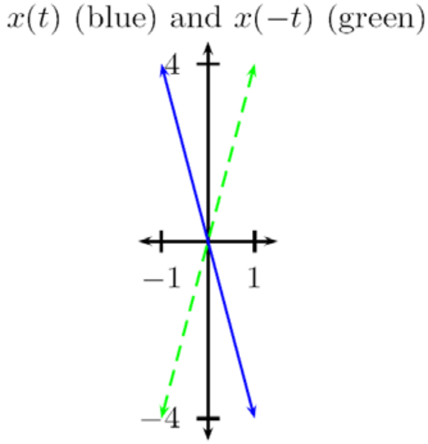


even + odd:

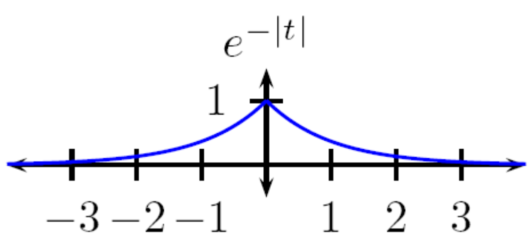


2.8

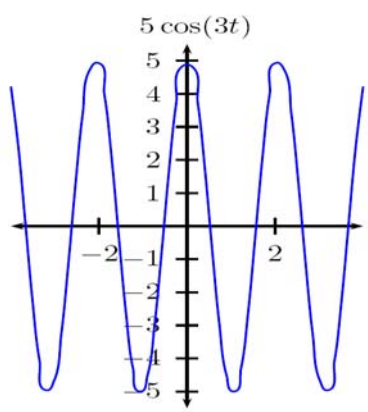
a) $-4t = -\left(-4(-t)\right)$ so it is odd.



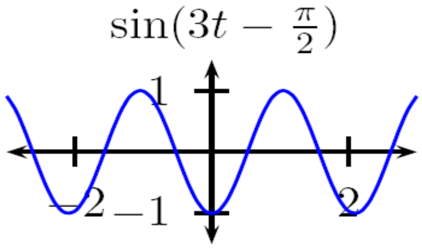
b) $e^{-|t|} = e^{-|-t|}$ so it is even $(|t| = |-t|)$.



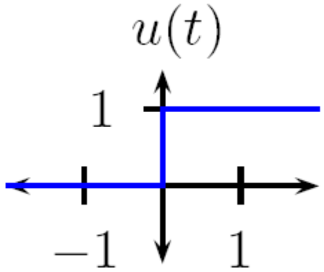
c) Since $\cos(t)$ is even, $5\cos(3t)$ is also even.



d) $\sin(3t - \frac{\pi}{2}) = -\cos(3t)$ which is even:



e) $u(t)$ is neither even nor odd; for example $u(3) = 1$ but $u(-3) = 0 \neq -u(3), \ \neq u(3)$.



2.9 (a) $\int_{-T}^{T} x_o(t) = \int_{-T}^{0} x_o(t)dt + \int_{0}^{T} x_o(t)dt$; $x_o(t) = -x_o(-t)$

$\therefore \int_{-T}^{0} x_o(t)dt = -\int_{-T}^{0} x_o(-t)dt \Big|_{t=-\tau} = \int_{-T}^{0} x_o(\tau)d\tau = -\int_{0}^{T} x_o(\tau)d\tau$

$\therefore \int_{-T}^{T} x_o(t)dt = 0$

(b) $\int_{-T}^{T} x(t)dt = \int_{-T}^{T} [x_e(t) + x_o(t)]dt = \int_{-T}^{T} x_e(t)dt$

and $A_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_e(t)d\tau$

(c) $x_o(0)=-x_o(-0)= -x_o(0)$. The only number with a=-a is a=0 so this implies $x_o(0)=0$.
$x(0)=x_e(0)+x_o(0)=x_e(0)$.

2.10

(a) Let $z(t)$ be the sum of two even functions $x_1(t)$ and $x_2(t)$. To show that $z(t)$ is even, we need to show that $z(t) = z(-t)$ for all t. This is easy to show, since $z(t) = x_1(t)+x_2(t)$ and $z(-t) = x_1(-t)+x_2(-t)$ (since to get $z(-t)$ we just plug in $-t$ everywhere for t, which amounts to just plugging in $-t$ in $x_1(t)$ and $x_2(t)$). Now since $x_1(t)$ and $x_2(t)$ are even, by definition $x_1(t) = x_1(-t)$ and $x_2(t) = x_2(-t)$ so $x_1(t) + x_2(t) = x_1(-t) + x_2(-t)$ so $z(t) = z(-t)$.

