

1 Introduction

1.1 Equation (a) of the problem statement is used to solve for h as

$$h = \frac{\dot{Q}}{A(T - T_{\infty})} \quad (a)$$

The Principle of Dimensional Homogeneity is used to determine the dimensions of the heat transfer coefficient. Using the F-L-T system dimensions of the quantities in Equation (a) are

$$[\dot{Q}] = \left[\frac{F \cdot L}{T} \right] \quad (b)$$

$$[A] = [L^2] \quad (b)$$

$$[T - T_{\infty}] = [\Theta] \quad (c)$$

Thus from Equations (a)-(d) the dimensions of the heat transfer coefficient are

$$\begin{aligned} [h] &= \left[\frac{F \cdot L}{T \cdot \Theta \cdot L^2} \right] \\ &= \left[\frac{F}{T \cdot \Theta \cdot L} \right] \end{aligned} \quad (d)$$

Possible units for the heat transfer coefficient using the SI system are $\frac{N}{m \cdot s \cdot K}$ while

possible units using the English system are $\frac{lb}{ft \cdot s \cdot R}$.

1.2 The Reynolds number is defined as

$$Re = \frac{\rho V D}{\mu} \quad (a)$$

The dimensions of the quantities on the left-hand side of Equation (a) are obtained using Table 1.2 as

$$[\rho] = \left[\frac{M}{L^3} \right] \quad (b)$$

$$[V] = \left[\frac{L}{T} \right] \quad (c)$$

$$[D] = [L] \quad (d)$$

$$[\mu] = \left[\frac{M}{L \cdot T} \right] \quad (e)$$

Substituting Equations (b)-(e) in Equation (a) leads to

$$\begin{aligned}
 [\text{Re}] &= \left[\frac{\frac{\text{M}}{\text{L}^3} \cdot \frac{\text{L}}{\text{T}} \cdot \text{L}}{\frac{\text{M}}{\text{L} \cdot \text{T}}} \right] \\
 &= \left[\frac{\text{M} \cdot \text{L}^3 \cdot \text{T}}{\text{M} \cdot \text{L}^3 \cdot \text{T}} \right] \\
 &= [1]
 \end{aligned} \tag{f}$$

Equation (f) shows that the Reynolds number is dimensionless.

1.3 The capacitance of a capacitor is defined by

$$C = \frac{i}{\frac{dv}{dt}} \tag{a}$$

The dimension of i is that of electric current, which is a basic dimension. The dimensions of electric potential are obtained from Table 1.2 as

$$[v] = \left[\frac{\text{F} \cdot \text{L}}{\text{i} \cdot \text{T}} \right] \tag{b}$$

Thus the dimensions of the time rate of change of electric potential are

$$\left[\frac{dv}{dt} \right] = \left[\frac{\text{F} \cdot \text{L}}{\text{i} \cdot \text{T}^2} \right] \tag{c}$$

Use of Equation (c) in Equation (a) leads to

$$\begin{aligned}
 [C] &= \left[\frac{\text{i}}{\frac{\text{F} \cdot \text{L}}{\text{i} \cdot \text{T}^2}} \right] \\
 &= \left[\frac{\text{i}^2 \cdot \text{T}^2}{\text{F} \cdot \text{L}} \right]
 \end{aligned} \tag{d}$$

1.4 (a) The natural frequency of a mass-spring system is

$$\omega_n = \sqrt{\frac{k}{m}} \tag{a}$$

where m is mass with dimension $[\text{M}]$ and k is stiffness with dimensions in the M-L-T system of $\left[\frac{\text{M}}{\text{T}^2} \right]$. Thus the dimensions of natural frequency are

$$\begin{aligned}
 [\omega_n] &= \left[\left(\frac{\frac{M}{T^2}}{M} \right)^{\frac{1}{2}} \right] \\
 &= \left[\frac{1}{T} \right] \quad (b)
 \end{aligned}$$

(b) The natural frequency of the system is 100 Hz, which for calculations must be converted to r/s,

$$\begin{aligned}
 \omega_n &= 20 \frac{\text{cycles}}{\text{s}} \\
 &= \left(20 \frac{\text{cycles}}{\text{s}} \right) \left(2\pi \frac{\text{r}}{\text{cycles}} \right) \\
 &= 125.7 \frac{\text{r}}{\text{s}} \quad (c)
 \end{aligned}$$

Equation (a) is rearranged as

$$k = m\omega_n^2 \quad (d)$$

Substitution of known values into Equation (d) leads to

$$\begin{aligned}
 k &= (0.1 \text{ kg}) \left(125.7 \frac{\text{r}}{\text{s}} \right)^2 \\
 &= 1.58 \times 10^3 \frac{\text{N}}{\text{m}} \quad (e)
 \end{aligned}$$

1.5 (a) The mass of the carbon nanotube is calculated as

$$\begin{aligned}
 m &= \rho AL = \rho(\pi r^2)L \\
 &= \left(1300 \frac{\text{kg}}{\text{m}^3} \right) \pi (0.34 \times 10^{-9} \text{ m})^2 (80 \times 10^{-9} \text{ m}) \\
 &= 3.78 \times 10^{-23} \text{ kg}
 \end{aligned}$$

(b) Conversion between TPa and psi leads to

$$\begin{aligned}
 E &= 1.1 \text{ TPa} = 1.1 \times 10^{12} \frac{\text{N}}{\text{m}^2} \\
 &= \left(1.1 \times 10^{12} \frac{\text{N}}{\text{m}^2} \right) \left(0.225 \frac{\text{lb}}{\text{N}} \right) \left(\frac{1 \text{ m}}{3.28 \text{ ft}} \right)^2 \left(\frac{1 \text{ ft}}{12 \text{ in}} \right)^2 \\
 &= 1.60 \times 10^8 \frac{\text{lb}}{\text{in}^2}
 \end{aligned}$$

(c) Calculation of the natural frequency leads to

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$$\begin{aligned}
 \omega &= 22.37 \sqrt{\frac{EI}{\rho AL^4}} \\
 &= 22.37 \sqrt{\frac{\left(1.1 \times 10^{12} \frac{\text{N}}{\text{m}^2}\right) \frac{\pi}{4} (0.34 \times 10^{-9} \text{ m})^4}{\left(1300 \frac{\text{kg}}{\text{m}^3}\right) \pi (0.34 \times 10^{-9})^2 (80 \times 10^{-9} \text{ m})^4}} \\
 &= 1.73 \times 10^{10} \frac{\text{r}}{\text{s}}
 \end{aligned}$$

