Chapter 1 - Solutions

1. Simulation is the imitation of a dynamic system using a computer model in order to evaluate and improve system performance.

2. The reasons for the increased popularity of computer simulation are:
    a. Increased awareness and understanding of simulation technology.
    b. Increased availability, capability and ease of use of simulation software.
    c. Increased computer memory and processing speeds; especially of PCs.
    d. Declining computer hardware and software costs.

3. Two specific questions simulation might help answer –
    a. In a bank
        i. What is the average time to process a loan?
        ii. How many tellers are needed during peak periods to keep waiting times under two minutes?
    b. In a manufacturing facility
        i. What is the throughput capacity of the facility?
        ii. What is the average work-in-process (WIP)?
    c. In a dental office
        i. What is the average patient waiting time?
        ii. What is the best way to schedule patients?

4. Three advantages that simulation has over alternative approaches to system design are:
    a. More accurate (accounts for interdependencies and variability)
    b. More visual with animation
    c. Shows behavior over time

5. No, simulation itself does not optimize a system design, but it allows alternative solutions to be compared. It can also be integrated with an optimizer (like SimRunner) to automatically find the best solution using special goal seeking algorithms.

6. Simulation follows the scientific method because in simulation we first formulate a hypothesis about what design or operating policies work best, then we set an experiment in the form of a simulation model and we conduct multiple replications of the simulation to test our hypothesis. Finally, we analyze the simulation results and draw conclusions about our hypothesis.

7. Some of the questions to ask include:
    a. Is the cost impact of the decision greater than the cost of doing simulation?
    b. Is the cost to experiment on the actual system greater than the cost to do a simulation?
    c. Is there any benefit to showing the simulation animation?

8. To develop an economic justification for using simulation, costs and benefits should be carefully assessed:
    a. Estimate the cost and time required to apply simulation to the current problem, in general, 1%-3% of the total project cost is required and less than 5% of the overall design time.
    b. Assess the risk of making poor design and operational decisions. Simulation allows designers to make mistakes at the design phase rather than at the implementation phase.
    c. Tie the benefits of simulation to management and organizational goals.

9. Even if no problems are encountered in analyzing the output of simulation, the exercise of developing a model is in itself beneficial because it forces one to think through the operational details of the process. Improvements often occur by going through the model building because we usually tend to ignore the details until the implementation phase when it is too late, simulation helps us to think about the details in advance. In addition, if the simulation uncovered no problems, this instills confidence that the design is a good one.

10. Before placing confidence in a simulation model you should know whether the model was valid and how the

experiment was conducted (how many replications, etc.).

11. Simulation can help by doing bottleneck analysis to find the process that causing the WIP. It can also help find ways to get increased and more balanced utilization.

12. A statistical background helps in doing simulation, but this does not mean that only statisticians can do simulation. A basic understanding of statistics is adequate for most simulation projects and it is not until precise analysis is required that a more in depth knowledge is necessary.

13. A programming background is useful in building simulation models and in debugging these models. It can also help develop abstract and logical thinking that are needed in model building. If a project requires integrating simulation with other applications, a computer program may need to be written.

14. Good project management and communication skills are important in simulation because a simulation study has all of the characteristics and objectives that are typical of a project -- multiple tasks to be coordinated, people to contact, schedules to meet, budgets to operate within and expectations to manage.

15. The process owner should be heavily involved in a simulation project because he is the most familiar with the design and operation of the system, and therefore is in the best position to recommend alternative solutions. Improvements to a design often suggest themselves in building the model, so the domain expert should be closely involved to be made aware of these improvement opportunities. He is also the one with much of the decision-making power so he needs to champion the solution.

16. For the situations listed simulation is

    a. Useful.
    b. Not useful (this is more a motivational and skill issue)
    c. Useful
    d. Not useful (this is directly measurable from the actual system)
    e. Useful
    f. Not useful (too costly for the decision being made and easier to experiment on the actual system)

17. It is important to have clearly defined objectives for a simulation project to avoid wasted efforts. It is also important that everyone understands what the purpose of the simulation is so expectations aren't disappointed.