(b) Let $x_1(t)$ and $x_2(t)$ be two odd functions. Then $x_1(-t) + x_2(-t) = -x_1(t) + (-x_2(t)) = -(x_1(t) + x_2(t))$ which shows that $x_1(t) + x_2(t)$ is odd.

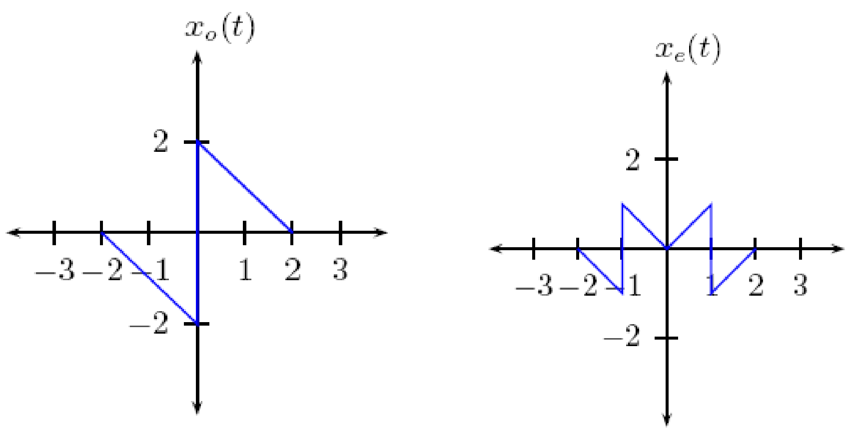
(c) Let $z(t) = x_1(t) + x_2(t)$ as in part a, where now $x_1(-t) = x_1(t)$ and $x_2(-t) = -x_2(t)$. We need to show that $z(t) \neq z(-t)$, $z(t) \neq -z(-t)$. Consider that $z(-t) = x_1(-t) + x_2(-t) = x_1(t) - x_2(t)$. In order to have $z(t)$ be even, we would therefore need to have $x_1(t) + x_2(t) = x_1(t) - x_2(t)$ for all t, which is equivalent to having $x_2(t) = -x_2(t)$ for all t, which is not possible for nonzero $x_2(t)$. Similarly, in order to have $z(t)$ be odd, we would need to have $z(t) = -z(t) \implies x_1(t) + x_2(t) = x_2(t) - x_1(t)$, which is not possible for nonzero $x_1(t)$. So the sum of an even and odd function must be neither even nor odd.

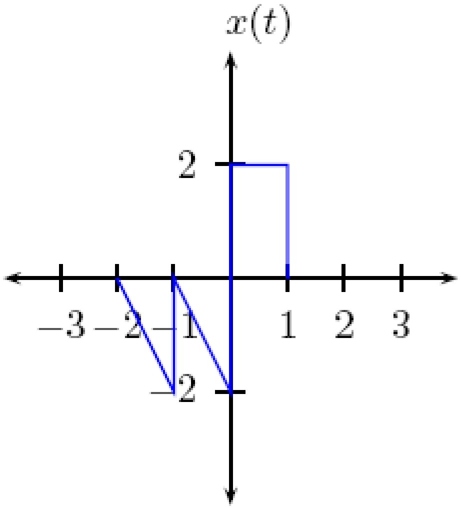
(d) Let $z(t) = x_1(t)x_2(t)$ where $x_1(t) = x_1(-t)$ and $x_2(t) = x_2(-t)$. Then $z(-t) = x_1(-t)x_2(-t) = x_1(t)x_2(t) = z(t)$ which shows that $z(t)$ is even.

(e) Let $z(t) = x_1(t)x_2(t)$, where $x_1(t) = -x_1(-t)$ and $x_2(t) = -x_2(-t)$. Clearly $z(t)$ is even because $z(-t) = x_1(-t)x_2(-t) = (-x_1(t))(-x_2(t)) = x_1(t)x_2(t) = z(t)$, which is the definition of evenness.

(f) Let $z(t) = x_1(t)x_2(t)$, where $x_1(t) = -x_1(-t)$ and $x_2(t) = x_2(-t)$. Clearly $z(t)$ is odd because $z(-t) = x_1(-t)x_2(-t) = (-x_1(t))\,x_2(t) = -x_1(t)x_2(t) = -z(t)$, which is the definition of oddness.

2.11





The plot of $x_o(t)$ is determined by $x_o(-t) = -x_o(t)$, the plot of $x_e(t)$ is determined by $x_e(t) = x(t) - x_o(t)$, and the plot of $x(t)$ is determined by $x(t) = x_e(t) + x_o(t)$.

2.12

(a) $\sin(t) = \sin(t + n2\pi)$ for any integer n, so $7\sin(3t) = 7\sin(3t + n2\pi) = 7\sin\left(3(t + n\frac{2\pi}{3})\right)$; therefore $x(t)$ is periodic with fundamental period $T_0 = \frac{2\pi}{3}$ and fundamental frequency $\omega_0 = \frac{2\pi}{T_0} = 3$.