Converting to Hz gives

$$\begin{aligned}
 \omega &= \left(1.73 \times 10^{10} \frac{\text{r}}{\text{s}}\right) \left(\frac{1 \text{ cycle}}{2\pi \text{ r}}\right) \\
 &= 2.75 \times 10^9 \text{ Hz}
 \end{aligned}$$

1.6 The power of the motor is calculated as

$$\begin{aligned}
 P &= \frac{900 \text{ kW} \cdot \text{hr}}{24 \text{ hr}} \\
 &= 37.5 \text{ kW}
 \end{aligned} \tag{a}$$

The power is converted to English units using the conversions of Table 1.1

$$\begin{aligned}
 P &= 37.5 \times 10^3 \text{ W} \\
 &= 37.5 \times 10^3 \frac{\text{N} \cdot \text{m}}{\text{s}} \\
 &= 37.5 \times 10^3 \frac{\text{N} \left(\frac{0.225 \text{ lb}}{\text{N}}\right) \cdot \text{m} \left(\frac{3.28 \text{ ft}}{\text{m}}\right)}{\text{s}} \\
 &= 2.77 \times 10^4 \frac{\text{ft} \cdot \text{lb}}{\text{s}}
 \end{aligned} \tag{b}$$

Conversion to horsepower leads to

$$\begin{aligned}
 P &= 2.77 \times 10^4 \frac{\text{ft} \cdot \text{lb}}{\text{s}} \left(\frac{1 \text{ hp}}{550 \frac{\text{ft} \cdot \text{lb}}{\text{s}}}\right) \\
 &= 50.3 \text{ hp}
 \end{aligned} \tag{c}$$

1.7 The conversion of density from English units to SI units is

$$\begin{aligned}
 \rho &= 1.94 \frac{\text{slugs}}{\text{ft}^3} \\
 &= 1.94 \frac{\text{slugs}}{\text{ft}^3} \left(\frac{1 \text{ kg}}{0.00685 \text{ slugs}} \right) \left(\frac{3.28 \text{ ft}}{1 \text{ m}} \right)^3 \\
 &= 9.99 \times 10^3 \frac{\text{kg}}{\text{m}^3}
 \end{aligned} \tag{a}$$

1.8 The constant acceleration of the train is

$$a = -6 \frac{\text{m}}{\text{s}^2} \tag{a}$$

The velocity is obtained using Equation (a) as

$$v(t) = -6t + C \tag{b}$$

The constant of integration is evaluated by requiring

$$\begin{aligned}
 v(t = 0) &= 180 \frac{\text{km}}{\text{hr}} \\
 &= 180 \frac{\text{km}}{\text{hr}} \left(\frac{1000 \text{ m}}{\text{km}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) \\
 &= 50 \frac{\text{m}}{\text{s}}
 \end{aligned} \tag{c}$$

Using Equation (c) in Equation (b) leads to

$$v(t) = -6t + 50 \frac{\text{m}}{\text{s}} \tag{d}$$

The train stops when its velocity is zero,

$$\begin{aligned}
 0 &= -6t + 50 \\
 t &= 8.33 \text{ s}
 \end{aligned} \tag{e}$$

The distance traveled is obtained by integrating Equation (d) and assuming $x(0)=0$, leading to

$$x(t) = -3t^2 + 50t \tag{f}$$

The distance traveled before the train stops is

$$\begin{aligned}
 x(8.33) &= -3(8.33)^2 + 50(8.33) \\
 &= 208.3 \text{ m}
 \end{aligned} \tag{g}$$

1.9 The differential equation for the angular velocity of a shaft is

$$J \frac{d\omega}{dt} + c_t \omega = T \tag{a}$$

Each term in Equation (a) has the same dimensions, those of torque or $[F \cdot L]$. The dimensions of angular velocity are $\left[\frac{1}{T} \right]$. Thus the dimensions of c_t are

$$[c_t] = \left[\frac{F \cdot L}{\frac{1}{T}} \right] = [F \cdot L \cdot T] \quad (b)$$

1.10 The equation for the torque applied to the armature is

$$T = K_a i_a i_f \quad (a)$$

Equation (a) is rearranged as

$$K_a = \frac{T}{i_a i_f} \quad (b)$$

The dimensions of torque are $[F \cdot L]$ thus the dimensions of the constant are

$$[K_a] = \left[\frac{F \cdot L}{i^2} \right] \quad (c)$$

The equation for the back emf is

$$v = K_v i_f \omega \quad (d)$$

Equation (d) is rearranged as

$$K_v = \frac{v}{i_f \omega} \quad (e)$$

The dimensions of voltage are $\left[\frac{F \cdot L}{i \cdot T} \right]$ and the dimensions of angular velocity are $\left[\frac{1}{T} \right]$.

The dimensions of the constant K_v are

$$[K_v] = \left[\frac{\frac{F \cdot L}{i \cdot T}}{i \frac{1}{T}} \right] = \left[\frac{F \cdot L}{i^2} \right] \quad (f)$$

It is clear from Equations (c) and (f) that the dimensions of $[K_a]$ and $[K_v]$ are the same.

These dimensions are the same as those of inductance (Table 1.2).