18. A possible simulation objective for the following systems:

    a. Manufacturing cell: finding the optimum number of operators.
    b. Fast food restaurant: finding the optimum number of tables.
    c. Emergency room: finding the expected waiting time for a given arrival rate.
    d. Tire distribution center: finding the optimum inventory level of tires.
    e. Public bus system: finding the optimum number of buses for a route.
    f. Post office: finding the expected waiting time for customers during peak periods.
    g. Elevators: finding the optimum number of elevators to minimize waiting.
    h. Computer system: finding the number of transactions that can be handled per time period.
    i. Car rental agency: finding the optimum number of cars to have.

Case study A: AST Research Inc.

1. The objectives of the simulation were to
    a. Minimize the time needed to assemble a computer
    b. Maximize productivity
    c. Optimize their assembly lines
    d. Gain confidence that proposed changes in the assembly process would work.

2. In comparison to traditional methods such as: gathering time-and-motion data via stopwatches and video, performing simple arithmetic calculations to obtain information about the operation and performance of the assembly line, and using seat-of-the-pants guesstimates to "optimize" assembly line output and labor utilization, simulation helped AST make fewer missteps in terms of implementing changes, which could have impaired their output. Engineering was able to try multiple scenarios in their efforts to improve productivity and efficiency at comparatively low cost and risk. All of these benefits were accomplished at minimal cost.

3. Use of simulation helped AST avoid making changes that were thought to be common sense, but when simulated turned out to be ineffective. These include the supposition that increasing throughput 30 percent would require additional equipment. There was also skepticism that having only five operators in a cell would work..

4. One side benefit was that AST "learned that the best simulation efforts invite participation by more disciplines in the factory, which helps in terms of team-building."

5. Insights on the use of simulation might include
   a. Simulation can be used to achieve multiple objectives.
   b. Often simulation can give results that are non-intuitive.
   c. Simulation is a powerful tool for convincing others of a solution.

Case study B: Durham Regional Hospital.

6. This was a good application for simulation because the process to be simulated was of an operational nature and repetitive, the process information was available, the cost impact of the simulation was high compared with the costs of the simulation, and the management was willing to support the simulation project.

7. The key elements of the study that made the project successful were:
   a. The availability of reliable information about the process.
   b. The confidence the management had with the simulation approach.

8. The specific decisions that were made as a result of the simulation study were:
   a. Convenient Care patients should be accepted only after 5:00 p.m.
   b. The Express Services should be closed on the weekends.
   c. The staffing levels should be lower.

9. As a result of the simulation project, the projected savings were $148,762 annually.

10. It should be recognized from the study that simulation is a very powerful tool if used for the appropriate project. It can help decision makers in their job and saves a lot of time, money, and effort.

Chapter 2 - Solutions

1. Simulation is merely a tool to help make system design and operational decisions. It only evaluates a system design or operating strategy. It is up to the modeler to make good design decisions based on an understanding of cause-and-effect relationships and the impact of key decision variables.

2. A system is a collection of elements that function together to achieve a desired goal.

3. The elements of a system from a simulation perspective are:
    a. Entities -- the items processed through the system, such as customers in a bank.
    b. Activities -- the tasks performed in the system that are either directly or indirectly involved processing of entities, such as treating a patient in a hospital.
    c. Resources -- the means by which activities are performed, such as a machine tool in a job shop or a nurse in a hospital.
    d. Controls -- the rules that specify who, where, when, what, and how of entity processing, such as instruction sheets in an assembly line or the check processing sequence in a bank.

4. The two characteristics of systems that make them so complex are:
    a. Interdependencies between elements in which every element affects other elements in the system.
    b. Variability in element behavior that produces uncertainty.

5. Decision variables specify the defining characteristics of the system (number and types of resources, activity times, etc.). Response variables measure the performance of the system in response to the values assigned to the decision variables. In short, decision variables define how the system works while response variables indicate how a system performs.

6. Five decision variables of a manufacturing or service system that tend to be random in nature are:
    a. The likelihood that an entity will have a particular attribute (defective, urgent, etc.).
    b. Operation or processing times.
    c. Break times.
    d. The rate of demand for a product or service.
    e. The time between equipment failures.

7. Two examples of state variables are:
    a. The current status (busy or idle) of the milling machine in a job shop.
    b. The current number of customers waiting to be served in a fast food restaurant.

8. Three performance metrics important for a computer assembly line are:
    a. The average flow time for parts.
    b. The average number of parts in the system.
    c. Average throughput.

