

Chapter 1

1. (a) Applying formula (1.1)

$$A(t) = t^2 + 2t + 3 \quad \text{and} \quad A(0) = 3$$

so that

$$a(t) = \frac{A(t)}{k} = \frac{A(t)}{A(0)} = \frac{1}{3}(t^2 + 2t + 3).$$

- (b) The three properties are listed on p. 2.

$$(1) \quad a(0) = \frac{1}{3}(3) = 1.$$

$$(2) \quad a'(t) = \frac{1}{3}(2t + 2) > 0 \quad \text{for} \quad t \geq 0,$$

so that $a(t)$ is an increasing function.

$$(3) \quad a(t) \text{ is a polynomial and thus is continuous.}$$

- (c) Applying formula (1.2)

$$\begin{aligned} I_n &= A(n) - A(n-1) = [n^2 + 2n + 3] - [(n-1)^2 + 2(n-1) + 3] \\ &= n^2 + 2n + 3 - n^2 + 2n - 1 - 2n + 2 - 3 \\ &= 2n + 1. \end{aligned}$$

2. (a) Applying formula (1.2)

$$\begin{aligned} I_1 + I_2 + \dots + I_n &= [A(1) - A(0)] + [A(2) - A(1)] + \dots + [A(n) - A(n-1)] \\ &= A(n) - A(0). \end{aligned}$$

- (b) The LHS is the increment in the fund over the n periods, which is entirely attributable to the interest earned. The RHS is the sum of the interest earned during each of the n periods.

3. Using ratio and proportion

$$\frac{5000}{11,130}(12,153.96 - 11,575.20) = \$260.$$

4. We have $a(t) = at^2 + b$, so that

$$\begin{aligned} a(0) &= & b &= 1 \\ a(3) &= 9a + & b &= 1.72. \end{aligned}$$

Solving two equations in two unknowns $a = .08$ and $b = 1$. Thus,

$$a(5) = 5^2(.08) + 1 = 3$$

$$a(10) = 10^2(.08) + 1 = 9.$$

and the answer is $100 \frac{a(10)}{a(5)} = 100 \frac{9}{3} = 300$.

5. (a) From formula (1.4b) and $A(t) = 100 + 5t$

$$i_5 = \frac{A(5) - A(4)}{A(4)} = \frac{125 - 120}{120} = \frac{5}{120} = \frac{1}{24}.$$

$$(b) \quad i_{10} = \frac{A(10) - A(9)}{A(9)} = \frac{150 - 145}{145} = \frac{5}{145} = \frac{1}{29}.$$

$$A(t) = 100(1.1)^t \quad \text{and}$$

$$6. (a) \quad i_5 = \frac{A(5) - A(4)}{A(4)} = \frac{100[(1.1)^5 - (1.1)^4]}{100(1.1)^4} = 1.1 - 1 = .1.$$

$$(b) \quad i_{10} = \frac{A(10) - A(9)}{A(9)} = \frac{100[(1.1)^{10} - (1.1)^9]}{100(1.1)^9} = 1.1 - 1 = .1.$$

7. From formula (1.4b)

$$i_n = \frac{A(n) - A(n-1)}{A(n-1)}$$

so that

$$A(n) - A(n-1) = i_n A(n-1)$$

and

$$A(n) = (1 + i_n) A(n-1).$$

8. We have $i_5 = .05$, $i_6 = .06$, $i_7 = .07$, and using the result derived in Exercise 7

$$\begin{aligned} A(7) &= A(4)(1 + i_5)(1 + i_6)(1 + i_7) \\ &= 1000(1.05)(1.06)(1.07) = \$1190.91. \end{aligned}$$

9. (a) Applying formula (1.5)

$$615 = 500(1 + 2.5i) = 500 + 1250i$$

so that

$$1250i = 115 \quad \text{and} \quad i = 115/1250 = .092, \quad \text{or} \quad 9.2\%.$$

(b) Similarly,

$$630 = 500(1 + .078t) = 500 + 39t$$

so that

$$39t = 130 \quad \text{and} \quad t = 130/39 = 10/3 = 3\frac{1}{3} \text{ years.}$$