1.11 (a) The dimensions of $\dot{Q}$ are determined from Equation (a)

$$\dot{Q} = \sigma A \varepsilon (T^4 - T_b^4) \quad (a)$$

$$\left[\frac{F \cdot L}{L^2 \cdot T \cdot \Theta^4} \right] [L^2] [\Theta^4] = \left[\frac{F \cdot L}{T} \right] \quad (b)$$

(b) The differential equations governing the temperature in the body is

$$\rho c \frac{dT}{dt} + \sigma \varepsilon (T^4 - T_b^4) = 0 \quad (c)$$

The perturbation in temperature in the radiating body is defined by

$$T_b = T_{bs} + T_{b1} \quad (d)$$

This leads to a perturbation in the temperature of the receiving body defined as

$$T = T_s + T_1 \quad (e)$$

Substitution of equations (d) and (e) in Equation (c) leads to

$$\rho c \frac{d}{dt}(T_s + T_1) + \sigma \varepsilon \left[(T_s + T_1)^4 - (T_{bs} + T_{b1})^4 \right] = 0 \quad (f)$$

Simplifying Equation (f) gives

$$\rho c \frac{dT_1}{dt} + \sigma \varepsilon \left[T_s^4 \left(1 + \frac{T_1}{T_s} \right)^4 - T_{bs}^4 \left(1 + \frac{T_{b1}}{T_{bs}} \right)^4 \right] = 0 \quad (g)$$

Expanding the nonlinear terms, keeping only through the linear terms and noting that $T_s = T_{bs}$

$$\begin{aligned} \rho c \frac{dT_1}{dt} + \sigma \varepsilon \left[T_s^4 \left(4 \frac{T_1}{T_s} \right) - T_{bs}^4 \left(4 \frac{T_{b1}}{T_{bs}} \right) \right] &= 0 \\ \rho c \frac{dT_1}{dt} + 4\sigma \varepsilon T_s^3 T_1 &= 4\sigma \varepsilon T_{bs}^3 T_{b1} \end{aligned} \quad (h)$$

1.12 The differential equation is linearized by using the small angle assumption which implies $\sin \theta \approx \theta$ and $\cos \theta \approx 1$. Using these approximations in the differential equation leads to the linearized approximation as

$$\frac{1}{3} mL^2 \ddot{\theta} + \frac{1}{4} cL^2 \dot{\theta} + kL^2 \theta = 0 \quad (a)$$

1.13 The differential equation is linearized by using the small angle assumption which implies $\sin \theta \approx \theta$ and $\cos \theta \approx 1$. Using these approximations in the differential equation leads to the linearized approximation as

$$\frac{1}{3} mL^2 \ddot{\theta} + \left(mg \frac{L}{2} + \ddot{y} \right) \theta = L \ddot{x} \quad (a)$$

1.14 The nonlinear differential equations governing the concentration of the reactant and temperature are

$$V \frac{dC_A}{dt} + (q + \alpha V e^{-E/(RT)}) C_A = q C_{Ai} \quad (a)$$

$$\rho q c_p T_i - \rho q c_p T - \dot{Q} + \lambda V \alpha e^{-E/(RT)} C_A = \rho V c_p \frac{dT}{dt} \quad (b)$$

The reactor is operating at a steady-state when a perturbation in flow rate occurs according to

$$q = q_s + q_p(t) \quad (c)$$

The flow rate perturbation induces perturbations in concentration and temperature according to

$$C_A = C_{As} + C_{Ap}(t) \quad (d)$$

$$T = T_s + T_p(t) \quad (e)$$

The steady-state conditions are defined by setting time derivatives to zero in Equation (a) leading to

$$(q_s + \alpha V e^{-E/(RT_s)}) C_{As} = q_s C_{Ai} \quad (f)$$

$$\rho q c_p T_i - \rho q_s c_p T_s - \dot{Q} + \lambda V \alpha e^{-E/(RT_s)} C_{As} = 0 \quad (g)$$

Substitution of Equations (d) and (e) into Equations (a) and (b) leads to

$$V \frac{dC_{Ap}}{dt} + (q_s + q_p + \alpha V e^{-E/[R(T_s+T_p)]}) (C_{As} + C_{Ap}) = (q_s + q_p) C_{Ai} \quad (h)$$

$$\rho(q_s + q_p) c_p T_i - \rho(q_s + q_p) c_p (T_s + T_p) - \dot{Q} + \lambda V \alpha e^{-E/[R(T_s+T_p)]} (C_{As} + C_{Ap}) = \rho V c_p \frac{dT_p}{dt} \quad (i)$$

It is noted from Equation (f) of Example (1.6) that a linearization of the exponential terms in Equations (h) and (i) is

$$e^{-\frac{E}{R(T_s+T_p)}} = e^{-\frac{E}{RT_s}} + \frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p \quad (j)$$

Use of Equation (j) in Equations (h) and (i) and rearrangement leads to

$$V \frac{dC_{Ap}}{dt} + \left[q_s + q_p + \alpha V \left(e^{-\frac{E}{RT_s}} + \frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p \right) \right] (C_{As} + C_{Ap}) = (q_s + q_p) C_{Ai} \quad (k)$$

$$\begin{aligned} \rho(q_s + q_p) c_p T_i - \rho(q_s + q_p) c_p (T_s + T_p) - \dot{Q} + \lambda V \alpha \left[e^{-\frac{E}{RT_s}} + \frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p \right] (C_{As} + C_{Ap}) \\ = \rho V c_p \frac{dT_p}{dt} \end{aligned} \quad (l)$$

Equations (g) and (h) are used to simplify Equations (k) and (l) to

$$\begin{aligned} V \frac{dC_{Ap}}{dt} + q_s C_{Ap} + q_p C_{As} + q_p C_{Ap} + \alpha V e^{-\frac{E}{RT_s}} C_{Ap} + \alpha V \left(\frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p \right) (C_{As} + C_{Ap}) \\ = q_p C_{Ai} \end{aligned} \quad (m)$$

$$\begin{aligned} \rho q_p c_p T_i - \rho c_p (q_p T_s + q_s T_p + q_p T_p) + \lambda V \alpha e^{-\frac{E}{RT_s}} C_{Ap} + \lambda V \alpha \left[\frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p \right] (C_{As} + C_{Ap}) \\ = \rho V c_p \frac{dT_p}{dt} \end{aligned} \quad (n)$$

Neglecting products of perturbations Equations (m) and (n) are rearranged as

$$V \frac{dC_{Ap}}{dt} + q_s C_{Ap} + q_p C_{Ap} + \alpha V e^{-\frac{E}{RT_s}} C_{Ap} + \alpha V \left(\frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p \right) C_{As} = q_p C_{Ai} - q_p C_{As} \quad (o)$$

$$\rho V c_p \frac{dT_p}{dt} - \rho q_p c_p T_i + \rho c_p (q_p T_s + q_s T_p) + \lambda V \alpha e^{-\frac{E}{RT_s}} C_{Ap} + \lambda V \alpha \left[\frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p \right] C_{As} = 0 \quad (p)$$