9. Three performance metrics useful for a hospital emergency room are:
    a. Patient waiting times.
    b. Staff utilization.
    c. Average time to be treated.

10. Optimization is finding the best feasible combination of decision variable settings that either maximizes or minimizes some response variable or objective function.

11. Maximizing resources utilization is not a good overriding performance objective for a manufacturing system because it ignores larger objectives such as minimizing work in process and cycle times. Increasing the utilization of a non-bottleneck resource, for example, often only creates excessive inventories and slows cycle times without creating additional throughput.

12. A systems approach to problem solving is one that looks at overall objectives and considers how elements relate

to each other and to the system as a whole.

13. Simulation is helps evaluate the performance of the system as a whole. It also helps make system improvements by encourages thinking in radical new ways (thinking outside the box). Simulation is also very helpful in the process of evaluating alternative solutions to insure that the best solution is obtained.

14. Analytical techniques like hand calculations or spreadsheets (even if they are not as powerful as simulation) are handy for solving simple systemic problems where little interdependency and variability are involved. An example might be determining the number of machines required to perform a standalone, parallel operation.

15. An analytical model may be used to provide an initial, ballpark estimate for the number of lift trucks needed. Simulation can then be used to fine-tune the estimate. After a simulation an analytical model can be used to validate the results of simulation by comparing the results of simulation with results obtained using the analytical model.

16. The advantages of simulation over the other techniques used in systems analysis are:
    a. Simulation enables the planner to accurately predict the expected performance of a system design (regardless of how complex it is), while other techniques tend to make oversimplifying assumptions.
    b. It provides a dynamic model where other techniques are predominately static calculations.
    c. It fully accounts for interdependencies and variability of systems.
    d. It is more visually appealing and convincing.
    e. It provides measures on all areas of performance.
    f. It is more versatile where other methods tend to be limited to a narrow class of problems.

17. The expected number of students waiting to receive help (L_q) is calculated by first identifying the arrival rate (λ), the service rate (μ), and (ρ).

$$\lambda = 6 \text{ per hour}$$
$$\mu = 8.57 \text{ per hour}$$
$$\rho = \lambda/\mu = .7$$

Solving for L_q

$$\begin{aligned} L_q &= \rho^2 / (1 - \rho) \\ &= .7^2 / (1 - .7) \\ &= .49 / .3 \\ &= 1.63 \text{ students} \end{aligned}$$

Solving for the expected waiting time in the queue (W_q) using Little's formula

$$\begin{aligned} W_q &= \lambda / L_q \\ &= 6 / 1.63 \\ &= 3.68 \text{ minutes} \end{aligned}$$

The percentage of time there were more than 2 students waiting is given by solving for the probability

$$P(n > 2) = 1 – P(n = 0) – P(n = 1) – P(n = 2)$$
$$P(n = 0) = (1 - .7) = .3$$
$$P(n = 1) = (1 - .7)\ .7 = .21$$
$$P(n = 2) = (1 - .7)\ .49 = .147$$
$$P(n > 2) = 1 – .657 = .343 \text{ or } 34.3\%$$

Chapter 3 – Solutions

1. A stochastic model is one in which one or more input variables are random and, therefore, the output is also random. For stochastic models, multiple replications are needed to obtain a statistically average response. A deterministic model is one that has no random input variables, so there is no randomness in the output. Consequently, only one replication is needed since the result is always the same.

2. An example of a discrete-change state variable is the number of customers in a queue. An example of a continuous-change state variable is the level of oil in an oil tanker that is being loaded.

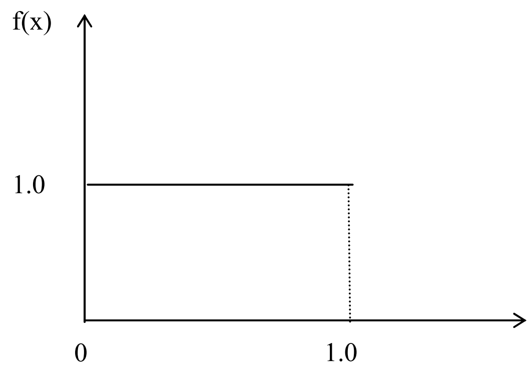
3. Discrete and continuous state variables for each system are as follows:
   a. Discrete: the presence of a tanker truck. Continuous: the oil level in a tank.
   b. Discrete: number of cases waiting to be shipped. Continuous: the beverage level in a tank.
   c. Discrete: the stop light (semaphore) color. Continuous: The acceleration of cars.