(b) $\sin(8(t + \frac{2\pi}{8}) + 30) = \sin(8t + 2\pi + 30) = \sin(8t + 30)$.
$\omega_0 = 8$ and $T_0 = \frac{2\pi}{8} = \frac{\pi}{4}$.

(c) $e^{jt} = \cos(t) + j\sin(t)$ is periodic with fundamental period 2π, so e^{j2t} is periodic with fundamental period $\frac{2\pi}{2} = \pi$, and fundamental frequency $\omega_0 = 2$.

(d) $\cos(t) = \cos(t+n2\pi)$ for any integer n, and $\sin(2t) = \sin(2(t+m\pi))$ for any integer m, so $\cos(t)+\sin(2t)$ will be periodic with period T_0 if $\cos(t) + \sin(2t) = \cos(t + T_0) + \sin(2(t + T_0))$. This will hold as long as $T_0 = n2\pi$ and $T_0 = m\pi$ for some integers n and m, and the fundamental period is the smallest value for which this holds, which is $T_0 = 2\pi$, with fundamental frequency $\omega_0 = 1$.

(e) $e^{j(5t+\pi)} = e^{j\pi}e^{j5t}$. So the phase shift of π just means a complex constant (constant with respect to time) out front and does not effect periodicity of the signal e^{j5t}, which has fundamental period $T_0 = \frac{2\pi}{5}$ and $\omega_0 = 5$.

(f) e^{-j10t} and e^{j15t} are both periodic with periods $\frac{\pi}{5}, \frac{2\pi}{15}$ and their sum is periodic with period $T_0 = LCM(\frac{\pi}{5}, \frac{2\pi}{15}) = \frac{2\pi}{5}$ and $\omega_0 = 5$:
$e^{-j10(t+\frac{2\pi}{5})} + e^{j15(t+\frac{2\pi}{5})} = e^{-j10t}e^{-j4\pi} + e^{j15t}e^{j6\pi}$ and since $e^{-j4\pi} = 1$ and $e^{j6\pi} = 1$ this $= e^{-j10t} + e^{j15t}$.

2.13

(a) periodic, $T_0 = 2\pi$, $\omega_0 = 1$
(b) periodic, $T_0 = \pi$, $\omega_0 = 2$
(c) not periodic since 1 and π do not have any common factors (the only factor of 1 is 1, but since π is irrational, it cannot be an integer times 1)
(d) periodic, $T_0 = 12$, $\omega_0 = \frac{\pi}{6}$

2.14

(a) periodic, $T_0 = \frac{\pi}{2}$, $\omega_0 = 4$
(b) periodic, $T_0 = \frac{\pi}{2}$, $\omega_0 = 4$
(c) not periodic, since 2π and 6 do not have a common factor
(d) periodic; $x_1(t)$ has period 2, $x_2(t)$ has period 1, and $x_3(t)$ has period $\frac{12}{5}$ so the sum has period $T_0 = LCM(2, 1, \frac{12}{5}) = 12$ and fundamental frequency $\omega_0 = \frac{\pi}{6}$.

2.15

(a) For $x_1(t) + x_2(t)$ to be periodic we need some number T such that $x_1(t+T) + x_2(t+T) = x_1(t) + x_2(t)$ for all t. This can only be true if $x_1(t+T) = x_1(t)$ and $x_2(t+T) = x_2(t)$, which can only be true if $T = k_1 T_1$ and $T = k_2 T_2$ (T is an integer multiple of both the periods). So we need there to be some integers k_1 and k_2 such that $k_1 T_1 = k_2 T_2 \implies \frac{T_1}{T_2} = \frac{k_2}{k_1}$.
(b) Put $\frac{k_2}{k_1}$ in its most reduced form $\frac{n}{m}$ by canceling any common terms in the numerator and denominator; then $T_0 = nT_2 = mT_1$.