1.15 The specific heat is related to temperature by

$$c_p = A_1 + A_2 T^{1.5} + A_3 T^{2.6} \quad (a)$$

The transient temperature is the steady-state temperature plus a perturbation,

$$T = T_s + T_p \quad (b)$$

Substituting Equation (b) into Equation (a) leads to

$$\begin{aligned} c_p &= A_1 + A_2 (T_s + T_p)^{1.5} + (T_s + T_p)^{2.6} \\ &= A_1 + A_2 T_s^{1.5} \left(1 + \frac{T_p}{T_s} \right)^{1.5} + A_3 T_s^{2.6} \left(1 + \frac{T_p}{T_s} \right)^{2.6} \end{aligned} \quad (c)$$

Using the binominal expansion to linearize Equation (c) leads to

$$c_p = A_1 + A_2 T_s^{1.5} \left(1 + 1.5 \frac{T_p}{T_s} \right) + A_3 T_s^{2.6} \left(1 + 2.6 \frac{T_p}{T_s} \right) \quad (d)$$

The differential equation for the time-dependent temperature is

$$c_p \frac{dT}{dt} + \frac{1}{R} T = \frac{1}{R} T_\infty \quad (e)$$

Substituting Equations (b) and (d) into Equation (e) along with $T_\infty = T_{\infty s} + T_{\infty p}$ leads to

$$\left[A_1 + A_2 T_s^{1.5} \left(1 + 1.5 \frac{T_p}{T_s} \right) + A_3 T_s^{2.6} \left(1 + 2.6 \frac{T_p}{T_s} \right) \right] \frac{d}{dt} (T_s + T_p) + \frac{1}{R} (T_s + T_p) = \frac{1}{R} (T_{\infty s} + T_{\infty p}) \quad (f)$$

Noting that the steady-state is defined by $\frac{dT_s}{dt} = 0$ and $T_s = T_{\infty s}$ reduces Equation (f) to

$$\left[A_1 + A_2 T_s^{1.5} \left(1 + 1.5 \frac{T_p}{T_s} \right) + A_3 T_s^{2.6} \left(1 + 2.6 \frac{T_p}{T_s} \right) \right] \frac{dT_p}{dt} + \frac{1}{R} T_p = \frac{1}{R} T_{\infty p} \quad (g)$$

Terms such as $T_p \frac{dT_p}{dt}$ are nonlinear. Equation (g) is linearized by noting that $\left| \frac{T_p}{T_s} \right| \ll 1$

$$(A_1 + A_2 T_s^{1.5} + A_3 T_s^{2.6}) \frac{dT_p}{dt} + \frac{1}{R} T_p = \frac{1}{R} T_{\infty p} \quad (h)$$

1.16 The force acting on the piston at any instant is

$$F = pA \quad (a)$$

where A is the area of the piston head. The pressure is related to the density by

$$p = C \rho^\gamma \quad (b)$$

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The mass of air in the cylinder is constant and is calculated when the piston is in equilibrium as

$$m = \rho_0 Ah \quad (c)$$

where ρ_0 is the density of the air in equilibrium. Using Equation (b) in Equation (c) leads to

$$m = \left(\frac{p_0}{C} \right)^{\frac{1}{\gamma}} Ah \quad (d)$$

where p_0 is the pressure in the cylinder when the piston is in equilibrium. At any instant the mass is calculated as

$$\begin{aligned} m &= \rho A(h - x) \\ &= \left(\frac{p}{C} \right)^{\frac{1}{\gamma}} A(h - x) \end{aligned} \quad (e)$$

Since the mass is constant, Equations (d) and (e) are equated leading to

$$p = p_0 \left(\frac{h}{h - x} \right)^{\gamma} \quad (f)$$

Substitution of Equation (f) into Equation (a) leads to

$$F = p_0 A \left(\frac{h}{h - x} \right)^{\gamma} \quad (g)$$

(b) Equation (g) is rearranged as

$$F = p_0 A \left(1 - \frac{x}{h} \right)^{-\gamma} \quad (h)$$

Since $\frac{x}{h} < 1$ a binomial expansion can be used on the right-hand side of Equation (h).

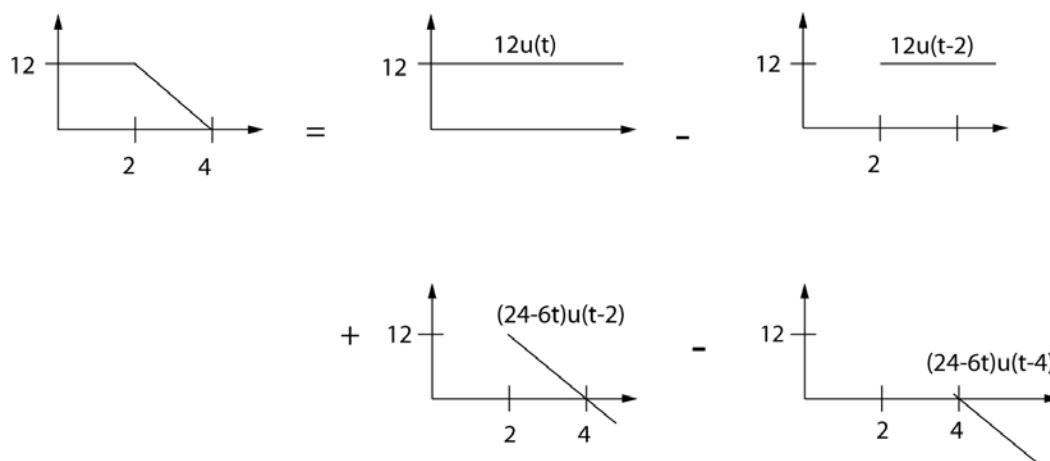
Using the binomial expansion keeping only through the linear term leads to