4. A random number generator produces uniformly distributed values between 0 and 1 such that $0 \leq x \leq 1$. The continuous uniform probability density function f(x), graph, mean, and variance are shown below.

$$f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{mean} = \mu = \frac{1}{2}$$

$$\text{variance} = \sigma^2 = \frac{1}{12}$$



5. The numbers produced by the random number generator must be (1) independent and (2) uniformly distributed between 0 and 1 as shown in number 4 above.

6.
   a. The fist LCG $\{Z_i = (9Z_{i-1} + 3) \bmod (32)\}$ will achieve its maximum cycle length of 32. The second LCG will not achieve the maximum cycle length because it violates the third condition given on page 66 of the textbook.
   b. LCG $Z_i = (12Z_{i-1}+5) \bmod (32)$ with $Z_0 = 29$

i	Z_i
0	29
1	1
2	17
3	17
4	17
5	17

7. A random variate is a sampled value or individual realization of a random variable. To generate a random variate, a random number is generated and plugged into a transform equation that converts the random number to a value that conforms to the probability distribution defining the random variable.

8. Applying the Inverse Transformation Method yields:

a.

$$x_i = (\beta - \alpha)U_i + \alpha$$
$$x_1 = (7-4)0.10 + 4 = 4.3$$
$$x_2 = (7-4)0.53 + 4 = 5.59$$
$$x_3 = (7-4)0.15 + 4 = 4.45$$

b.

$$x_1 = 2 \text{ for } U_1 = 0.10$$
$$x_2 = 4 \text{ for } U_2 = 0.53$$
$$x_3 = 2 \text{ for } U_3 = 0.15$$

9. A random number uniformly distributed between 0 and 1 is generated, if it is less than or equal to 0.12 the part is rejected, otherwise the part is accepted.

10. The spreadsheet is reproduced below.

Spreadsheet Simulation of Automatic Teller Machine (ATM)

	Arrivals to ATM			ATM Processing Time			ATM Simulation Logic						
	Stream 1	Random Number	Interarrival Time	Stream 2	Random Number	Service Time	Customer Number	Arrival Time	Begin Service Time	Service Time	Departure Time	Time in Queue	Time in System
i	*($Z1_i$)*	*($U1_i$)*	*($X1_i$)*	*($Z2_i$)*	*($U2_i$)*	*($X2_i$)*	(1)	(2)	(3)	(4)	(5)=(3)+(4)	(6)=(3)-(2)	(7)=(5)-(2)
0	29			92									
1	100	0.781	4.56	15	0.117	0.30	1	4.56	4.56	0.30	4.86	0.00	0.30
2	55	0.430	1.69	62	0.484	1.59	2	6.25	6.25	1.59	7.84	0.00	1.59
3	6	0.047	0.14	25	0.195	0.52	3	6.39	7.84	0.52	8.36	1.45	1.97
4	1	0.008	0.02	16	0.125	0.32	4	6.41	8.36	0.32	8.68	1.95	2.27
5	24	0.188	0.62	83	0.648	2.51	5	7.03	8.68	2.51	11.19	1.65	4.16
6	123	0.961	9.73	82	0.641	2.46	6	16.76	16.76	2.46	19.22	0.00	2.46
7	26	0.203	0.68	61	0.477	1.56	7	17.44	19.22	1.56	20.78	1.78	3.34
8	37	0.289	1.02	4	0.031	0.08	8	18.46	20.78	0.08	20.86	2.32	2.40
9	12	0.094	0.30	87	0.680	2.73	9	18.76	20.86	2.73	23.59	2.10	4.83
10	127	0.992	14.48	38	0.297	0.85	10	33.24	33.24	0.85	34.09	0.00	0.85
11	110	0.859	5.88	33	0.258	0.72	11	39.12	39.12	0.72	39.84	0.00	0.72
12	9	0.070	0.22	56	0.438	1.38	12	39.34	39.84	1.38	41.22	0.50	1.88
13	64	0.500	2.08	27	0.211	0.57	13	41.42	41.42	0.57	41.99	0.00	0.57
14	67	0.523	2.22	58	0.453	1.45	14	43.64	43.64	1.45	45.09	0.00	1.45
15	2	0.016	0.05	69	0.539	1.86	15	43.69	45.09	1.86	46.95	1.40	3.26
16	45	0.352	1.30	44	0.344	1.01	16	44.99	46.95	1.01	47.96	1.96	2.97
17	52	0.406	1.56	31	0.242	0.66	17	46.55	47.96	0.66	48.62	1.41	2.07
18	71	0.555	2.43	14	0.109	0.28	18	48.98	48.98	0.28	49.26	0.00	0.28
19	86	0.672	3.34	41	0.320	0.93	19	52.32	52.32	0.93	53.25	0.00	0.93
20	17	0.133	0.43	96	0.750	3.33	20	52.75	53.25	3.33	56.58	0.50	3.83
21	104	0.813	5.03	99	0.773	3.56	21	57.78	57.78	3.56	61.34	0.00	3.56
22	11	0.086	0.27	34	0.266	0.74	22	58.05	61.34	0.74	62.08	3.29	4.03
23	106	0.828	5.28	77	0.602	2.21	23	63.33	63.33	2.21	65.54	0.00	2.21
24	53	0.414	1.60	84	0.656	2.56	24	64.93	65.54	2.56	68.10	0.61	3.17
25	92	0.719	3.81	103	0.805	3.92	25	68.74	68.74	3.92	72.66	0.00	3.92
											Average	0.84	2.36