10. We have

$$1110 = 1000(1 + it) = 1000 + 1000it$$

$$1000it = 110 \quad \text{and} \quad it = .11$$

so that

$$\begin{aligned} 500 \left[1 + \left(\frac{3}{4} \right) (i)(2t) \right] &= 500[1 + 1.5it] \\ &= 500[1 + (1.5)(.11)] = \$582.50. \end{aligned}$$

11. Applying formula (1.6)

$$i_n = \frac{i}{1 + i(n-1)} \quad \text{and} \quad .025 = \frac{.04}{1 + .04(n-1)}$$

so that

$$.025 + .001(n-1) = .04, \quad .001n = .016, \quad \text{and} \quad n = 16.$$

12. We have

$$i_1 = .01 \quad i_2 = .02 \quad i_3 = .03 \quad i_4 = .04 \quad i_5 = .05$$

and adapting formula (1.5)

$$1000 \left[1 + (i_1 + i_2 + i_3 + i_4 + i_5) \right] = 1000(1.15) = \$1150.$$

13. Applying formula (1.8)

$$600(1 + i)^2 = 600 + 264 = 864$$

which gives

$$(1 + i)^2 = 864/600 = 1.44, \quad 1 + i = 1.2, \quad \text{and} \quad i = .2$$

so that

$$2000(1 + i)^3 = 2000(1.2)^3 = \$3456.$$

14. We have

$$\frac{(1 + i)^n}{(1 + j)^n} = (1 + r)^n \quad \text{and} \quad 1 + r = \frac{1 + i}{1 + j}$$

so that

$$r = \frac{1+i}{1+j} - 1 = \frac{(1+i) - (1+j)}{1+j} = \frac{i-j}{1+j}.$$

This type of analysis will be important in Sections 4.7 and 9.4.

15. From the information given:

$$\begin{aligned} (1+i)^a &= 2 & (1+i)^a &= 2 \\ 2(1+i)^b &= 3 & (1+i)^b &= 3/2 \\ 3(1+i)^c &= 15 & (1+i)^c &= 5 \\ 6(1+i)^n &= 10 & (1+i)^n &= 5/3. \end{aligned}$$

By inspection $\frac{5}{3} = 5 \cdot \frac{2}{3} \cdot \frac{1}{2}$. Since exponents are additive with multiplication, we have $n = c - a - b$.

16. For one unit invested the amount of interest earned in each quarter is:

Quarter:	1	2	3	4
Simple:	.03	.03	.03	.03
Compound:	$1.03 - 1$	$(1.03)^2 - 1.03$	$(1.03)^3 - (1.03)^2$	$(1.03)^4 - (1.03)^3$

Thus, we have

$$\frac{D(4)}{D(3)} = \frac{\left[(1.03)^4 - (1.03)^3 \right] - .03}{\left[(1.03)^3 - (1.03)^2 \right] - .03} = 1.523.$$

17. Applying formula (1.12)

$$A: 10,000 \left[(1.06)^{-18} + (1.06)^{-19} \right] = 6808.57$$

$$B: 10,000 \left[(1.06)^{-20} + (1.06)^{-21} \right] = \underline{6059.60}$$

$$\text{Difference} = \$748.97.$$

18. We have

$$v^n + v^{2n} = 1$$

and multiplying by $(1+i)^{2n}$

$$(1+i)^n + 1 = (1+i)^{2n}$$

or

$$(1+i)^{2n} - (1+i)^n - 1 = 0 \quad \text{which is a quadratic.}$$

Solving the quadratic

$$(1+i)^n = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2} \quad \text{rejecting the negative root.}$$

Finally,

$$(1+i)^{2n} = \left(\frac{1+\sqrt{5}}{2} \right)^2 = \frac{1+2\sqrt{5}+5}{4} = \frac{3+\sqrt{5}}{2}.$$

19. From the given information $500(1+i)^{30} = 4000$ or $(1+i)^{30} = 8$.

The sum requested is

$$\begin{aligned} 10,000(v^{20} + v^{40} + v^{60}) &= 10,000(8^{-\frac{2}{3}} + 8^{-\frac{4}{3}} + 8^{-2}) \\ &= 10,000\left(\frac{1}{4} + \frac{1}{16} + \frac{1}{64}\right) = \$3281.25. \end{aligned}$$