$$F = p_0 A + \frac{\gamma p_0 A}{h} x \quad (e)$$

The linear stiffness is obtained from Equation (e) as

$$k = \frac{\gamma p_0 A}{h} \quad (f)$$

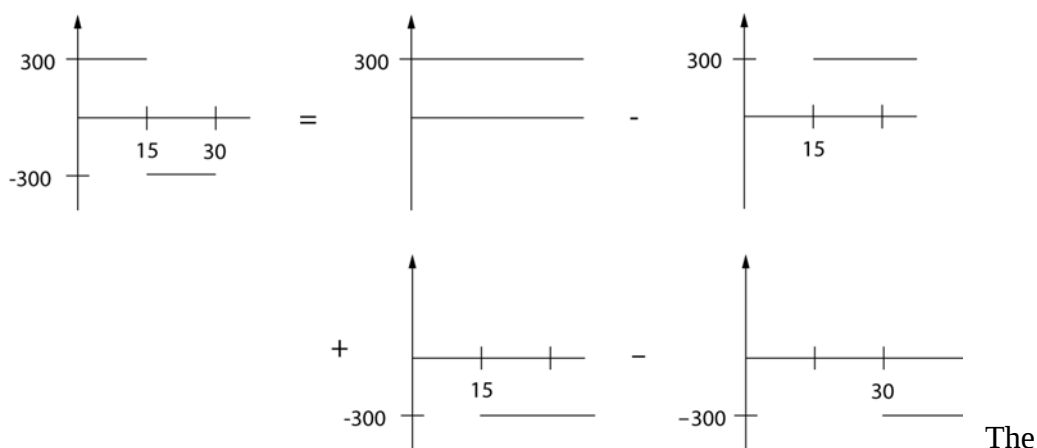
1.17 The appropriate superposition of the voltage in Figure P1.17 is illustrated below



The mathematical representation of the voltage source is

$$v(t) = 12[u(t) - u(t-2)] + (24-6t)[u(t-2) - u(t-4)]$$

1.18 The superposition of the force of Figure P1.18 is illustrated below.

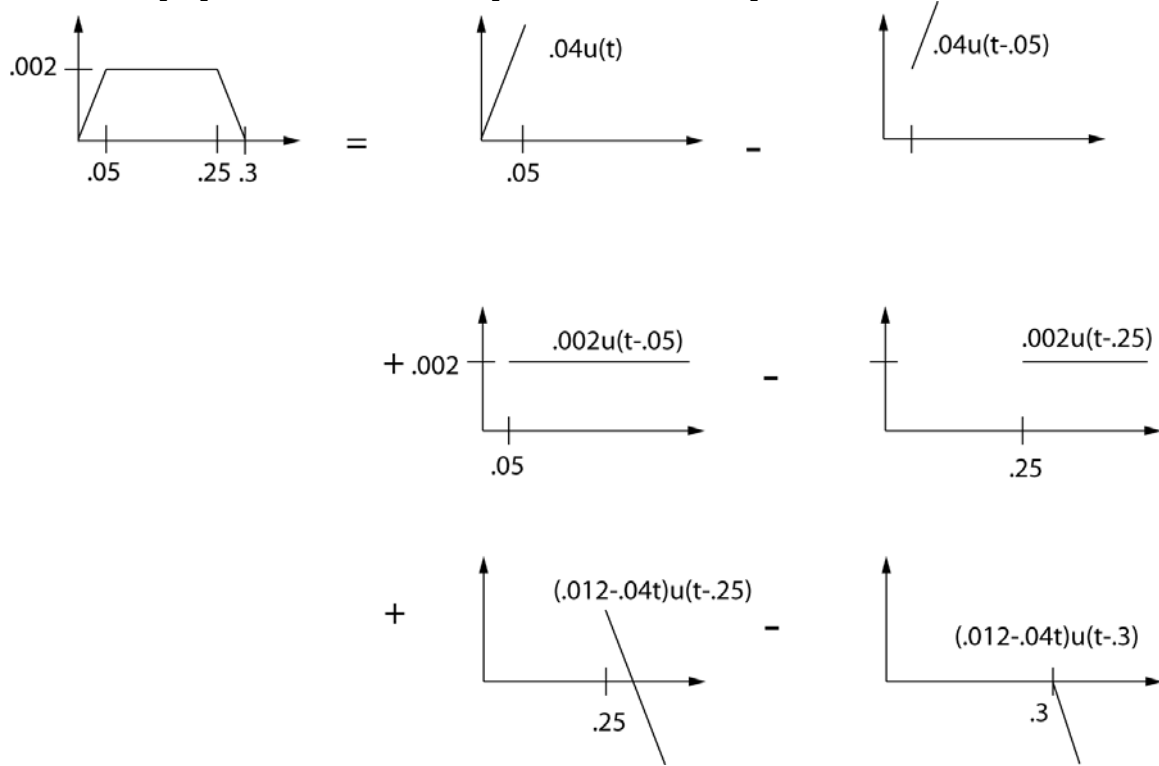


The mathematical representation of the force is

$$F(t) = 300[u(t) - u(t-15)] - 300[u(t-15) - u(t-30)] \quad (a)$$

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1.19 The superposition of the cam displacement over one period is shown below



(a) The mathematical representation of the displacement over one period is

$$x(t) = 0.04[u(t) - u(t - 0.05)] + 0.002[u(t - 0.05) - u(t - 0.25)] + (0.012 - 0.04t)[u(t - 0.25) - u(t - 0.3)] \quad (\text{a})$$

(b) The period of the cycle is 0.5 s. Thus the displacement over the second period is obtained by replacing t by $t+0.5$ in Equation (a). The displacement over the k th period is obtained by replacing t by $t+(k-1)(0.5)$ in Equation (a). The total displacement is obtained by summing over all periods

$$x(t) = \sum_{k=1}^K \{0.04[u(t - .5k + .5) - u(t - .5k + .45)] + .002[u(t - 0.5k + .45) - u(t - .5k + .25)] + (0.02k - 0.008 - 0.04t)[u(t - 0.5k + .25) - u(t - 0.5k + 0.2)]\} \quad (\text{b})$$

where K is the smallest integer greater than $t/(0.05)$.