a. Average values in the spreadsheet match those given in Table 3.3 of the textbook.

b. By inspection, random number values for Stream 1 and Stream 2 in the above table are completely different than the values in Table 3.2 of the textbook. Yes, this is a requirement for a new replication. Note that the $Z1_0 = 29$ is the value of $Z2_{25}$ in Table 3.2. Thus, the random number generator starts the new replication above with the next number on its stream, which guarantees that the streams will not overlap in the two replications.

11. Given min = *a* and max = *b*, the resulting equation for generating variate x from the uniform (*a*, *b*) distribution is given by $x = a + (b - a)U$, where U is uniform (0, 1).

Given $a = 1$ and $b = 5$, the resulting equation for generating variate x from the uniform (1, 5) distribution is given by $x = 1 + (5 - 1)U$, where U is uniform (0, 1). This equation is used in the Excel spreadsheet under the Interarrival Time column within the Arrivals to ATM section. The spreadsheet is shown below.

Both the average time in queue and the average time in the system decreased with the interarrival time simulated using the uniform distribution in place of the exponential distribution. Although both the exponential distribution and the uniform distribution used for the interarrival time had the same mean value of 3.0 minutes, the different distributions produced different output for the system. A key to building valid simulation models is identifying the distribution that best describes the actual situation—in this case, the interarrival time pattern.

Spreadsheet Simulation of Automatic Teller Machine (ATM)