2.16

Let $u = at$ so performing u substitution gives:

$$\begin{aligned}\int_{-\infty}^{\infty} \delta(at-b)\sin^2(t-4)dt &= \int_{-\infty}^{\infty} \delta(u-b)\sin^2(\tfrac{u}{a}-4)\tfrac{du}{a} \\ &= \sin^2(\tfrac{b}{a}-4)\tfrac{1}{a}\end{aligned}$$

2.17 By sifting property, y(t) = 1/2 x(2) + 1/2 x(-2)

2.18

(a) $x_1(t) = 2t\,u(t) - 4(t-1)\,u(t-1) + 2(t-2)\,u(t-2)$

(b) $t<0, \quad x_1(t)=0$

$0<t<1, \quad x_1(t) = 2t$

$1<t<2, \quad x_1(t) = 2t-4t+4 = 4-2t$

$2<t, \quad x_1(t) = 4-2t+2t-4=0$

(c) $x(t) = \sum_{k=-\infty}^{\infty} x_1(t-kT_0) = \sum_{k=-\infty}^{\infty} x_1(t-2k)$

2.19

(a) $x_1(t) = 5tu(t) - 5tu(t-1) + 5u(t-1) - 5u(t-3)$

(b)

$$
\begin{aligned}
t < \quad & 0, f(t) = 0-0+0-0=0 \\
0 < t < \quad & 1, f(t) = 5t-0+0=0=5t \\
1 < t < \quad & 3, f(t) = 5t-5t+5-0=5 \\
3 < \quad & t, f(t) = 5t-5t+5-5=0
\end{aligned}
$$

(c) $x_2(t) = \sum_{k=-\infty}^{\infty} x_1(t-k4)$

2.20. (a) let $at = \tau$, $\therefore \int_{-\infty}^{\infty} \delta(at)\,dt = \int_{-\infty}^{\infty} \delta(\tau)\frac{d\tau}{a}$

$$= \frac{1}{a}\int_{-\infty}^{\infty} \delta(\tau)\,d\tau \Rightarrow \underline{\delta(at) = \frac{1}{a}\delta(t),\ a>0}$$

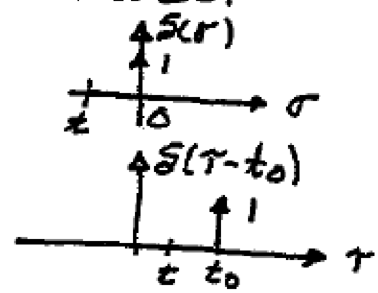
For $a<0$, $at=\tau \Rightarrow -|a|t = \tau$, $dt = -\frac{d\tau}{|a|}$

$$\therefore \int_{-\infty}^{\infty} \delta(at)\,dt = \int_{\infty}^{-\infty} \delta(\tau)\frac{-d\tau}{|a|} = \frac{1}{|a|}\int_{-\infty}^{\infty} \delta(\tau)\,d\tau$$

$\therefore \underline{\delta(at) = \frac{1}{|a|}\delta(t)}$ for the general case.

(b) $\int_{-\infty}^{t} \delta(\sigma)\,d\sigma = u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$

$$\therefore \int_{-\infty}^{t} \delta(\tau - t_0)\,d\tau = u(t-t_0)$$



(c) $\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$

(continued)…

2.20 (c)

Recall the rules about integrating delta functions: $\delta(t)$ is nonzero only at $t = 0$, so $x(t)\delta(t) = x(0)\delta(t)$, and $\int_{-\infty}^{\infty} \delta(t)dt = 1$, so $\int_{-\infty}^{\infty} x(t)\delta(t)dt = \int_{-\infty}^{\infty} x(0)\delta(t)dt = x(0)\int_{-\infty}^{\infty} \delta(t)dt = x(0)$. We can time-shift the delta function: $\delta(t-t_0)$ is nonzero only at $t = t_0$, so $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$ and $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$.

i) $\int_{-\infty}^{\infty} \cos(2t)\delta(t)dt = \cos(2 \cdot 0)\int_{-\infty}^{\infty} \delta(t)dt = 1$.

ii) $\delta(t - \frac{\pi}{4})$ is a time-shifted version of $\delta(t)$, and is nonzero only at $t = \frac{\pi}{4}$. So:

$$\begin{aligned}\int_{-\infty}^{\infty} \sin(2t)\delta(t - \frac{\pi}{4})dt = & \int_{-\infty}^{\infty} \sin(2 \cdot \tfrac{\pi}{4})\delta(t - \tfrac{\pi}{4})dt \\ = & \sin(\tfrac{\pi}{2})\int_{-\infty}^{\infty} \delta(t - \tfrac{\pi}{4})dt = \sin(\tfrac{\pi}{2}) = 1\end{aligned}$$

iii) $\cos(2(t - \frac{\pi}{4}))\delta(t - \frac{\pi}{4}) = \cos\left(2(\frac{\pi}{4} - \frac{\pi}{4})\right)\delta(t - \frac{\pi}{4}) = 1 \cdot \delta(t - \frac{\pi}{4})$, so the integral of this is 1.

iv) $\delta(t-2)$ is nonzero only at $t = 2$. Therefore $\int_{-\infty}^{\infty} \sin\left((t-1)\right)\delta(t-2)dt = \sin\left(2-1\right) = \sin(1) = 0.8414....$

v) $\delta(2t-4)$ is nonzero at $2t - 4 = 0 \implies t = 2$. So:

$$\int_{-\infty}^{\infty} \sin\left(t-1\right)\delta(2t-4)dt = \sin\left(2-1\right)\int_{-\infty}^{\infty} \delta(2t-4)dt$$