1.20 Integration of Newton's second law with respect to time leads to the principle of impulse and momentum

$$I = \int_{t_1}^{t_2} F dt = m(v_2 - v_1) \quad (\text{a})$$

where the total impulse applied between t_1 and t_2 is $\int_{t_1}^{t_2} F dt$. The 12 N·s impulse is

applied instantaneously to the 4-kg particle when it is at rest. Application of the principle of impulse and momentum leads to

$$\begin{aligned}
 12 \text{ N} \cdot \text{s} &= (4 \text{ kg})v_2 \\
 v_2 &= \frac{12 \text{ N} \cdot \text{s}}{4 \text{ kg}} \\
 &= 3 \frac{\text{m}}{\text{s}}
 \end{aligned} \tag{b}$$

1.21 The equation for the voltage drop across an inductor is

$$v = L \frac{di}{dt} \tag{a}$$

Integration of Equation (a) with respect to time leads to

$$\int_0^t v dt = L(i_2 - i_1) \tag{b}$$

The initial current is zero. Solving Equation (b) for i_2 leads to

$$\begin{aligned}
 i_2 &= \frac{\int_0^t v dt}{L} \\
 &= \frac{20 \text{ V} \cdot \text{s}}{0.4 \text{ H}} \\
 &= 50 \text{ A}
 \end{aligned} \tag{c}$$

1.22 The mathematical representation of the force is

$$F(t) = 100\delta(t) + 150\delta(t - 2.5) + 50\delta(t - 3.8) \tag{a}$$

1.23 The MATLAB file Problem1_23 which determines the steady-state response of a series LRC circuit is listed below

```

% Problem1_23.m
% Steady-state response of series LRC circuit
clear
disp('Steady-state response of series LRC circuit')
% Input parameters
disp('Input resistance in ohms')
R=input('>> ')
disp('Input capacitance in farads')
C=input('>> ')
disp('Input inductance in henrys')
L=input('>> ')
disp('Input source frequency in r/s')
om=input('>> ')
disp('Input source amplitude in V')
V0=input('>> ')
% Calculates parameters
disp('Natural frequency in r/s =')
```

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```
omn=1/(L*C)^0.5
disp('Dimensionless damping ratio =')
zeta=R/2*(C/L)^0.5
disp('Phase angle in rad=')
C1=om^2-omn^2;
C2=2*zeta*om*omn;
phi=atan2(C1,C2)
disp('Steady-state amplitude in A =')
C3=V0*om/L;
C4=1/(C1^2+C2^2)^0.5;
I=C3*C4
tf=10*pi/om;
dt=tf/200;
for k=1:201
    t(k)=(k-1)*dt;
    i(k)=I*sin(om*t(k)+phi);
end
plot(t,i)
xlabel('t (s)')
ylabel('i (A)')
title('Steady-state response of series LRC circuit')
str1=['R=',num2str(R),' \Omega'];
str2=['C=',num2str(C),' F'];
str3=['L=',num2str(L),' H'];
str4=['\omega=',num2str(om),' r/s'];
str5=['V_0=',num2str(V0),' V'];
text(0.9*tf,I,str1)
text(0.9*tf,0.8*I,str2)
text(0.9*tf,0.6*I,str3)
text(0.9*tf,0.4*I,str4)
text(0.9*tf,0.2*I,str5)
```

The MATLAB workspace from a sample execution of Problem1_23.m is

```
>> Problem1_23
Steady-state response of series LRC circuit
Input resistance in ohms
>> 100

R =

    100

Input capacitance in farads
>> 0.2e-6
```

C =

2.0000e-007

Input inductance in henrys

>> 0.5

L =

0.5000

Input source frequency in r/s

>> 2000

ω =

2000

Input source amplitude in V

>> 120

V0 =

120

Natural frequency in r/s =

ω_n =

3.1623e+003

Dimensionless damping ratio =

ζ =

0.0316

Phase angle in rad=

ϕ =

-1.5042

Steady-state amplitude in A =

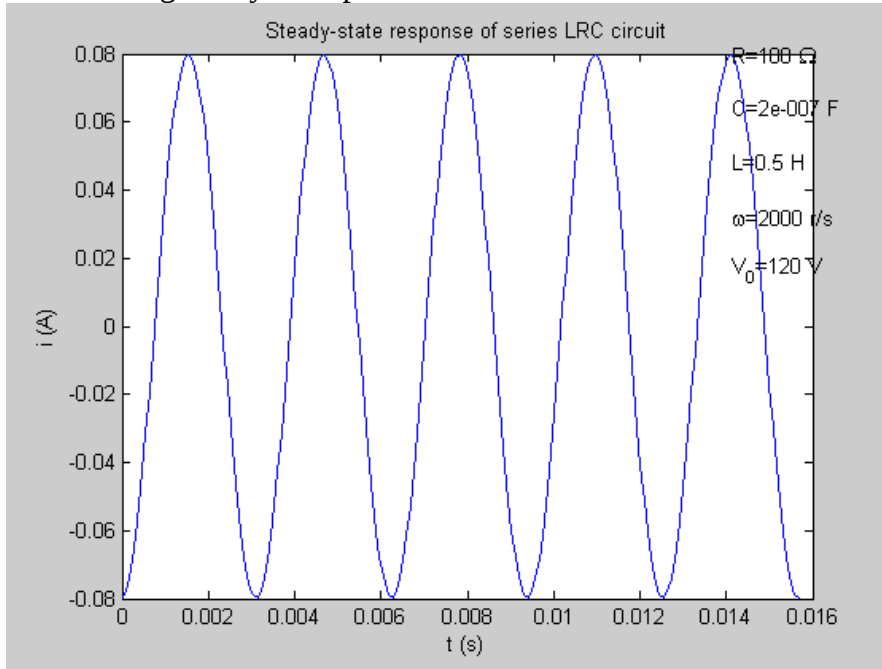
I =

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0.0798

>>

The resulting steady-state plot is



1.24 The MATABLB file Prolbem1_24.m is listed below

```
% Problem1_24.m
%(a) Input two five by five matrices
disp('Please input matrix A by row')
for i=1:5
    for j=1:5
        str={'Enter A(',num2str(i),num2str(j),')'};
        disp(str)
        A(i,j)=input('>> ');
    end
end
disp('Please input matrix B by row')
for i=1:5
    for j=1:5
        str={'Enter B(',num2str(i),num2str(j),')'};
        disp(str)
        B(i,j)=input('>> ');
    end
end
end
A
```