	Arrivals to ATM			ATM Processing Time			ATM Simulation Logic						
i	Stream 1 $(Z1_i)$	Random Number $(U1_i)$	Interarrival Time $(X1_i)$	Stream 2 $(Z2_i)$	Random Number $(U2_i)$	Service Time $(X2_i)$	Customer Number (1)	Arrival Time (2)	Begin Service Time (3)	Service Time (4)	Departure Time (5)=(3)+(4)	Time in Queue (6)=(3)-(2)	Time in System (7)=(5)-(2)
0	29			92									
1	100	0.781	4.12	15	0.117	0.30	1	4.12	4.12	0.30	4.42	0.00	0.30
2	55	0.430	2.72	62	0.484	1.59	2	6.84	6.84	1.59	8.43	0.00	1.59
3	6	0.047	1.19	25	0.195	0.52	3	8.03	8.43	0.52	8.95	0.40	0.92
4	1	0.008	1.03	16	0.125	0.32	4	9.06	9.06	0.32	9.38	0.00	0.32
5	24	0.188	1.75	83	0.648	2.51	5	10.81	10.81	2.51	13.32	0.00	2.51
6	123	0.961	4.84	82	0.641	2.46	6	15.65	15.65	2.46	18.11	0.00	2.46
7	26	0.203	1.81	61	0.477	1.56	7	17.46	18.11	1.56	19.67	0.65	2.21
8	37	0.289	2.16	4	0.031	0.08	8	19.62	19.67	0.08	19.75	0.05	0.13
9	12	0.094	1.38	87	0.680	2.73	9	21.00	21.00	2.73	23.73	0.00	2.73
10	127	0.992	4.97	38	0.297	0.85	10	25.97	25.97	0.85	26.82	0.00	0.85
11	110	0.859	4.44	33	0.258	0.72	11	30.41	30.41	0.72	31.13	0.00	0.72
12	9	0.070	1.28	56	0.438	1.38	12	31.69	31.69	1.38	33.07	0.00	1.38
13	64	0.500	3.00	27	0.211	0.57	13	34.69	34.69	0.57	35.26	0.00	0.57
14	67	0.523	3.09	58	0.453	1.45	14	37.78	37.78	1.45	39.23	0.00	1.45
15	2	0.016	1.06	69	0.539	1.86	15	38.84	39.23	1.86	41.09	0.39	2.25
16	45	0.352	2.41	44	0.344	1.01	16	41.25	41.25	1.01	42.26	0.00	1.01
17	52	0.406	2.62	31	0.242	0.66	17	43.87	43.87	0.66	44.53	0.00	0.66
18	71	0.555	3.22	14	0.109	0.28	18	47.09	47.09	0.28	47.37	0.00	0.28
19	86	0.672	3.69	41	0.320	0.93	19	50.78	50.78	0.93	51.71	0.00	0.93
20	17	0.133	1.53	96	0.750	3.33	20	52.31	52.31	3.33	55.64	0.00	3.33
21	104	0.813	4.25	99	0.773	3.56	21	56.56	56.56	3.56	60.12	0.00	3.56
22	11	0.086	1.34	34	0.266	0.74	22	57.90	60.12	0.74	60.86	2.22	2.96
23	106	0.828	4.31	77	0.602	2.21	23	62.21	62.21	2.21	64.42	0.00	2.21
24	53	0.414	2.66	84	0.656	2.56	24	64.87	64.87	2.56	67.43	0.00	2.56
25	92	0.719	3.88	103	0.805	3.92	25	68.75	68.75	3.92	72.67	0.00	3.92
											Average	0.15	1.67

12. Given $U_1 = 0.23$ and $U_2 = 0.61$ from a uniform (0, 1) distribution.

a. uniform (12, 20) continuous case

$a = \text{min} = 12$
$b = \text{max} = 20$

$x_i = a + (b - a)U_i$
$x_1 = 12 + (20 - 12)0.23 = 13.84$
$x_2 = 12 + (20 - 12)0.61 = 16.88$

b. triangular (12, 16, 20)

$a = \text{min} = 12$
$m = \text{mode} = 16$
$b = \text{max} = 20$

$$x_i = \begin{cases} a + \sqrt{(b-a)(m-a)U_i} & \text{for } 0 \le U_i \le \dfrac{m-a}{b-a} \\ b - \sqrt{(b-a)(b-m)(1-U_i)} & \text{for } \dfrac{m-a}{b-a} < U_i \le 1 \end{cases}$$

$x_1 = 12 + \sqrt{(20-12)(16-12)0.23} = 14.71 \quad \text{for } 0 \le U_1 \le 0.50$ case of $U_1 = 0.23$

$x_2 = 20 - \sqrt{(20-12)(20-16)(1-0.61)} = 16.47 \quad \text{for } 0.50 < U_2 \le 1$ case of $U_2 = 0.61$

c. normal (16, 10)

$\mu = 16$

$\sigma^2 = 10$

$$x_1 = \sqrt{-2\ln U_1}\cos 2\pi U_2$$

$$x_1 = \sqrt{-2\ln(0.23)}\cos 2\pi(0.61) = 1.71062$$

$$x_1^{'} = \mu + \sigma x_1$$

$$x_1^{'} = 16 + \sqrt{10}(1.71062) = 21.41 \text{ is a variate from normal } (16, 10) \text{ distribution}$$

and

$$x_2 = \sqrt{-2\ln U_1}\sin 2\pi U_2$$

$$x_2 = \sqrt{-2\ln(0.23)}\sin 2\pi(0.61) = 0.06684$$

$$x_2^{'} = \mu + \sigma x_2$$

$$x_2^{'} = 16 + \sqrt{10}(0.06684) = 16.21 \text{ is a variate from normal } (16, 10) \text{ distribution}$$