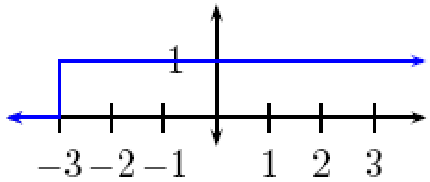
To figure out the integral, we can change variables—let $u = 2t$, so $dt = \frac{du}{2}$ and the $-\infty, \infty$ limits stay the same. This gives: $\int_{-\infty}^{\infty} \delta(2t-4)dt = \int_{-\infty}^{\infty} \delta(u-4)\frac{du}{2} = \frac{1}{2}$, so we get:

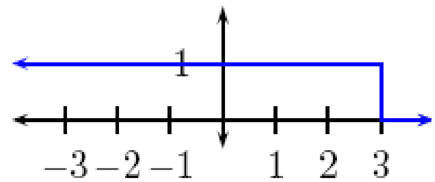
$$\int_{-\infty}^{\infty} \sin\left(t-1\right)\delta(2t-4)dt = 0.5\sin(1) = 0.4207...$$

2.21

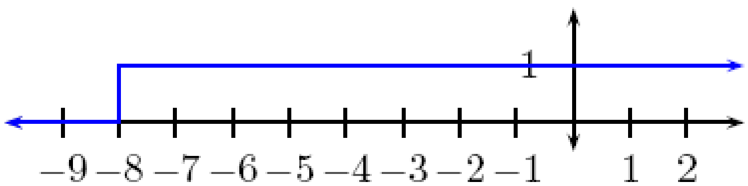
(a) $u(2t+6) = u(t+3)$

(b) $u(-2t+6) = u(-t+3)$

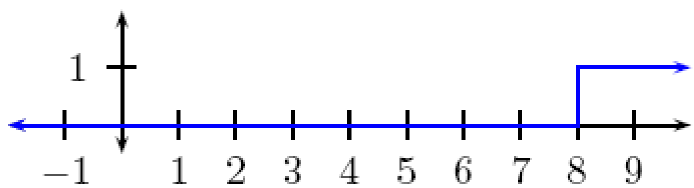


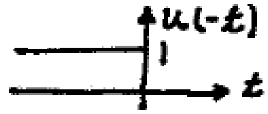


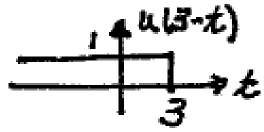
(c) $u(\frac{t}{4}+2) = u(t+8)$

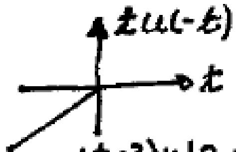


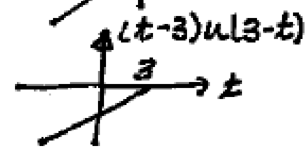
(d) $u(\frac{t}{4}-2) = u(t-8)$



2.22 (a)  $u(-t) = 1 - u(t)$

(b)  $u(3-t) = 1 - u(t-3)$

(c)  $t\,u(-t) = t\,[1 - u(t)]$

(d)  $(t-3)u(-t) = (t-3)[1 - u(t-3)]$

2.23 (a) $y_2(t) = T_2\left[T_1[x(t)]\right]$, $y_3(t) = T_3[T_1[x(t)]]$

$y(t) = T_2[T_1[x(t)]] + T_4\{T_3[T_1[x(t)]] + T_5[x(t)]$

(b) $y(t) = T_3\{T_2[T_1[x(t)]]\} + T_4\{T_2[T_1[x(t)]]\} + T_5[T_1[x(t)]]$

(c) $y(t) = T_2[T_1[x(t)] + T_4\{T_3[T_1[x(t)]] \times T_5[x(t)]\}$

(d) $y(t) = T_3\{T_2[T_1[x(t)]]\} \times T_4\{T_2[T_1[x(t)]]\} \times T_5[T_1[x(t)]]$

2.24 $y(t) = T_3[m(t) + T_1[x(t)]]$

$m(t) = T_2[x(t) - T_4[y(t)]]$

$\therefore\ y(t) = \underline{T_3\{T_2[x(t) - T_4[y(t)]] + T_1[x(t)]\}}$

2.25 $m(t) = T_1\{x(t) - T_4[y(t)]\} - T_3[y(t)]$

$y(t) = T_2[m(t)] = T_2\left[T_1\{x(t) - T_4[y(t)]\} - T_3[y(t)]\right]$

2.26

(a) (i) has memory; (ii) not invertible; (iii) stable; (iv) time invariant; (v) linear

(b) need $y(t_0)$ to only depend on $x(t)$ values as $t \le t_0 \implies t_0 + 1 - \alpha \le t_0 \implies$ causal for values of $\alpha \ge 1$.