20. (a) Applying formula (1.13) with $a(t) = 1+it = 1+.1t$, we have

$$d_5 = \frac{I_5}{A_5} = \frac{a(5) - a(4)}{a(5)} = \frac{1.5 - 1.4}{1.5} = \frac{.1}{1.5} = \frac{1}{15}.$$

- (b) A similar approach using formula (1.18) gives

$$a^{-1}(t) = 1 - dt = 1 - .1t$$

and

$$\begin{aligned} d_5 &= \frac{I_5}{A_5} = \frac{a(5) - a(4)}{a(5)} = \frac{(1-.5)^{-1} - (1-.4)^{-1}}{(1-.5)^{-1}} \\ &= \frac{1/.5 - 1/.6}{1 - 1/.5} = \frac{2 - 5/3}{2} = \frac{6-5}{2 \cdot 3} = \frac{1}{6}. \end{aligned}$$

21. From formula (1.16) we know that $v = 1 - d$, so we have

$$\begin{aligned} 200 + 300(1-d) &= 600(1-d)^2 \\ 6d^2 - 12d + 6 - 2 - 3 + 3d &= 0 \\ 6d^2 - 9d + 1 &= 0 \quad \text{which is a quadratic.} \end{aligned}$$

Solving the quadratic

$$d = \frac{9 \pm \sqrt{(-9)^2 - (4)(6)(1)}}{2 \cdot 6} = \frac{9 - \sqrt{57}}{12}$$

rejecting the root > 1 , so that

$$d = .1208, \text{ or } 12.08\%.$$

22. Amount of interest: $iA = 336$.

Amount of discount: $dA = 300$.

Applying formula (1.14)

$$i = \frac{d}{1-d} \quad \text{and} \quad \frac{336}{A} = \frac{300/A}{1-300/A} = \frac{300}{A-300}$$

so that

$$\begin{aligned} 336(A-300) &= 300A \\ 36A &= 100,800 \quad \text{and} \quad A = \$2800. \end{aligned}$$

23. Note that this Exercise is based on material covered in Section 1.8. The quarterly discount rate is $.08/4 = .02$, while 25 months is $8\frac{1}{3}$ quarters.

(a) The exact answer is

$$5000v^{25/3} = 5000(1-.02)^{25/3} = \$4225.27.$$

(b) The approximate answer is based on formula (1.20)

$$5000v^8 \left(1 - \frac{1}{3}d\right) = 5000(1-.02)^8 \left[1 - \left(\frac{1}{3}\right)(.02)\right] = \$4225.46.$$

The two answers are quite close in value.

24. We will algebraically change both the RHS and LHS using several of the basic identities contained in this Section.

$$\begin{aligned} \text{RHS} &= \frac{(i-d)^2}{1-v} = \frac{(id)^2}{d} = i^2d \quad \text{and} \\ \text{LHS} &= \frac{d^3}{(1-d)^2} = \frac{i^3v^3}{v^2} = i^3v = i^2d. \end{aligned}$$

25. Simple interest: $a(t) = 1 + it$ from formula (1.5).
Simple discount: $a^{-1}(t) = 1 - dt$ from formula (1.18).

Thus,

$$1 + it = \frac{1}{1 - dt}$$

and

$$\begin{aligned} 1 - dt + it - idt^2 &= 1 \\ it - dt &= idt^2 \\ i - d &= idt. \end{aligned}$$

26. (a) From formula (1.23a)

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = \left(1 + \frac{i^{(3)}}{3}\right)^3$$

so that

$$d^{(4)} = 4 \left[1 - \left(1 + \frac{i^{(3)}}{3}\right)^{-\frac{3}{4}} \right].$$

(b)

$$\left(1 + \frac{i^{(6)}}{6}\right)^6 = \left(1 - \frac{d^{(2)}}{2}\right)^{-2}$$

so that

$$i^{(6)} = 6 \left[\left(1 - \frac{d^{(2)}}{2}\right)^{-\frac{1}{3}} - 1 \right].$$

27. (a) From formula (1.24)

$$i^{(m)} - d^{(m)} = \frac{i^{(m)} d^{(m)}}{m}$$

so that

$$i^{(m)} = d^{(m)} \left(1 + \frac{i^{(m)}}{m} \right) = d^{(m)} (1 + i)^{\frac{1}{m}}.$$

(b) $i^{(m)}$ measures interest at the ends of m ths of a year, while $d^{(m)}$ is a comparable measure at the beginnings of m ths of a year. Accumulating $d^{(m)}$ from the beginning to the end of the m thly periods gives $i^{(m)}$.