d. uniform (12, 20) discrete case

a = min = 12

b = max = 20

$$x_i = a + \lfloor (b - a + 1)\, U_i \rfloor$$

$$x_1 = 12 + \lfloor (20 - 12 + 1) 0.23 \rfloor = 12 + \lfloor 2.07 \rfloor = 12 + 2 = 14$$

$$x_2 = 12 + \lfloor (20 - 12 + 1) 0.61 \rfloor = 12 + \lfloor 5.49 \rfloor = 12 + 5 = 17$$

e. bernoulli (0.50)

$p = 0.50$

$$x_i = \begin{cases} 1 & \text{for } U_i \le p \\ 0 & \text{elsewhere} \end{cases}$$

$$x_i = \begin{cases} 1 & \text{for } U_i \le 0.50 \\ 0 & \text{elsewhere} \end{cases}$$

$x_1 = 1$ case of $U_1 = 0.23$

$x_2 = 0$ case of $U_2 = 0.61$

Chapter 4 – Solutions

1. In discrete-event simulation, it is necessary to know the completion time of an activity before it begins so that the completion time can be scheduled in the event calendar.

2. An example of an activity whose completion is a scheduled event is a machining or service activity for which depends on no other system dependencies so that the completion time is definable by some probability distribution. An example of an activity whose completion is a conditional event is an assembly operation which is completed once all of the component parts become available.

3. The manual discrete-event simulation table solution is in Appendix A at the end of these solution chapters. The appendix also contains a blank table that can be given to students to save them time in completing the exercise.

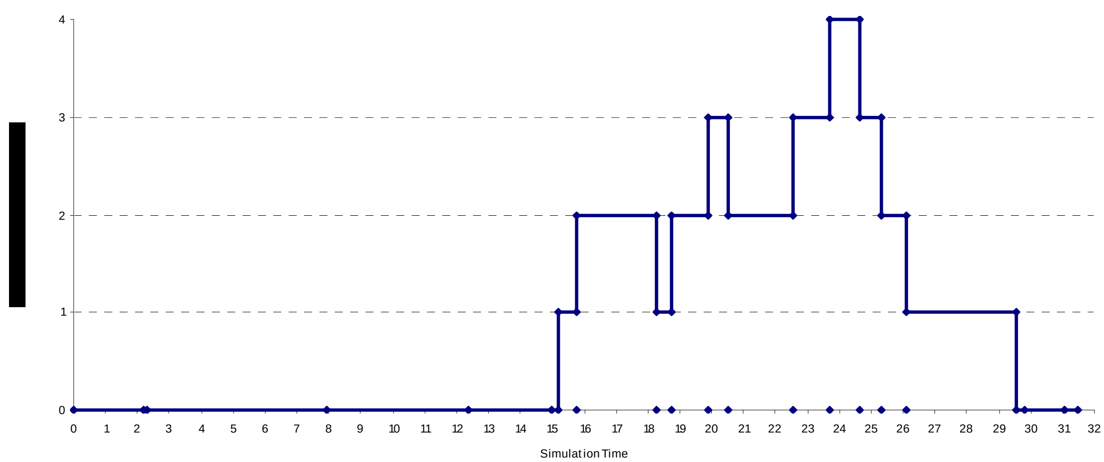
   a. Average time in system is a simple-average statistic. It is computed by taking the final cumulative time value under the Entities Processed Through System column and dividing by the total number of customers processed through the system (Total Processed).

   Average Time in System = 48.34 minutes / 10 customers = 4.834 minutes per customer.

   b. Average time in queue is a simple-average statistic. It is computed by taking the final cumulative time value under the Entities Processed Through Queue column and dividing by the total number of customers processed through the queue (Total Processed).

   Average Time in Queue = 28.54 minutes / 10 customers = 2.854 minutes per customer.

   c. The number of customers in queue plot follows:



The average number of customers in the queue is a time-average statistic (time-weighted statistic). It is computed by taking the final cumulative value under the Time-Weighted No. of Entities in Queue column and dividing by the simulation clock time at the end of the simulation (t_{20} = 31.49 minutes).

Average Number of Customers in Queue = 28.54 customers minutes / 31.49 minutes = 0.906319 customers