```

B
% (b) =A+B
C=A+B
% (c) D=A*B
D=A*B
% (d) det(A)
detA=det(A)
% eigenvalues and eigenvectors of A
[x,Y]=eigs(A);
disp('Eigenvalues of A')
Y
disp('Matrix of eigenvalues of A')
x

```

A sample output from execution of the file is shown below

```

>> clear
>> Problem1_24
Please input matrix A by row
'Enter A(11)'

>> 1
'Enter A(12)'

>> 0
'Enter A(13)'

>> 12
'Enter A(14)'

>> -1
'Enter A(15)'

>> 21
'Enter A(21)'

>> 14
'Enter A(22)'

>> -3
'Enter A(23)'

>> 2
'Enter A(24)'

>> 0

```

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```
'Enter A(25)'
>> -22
'Enter A(31)'
>> 11
'Enter A(32)'
>> 12
'Enter A(33)'
>> 10
'Enter A(34)'
>> -4
'Enter A(35)'
>> 12
'Enter A(41)'
>> 10
'Enter A(42)'
>> 11
'Enter A(43)'
>> 18
'Enter A(44)'
>> 12
'Enter A(45)'
>> 21
'Enter A(51)'
>> 10
'Enter A(52)'
>> 11
'Enter A(53)'
>> 31
'Enter A(54)'
>> 21
'Enter A(55)'
```

```
>> 11
Please input matrix B by row
'Enter B(11)'

>> 21
'Enter B(12)'

>> -21
'Enter B(13)'

>> 21
'Enter B(14)'

>> 10
'Enter B(15)'

>> 9
'Enter B(21)'

>> 8
'Enter B(22)'

>> 2
'Enter B(23)'

>> 2
'Enter B(24)'

>> 4
'Enter B(25)'

>> -5
'Enter B(31)'

>> 16
'Enter B(32)'

>> 12
'Enter B(33)'

>> 11
'Enter B(34)'

>> 18
'Enter B(35)'
```

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```
>> 11  
    'Enter B(41)'
```

```
>> 21  
    'Enter B(42)'
```

```
>> 32  
    'Enter B(43)'
```

```
>> 14  
    'Enter B(44)'
```

```
>> 19  
    'Enter B(45)'
```

```
>> 12  
    'Enter B(51)'
```

```
>> 12  
    'Enter B(52)'
```

```
>> 9  
    'Enter B(53)'
```

```
>> -5  
    'Enter B(54)'
```

```
>> 13  
    'Enter B(55)'
```

```
>> 21
```

```
A =
```

```
  1   0  12  -1  21  
 14  -3   2   0 -22  
 11  12  10  -4  12  
 10  11  18  12  21  
 10  11  31  21  11
```

B =

21	-21	21	10	9
8	2	2	4	-5
16	12	11	18	11
21	32	14	19	12
12	9	-5	13	21

C =

22	-21	33	9	30
22	-1	4	4	-27
27	24	21	14	23
31	43	32	31	33
22	20	26	34	32

D =

444	280	34	480	570
38	-474	420	-122	-299
547	-107	249	418	353
1090	601	493	969	818
1367	955	812	1244	859

detA =

-1171825

Eigenvalues of A

Y =

43.3949	0	0	0	0
0	-18.9247	0	0	0
0	0	-1.8896 - 11.6121i	0	0
0	0	0	-1.8896 + 11.6121i	0
0	0	0	0	10.3091

Matrix of eigenvalues of A

x =

-0.3664	-0.1632	-0.3379 - 0.4681i	-0.3379 + 0.4681i	0.2658
---------	---------	-------------------	-------------------	--------

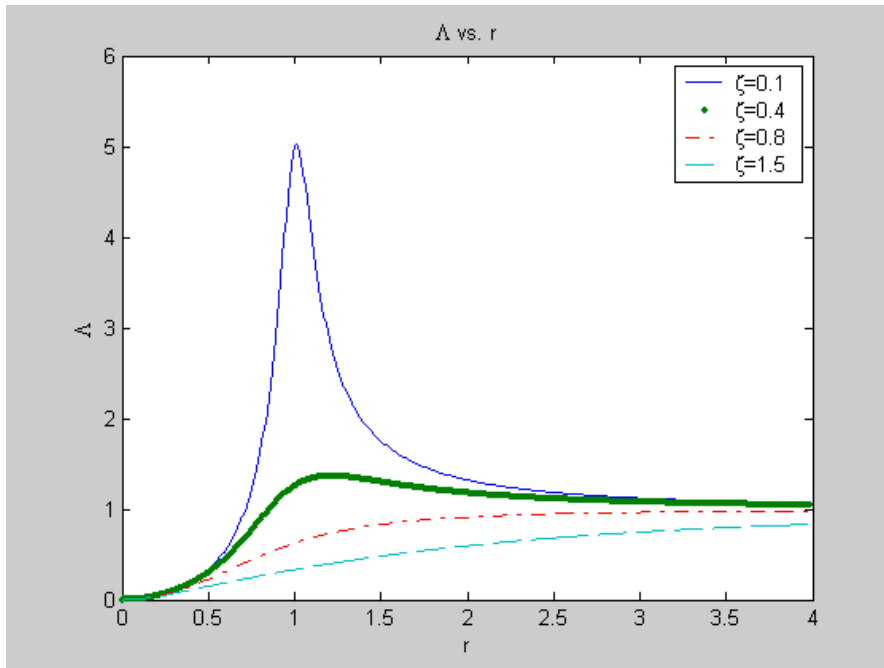
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0.1869	0.7510	0.7187	0.7187	-0.4106
-0.2133	-0.4463	0.0528 + 0.2510i	0.0528 - 0.2510i	-0.4042
-0.6060	-0.2256	-0.0845 - 0.0753i	-0.0845 + 0.0753i	0.6726
-0.6466	0.3991	-0.2465 + 0.1043i	-0.2465 - 0.1043i	0.3808

1.25 A MATLAB file to calculate and plot $\Lambda(r, \zeta)$ is given below