2.27

2.27(a) system is: $y(t) = \cos(x(t-1))$

i) Not memoryless: $y(t)$ depends on $x(t-1)$.

ii) Not invertible: for a counterexample of two input signals that give the same output signal at all points, take any $x(t)$ and $x(t) + 2\pi$.

iii) Causal; output at time t does not depend on input at times greater than t.

iv) Stable: clearly $|y(t)| \leq 1$ for any values of the input.

v) Time invariant: $y_d(t) = cos(x(t-1-t_0))$ and $y(t-t_0) = cos(x(t-t_0-1))$.

vi) Not linear: for example, violates the scaling property because $ay(t) \neq cos(ax(t-1))$ (if we input a scaled version of the input $ax(t)$ we don't get the output scaled by the same amount $ay(t)$). This system also violates additivity, the other necessary property for a system to be linear.

2.27(b)

i) not memoryless (at time t_0 output depends on input at time $3t_0$)

ii) invertible ($x(t) = \frac{1}{3}y(\frac{t-3}{3})$)

iii) not causal ($3t_0 > t_0$ for $t_0 > 0$)

iv) stable

v) not time invariant ($x(t-t_0) \to 3x(3t - t_0 + 3)$ but $y(t-t_0) = 3x(3(t-t_0)+3) = 3x(3t - 3t_0 + 3)$

vi) linear

2.27(c) system is: $y(t) = \ln(x(t))$

i) Memoryless;

ii) Invertible: $x(t) = e^{(y(t))}$

iii) Causal;

iv) Not stable: for example, $y(t) = -\infty$ whenever $x(t) = 0$

v) Time invariant;

vi) Not linear: for example, violates additivity: $\ln(x_1(t) + x_2(t)) \neq \ln(x_1(t)) + \ln(x_2(t))$ in general. Scaling doesn't work either.

2.27(d) System is: $y(t) = e^{tx(t)}$

i) Memoryless;

ii) $x(t) = \frac{\ln(y(t))}{t}$ except when $t = 0$ (we can't get back the value of $x(0)$.) This system would therefore be considered noninvertible but it is mostly invertible.

iii) Causal;

iv) Not stable: for example, if $x(t) = c$ (some constant $c > 0$) then $y(t) = e^{tc}$ which goes to ∞ as $t \to \infty$ (we can't find any number K such that $e^{tc} < K$ for all t). not memoryless, invertible, not causal, stable, not time invariant, linear

v) Not time invariant: if the input is $x(t-t_0)$ we get $y_d(t) = e^{(tx(t-t_0))} \neq y(t-t_0) = e^{((t-t_0)x(t-t_0))}$

vi) Not linear: doesn't satisfy either necessary property.

2.27(e) System is: $y(t) = 7x(t) + 6$

This system is memoryless, invertible, causal, stable, time invariant, but NOT linear: if we input $x_1(t) + x_2(t)$ we get out $7(x_1(t) + x_2(t)) + 6$, while if we input $x_1(t)$ and $x_2(t)$ separately and add them, we get $y_1(t) + y_2(t) = 7(x_1(t)) + 6 + 7(x_2(t)) + 6$, so the system violates additivity. Also violates scaling. Note that to show a system is linear you need to show it satisfies both properties (which you can do by showing that $ax_1(t) + bx_2(t) \to ay_1(t) + by_2(t)$), but to show that a system is NOT linear, you only need to show that it violates at least one of these properties.

2.27(f) System is: $y(t) = \int_{-\infty}^{t} x(5\tau)d\tau$

i), iii) Not memoryless, not causal: output at time t depends on both past values of $x(t)$ (because integrating from $-\infty$) and future values of t (because depends on $x(5t)$ and $5t > t$ for $t > 0$).

ii) invertible: $\frac{d}{dt}y(t) = x(5t) \implies x(t) = \frac{d}{dt}y(t)\mid_{t/5}$ (the function $y'(t)$ evaluated at $t/5$).

iv) Not stable: for instance, $x(t) = c$ (some constant) is a bounded input but the output is $y(t) = ct$, which goes to ∞ as t goes to ∞.

v) Not time-invariant: if the input is $x(t - t_0)$ we get $y_d(t) = \int_{-\infty}^{t} x(5\tau - t_0)d\tau$,but $y(t - t_0) = \int_{-\infty}^{t-t_0} x(5\tau)d\tau = \int_{-\infty}^{t} x(5(\tau - t_0))d\tau$.

vi) linear: if $x_1(t) \to y_1(t) = \int_{-\infty}^{t} x_1(5\tau)d\tau$ and $x_2(t) \to y_2(t) = \int_{-\infty}^{t} x_2(5\tau)d\tau$ then:

$$\begin{aligned} ax_1(t) + bx_2(t) \to \int_{-\infty}^{t} ax_1(5\tau) + bx_2(5\tau)d\tau &= a\int_{-\infty}^{t} x_1(5\tau)d\tau + b\int_{-\infty}^{t} x_2(5\tau)d\tau \\ &= ay_1(t) + by_2(t) \end{aligned}$$

2.27(g) System is: $y(t) = e^{-j\omega t}\int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}d\tau$.

i), iii) Not memoryless, not causal: depends on $x(t)$ values at all t from $-\infty$ to ∞.

ii) Not invertible

iv) Not stable: say $\omega = 0$ and the input is a constant c; the output is infinite.

v) NOT time-invariant:

$$\begin{aligned} x(t - t_0) \to y_d(t) = & \quad e^{-j\omega t}\int_{-\infty}^{\infty} x(\tau - t_0)e^{-j\omega\tau}d\tau \\ = & \quad e^{-j\omega t}\int_{-\infty}^{\infty} x(u)e^{-j\omega(u+t_0)}du = e^{-j\omega(t+t_0)}\int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}d\tau \end{aligned}$$

which comes from u-substitution, letting $u = t - t_0$. But $y(t - t_0) = e^{-j\omega(t-t_0)}\int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}d\tau$ which is not equal to the above.

vi) Linear; the integral and multiplication by $e^{-j\omega t}$ are both linear operations.

2.27(h)

i) Not memoryless ($y(t)$ depends on input over last second)

ii) not invertible (for example, $x(t) = 0$ and $x(t) = \cos(2\pi t)$ have the same output signal

iii) causal

iv) stable

v) time invariant (since $x(t - t_0) \to \int_{t-1}^{t} x(\tau - t_0)d\tau = \int_{t-t_0-1}^{t-t_0} x(\tau)d\tau = y(t - t_0)$)

vi) linear

2.28

(a) $x_2(t) = 2u(t + 1) - u(t) - u(t - 1) = x_1(t) + 2x_1(t + 1)$
so $y_2(t) = y_1(t) + 2y_1(t + 1)$

(b) $x_1(t) = 2u(t - 1) - u(t - 2) - u(t - 3)$ so $x_2(t) = x_1(t + 2)$ and $y_2(t) = y_1(t + 2)$

2.29

i) not memoryless unless $t_0=0$

ii) invertible: $x(t)=y(t+t_0)$

iii) If $t_0 \geq 0$ it is causal; otherwise not.

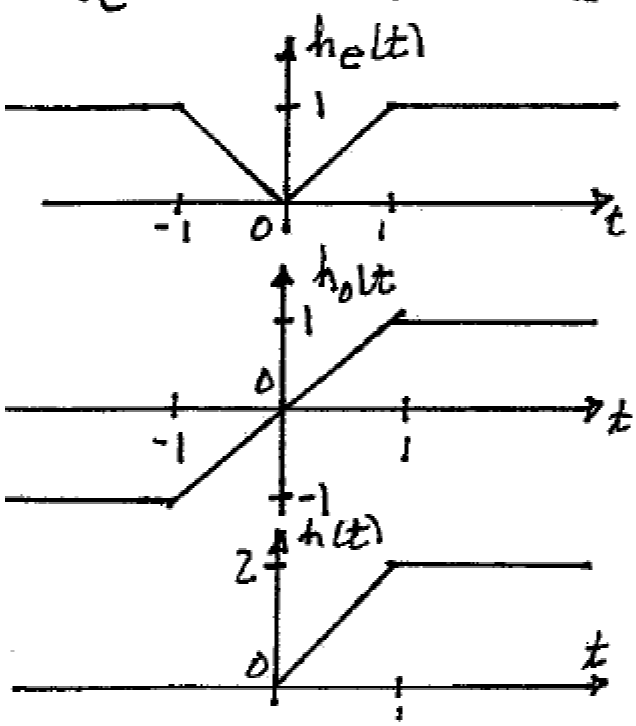
iv) stable; the output only takes value of the input so if the input is bounded the output will be too.

v) time invariant: let $y_d(t)$ be the output when $x(t-t_1)$ is the input. $x(t-t_1) \rightarrow y_d(t) = x(t-t_1-t_0)$ and $y(t-t_1) = x(t-t_1-t_0)$, so $y_d(t) = y(t-t_1)$.

vi) linear: scaling and adding two inputs $ax_1(t) + bx_2(t)$ gives output $ax_1(t-t_0) + bx_2(t-t_0)$, which is the same output we would get by putting $x_1(t)$ and $x_2(t)$ into the system separately and then scaling and adding the outputs.