```
% Plots the function LAMBDA(r,zeta) as a function of r for several values of
% zeta
% Specify four values of zeta
zeta1=0.1;
zeta2=0.4;
zeta3=0.8;
zeta4=1.5;
% Define values of r for calculations
for i=1:400
    r(i)=(i-1)*.01;
% Calculate function
LAMBDA1(i)=r(i)^2/((1-r(i)^2)^2+(2*zeta1*r(i))^2)^0.5;
LAMBDA2(i)=r(i)^2/((1-r(i)^2)^2+(2*zeta2*r(i))^2)^0.5;
LAMBDA3(i)=r(i)^2/((1-r(i)^2)^2+(2*zeta3*r(i))^2)^0.5;
LAMBDA4(i)=r(i)^2/((1-r(i)^2)^2+(2*zeta4*r(i))^2)^0.5;
end
plot(r,LAMBDA1,'-r,LAMBDA2','r,LAMBDA3','-r,LAMBDA4,'--')
xlabel('r')
ylabel('\Lambda')
str1=['\zeta=',num2str(zeta1)];
str2=['\zeta=',num2str(zeta2)];
str3=['\zeta=',num2str(zeta3)];
str4=['\zeta=',num2str(zeta4)];
legend(str1,str2,str3,str4)
title('\Lambda vs. r')
```

The resulting output from execution of the .m file is the following plot



1.26 The MATABL .m file Problem1_26 which determines and plots the step response of an underdamped mechanical system is shown below.

```
% Problem1_26.m
% Step response of an underdamped mechanical system
% Input natural frequency and damping ratio
clear
disp('Step response of underdamped mechanical system')
disp('Please input natural frequency in r/s')
om=input('>> ')
disp('Please input the dimensionless damping ratio')
zeta=input('>> ')
% Damped natural frequency
omd=om*(1-zeta^2)^0.5;
C1=zeta*om/omd;
C2=1/om^2;
C3=zeta*om;
tf=10*pi/omd;
dt=tf/500;
for i=1:501
t(i)=(i-1)*dt;
x(i)=C2*(1-exp(-C3*t(i)))*(C1*sin(omd*t(i))-cos(omd*t(i)));
end
plot(t,x)
xlabel('t (s)')
ylabel('x (m)')
```

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```
str1=['Step response of underdamped mechanical system with  
\omega_n=',num2str(om),'and \zeta=',num2str(zeta)]  
title(str1)  
str2=['x(t)=',num2str(C2),'[1-e^-^',num2str(C3),'^t(',num2str(C1),'sin(',num2str(omd),'t)-  
cos(',num2str(omd),'t))']']  
text(tf/4,C2/2,str2)
```

Output from execution of Problem1_26 follows

```
>> Problem1_26  
Step response of underdamped mechanical system  
Please input natural frequency in r/s  
>> 100
```

om =

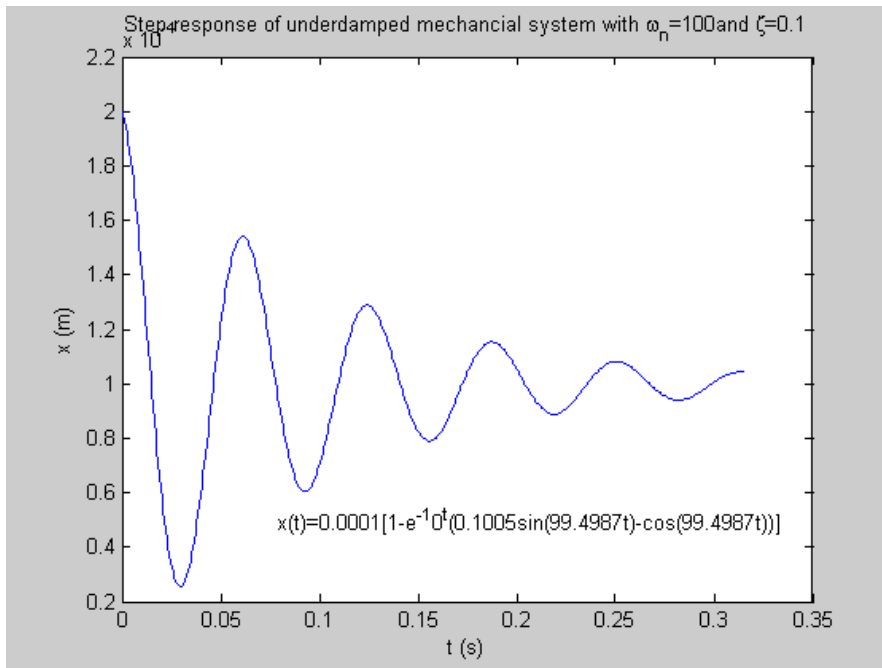
100

Please input the dimensionless damping ratio

```
>> 0.1
```

zeta =

0.1000



1.27 The perturbation in liquid level is

$$h(t) = qR(1 - e^{-t/(RA)}) \quad (a)$$

(a) Since the argument of a transcendental function must be dimensionless the dimensions of the product of resistance and area must be time. Thus the dimensions of resistance must be $\left[\frac{\mathbf{T}}{\mathbf{L}^2} \right]$

(b) Note that the steady-state value of the liquid-level perturbation is qR . The MATLAB file Problem1_27.m which calculates and plots $h(t)$ from $t=0$ until h is within 1 percent of its steady-state value is given below

```
disp('Please enter resistance in s/m^2 ')
R=input('>> ')
% Final value of h
hf=0.99*q*R;
dt=0.01*R*A;
h1=0;
h(1)=0;
t(1)=0;
i=1;
while h1<hf
    i=i+1;
    t(i)=t(i-1)+dt;
    h(i)=q*R*(1-exp(-t(i)/(R*A)));
    h1=h(i);
end
plot(t,h)
xlabel('t (s)')
ylabel('h (m)')
title('Perturbation flow rate vs time')
str1=['A=',num2str(A),' m^3/s']
str2=['R=',num2str(R),' s/m^2']
str3=['q=',num2str(q),' m^3/s']
text(0.5*t(i),0.5*h(i),str1);
text(0.5*t(i),0.4*h(i),str2);
text(0.5*t(i),0.3*h(i),str3);
```

Sample output from execution of Problem1_27.m is given below

```
>> Please enter tank area in m^2
>> 100

A =

    100
```

Please enter flow rate in m^3/s

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>> 0.2

q =

0.2000

Please enter resistance in s/m²

>> 15

R =

15

str1 =

A=100 m³/s

str2 =

R=15 s/m²

str3 =

q=0.2 m³/s

>>