2.30

$h_e(t) = t\,[u(t) - u(t-1)] + u(t-1)$



$h_e(t)$ even

$h_o(t) = h(t) - h_e(t)$

$\therefore h_o(t) = -h_e(t) \quad t<0$

and $h_o(t) = -h_o(-t)$

$\therefore h(t) = 2t\,u(t) - 2(t-1)u(t-1)$

$t<0, \; h(t) = 0$

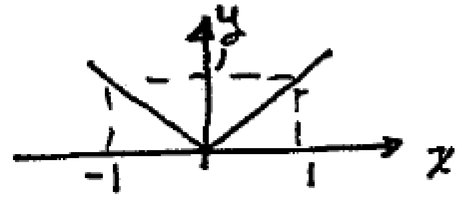
$0<t<1, \; h(t) = 2t$

$2<t, \; h(t) = 2t-2t+2 = 2$

2.31

(a) (i) memoryless

(ii) $y=1$ for $x=\pm 1$, not invertible



(iii) causal

(iv) stable

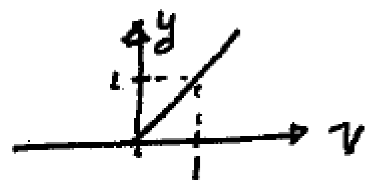
$y=|x|$

(v) time invariant

(vi) $|x_1+x_2| \neq |x_1|+|x_2|$ not linear

(b) (i) memoryless

(ii) $y=0$ for $x\leq 0$, not invertible



(iii) causal

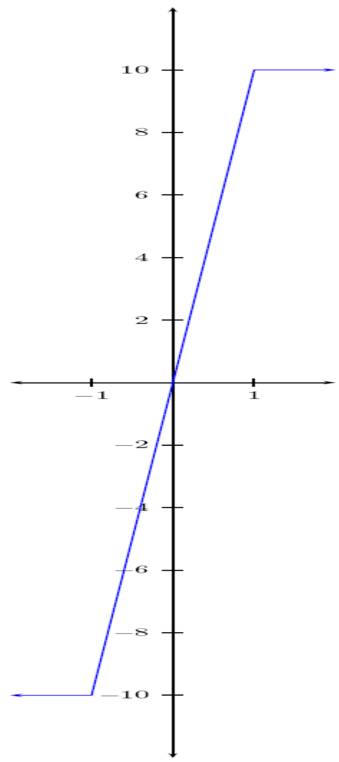
(iv) stable

(v) time invariant

(vi) $y\Big|_{\substack{x_1=1\\x_2=-1}} \neq y\Big|_{x_1+x_2}$, not linear

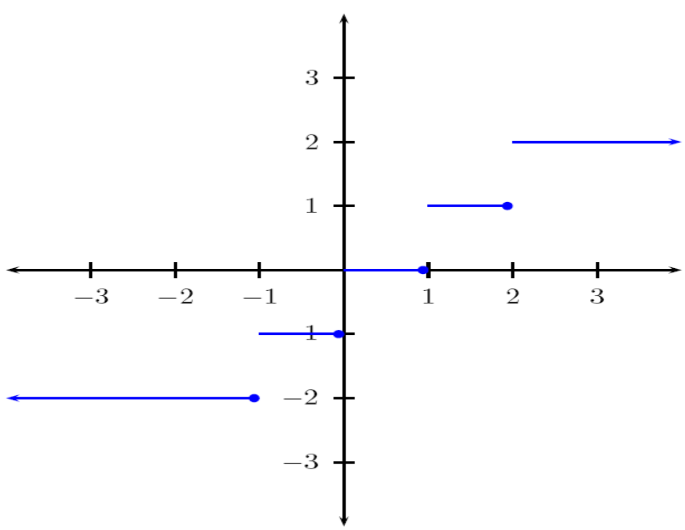
(parts c,d on next page)

(c)



The system is i) memoryless, ii) not invertible (output $= 10$ for all input values > 10, iii) causal, iv) stable ($|y(t)| \leq 10$ for any input), v) time invariant, vi) not linear (suppose $x(t) = 3$ then $y(t) = 3$ but $4x(t)$ has output $10 \neq 3(4) = 12$.

(d)



The system is i) memoryless, ii) not invertible (any input greater than 2 goes to the same output (2)), iii) causal, iv) stable, v) time invariant, vi) not linear ($x_1(t) = 2 \to 1$ and $x_2(t) = 1 \to 0$ but $x_1(t) + x_2(t) \to 2 \neq 1 + 0$.