28. (a) We have $j = \frac{i^{(4)}}{4} = \frac{.06}{4} = .015$ and $n = 2 \cdot 4 = 8$ quarters, so that the accumulated value is

$$100(1.015)^8 = \$112.65.$$

(b) Here we have an unusual and uncommon situation in which the conversion frequency is less frequent than annual. We have $j = 4(.06) = .24$ per 4-year period and $n = 2(1/4) = 1/2$ such periods, so that the accumulated value is

$$100(1 - .24)^{-.5} = 100(.76)^{-.5} = \$114.71.$$

29. From formula (1.24)

$$i^{(m)} - d^{(m)} = \frac{i^{(m)} d^{(m)}}{m}$$

so that

$$m = \frac{i^{(m)} d^{(m)}}{i^{(m)} - d^{(m)}} = \frac{(.1844144)(.1802608)}{.1844144 - .1802608} = 8.$$

30. We know that

$$1 + \frac{i^{(4)}}{4} = (1+i)^{\frac{1}{4}} \quad \text{and} \quad 1 + \frac{i^{(5)}}{5} = (1+i)^{\frac{1}{5}}$$

so that

$$\text{RHS} = (1+i)^{\frac{1}{4} - \frac{1}{5}} = (1+i)^{\frac{1}{20}}$$

$$\text{LHS} = (1+i)^{\frac{1}{n}} \quad \text{and} \quad n = 20.$$

31. We first need to express v in terms of $i^{(4)}$ and $d^{(4)}$ as follows:

$$v = 1 - d = \left(1 - \frac{d^{(4)}}{4}\right)^4 \quad \text{so that} \quad d^{(4)} = 4(1 - v^{25})$$

and

$$v = (1+i)^{-1} = \left(1 - \frac{i^{(4)}}{4}\right)^{-4} \quad \text{so that} \quad i^{(4)} = 4(v^{-.25} - 1).$$

Now

$$r = \frac{i^{(4)}}{d^{(4)}} = \frac{4(v^{-.25} - 1)}{4(1 - v^{25})} = v^{-.25} \quad \text{so that} \quad v^{25} = r^{-1} \quad \text{and} \quad v = r^{-4}.$$

32. We know that $d < i$ from formula (1.14) and that $d^{(m)} < i^{(m)}$ from formula (1.24). We also know that $i^{(m)} = i$ and $d^{(m)} = d$ if $m = 1$. Finally, in the limit $i^{(m)} \rightarrow \delta$ and $d^{(m)} \rightarrow \delta$ as $m \rightarrow \infty$. Thus, putting it all together, we have

$$d < d^{(m)} < \delta < i^{(m)} < i.$$

33. (a) Using formula (1.26), we have

$$A(t) = K a^t b^{t^2} d^{c^t}$$

$$\ln A(t) = \ln K + t \ln a + t^2 \ln b + c^t \ln d$$

and

$$\delta_t = \frac{d}{dt} \ln A(t) = \ln a + 2t \ln b + c^t \ln c \ln d.$$

(b) Formula (1.26) is much more convenient since it involves differentiating a sum, while formula (1.25) involves differentiating a product.

34. Fund A: $a^A(t) = 1 + .10t$ and $\delta_t^A = \frac{\frac{d}{dt}a^A(t)}{a^A(t)} = \frac{.10}{1 + .10t}$.

Fund B: $a^B(t) = (1 - .05t)^{-1}$ and $\delta_t^B = \frac{\frac{d}{dt}a^B(t)}{a^B(t)} = \frac{.05}{1 + .05t}$.

Equating the two and solving for t , we have

$$\frac{.10}{1 + .10t} = \frac{.05}{1 - .05t} \quad \text{and} \quad .10 - .005t = .05 + .005t$$

so that $.10t = .05$ and $t = .5$.

35. The accumulation function is a second degree polynomial, i.e. $a(t) = at^2 + bt + c$.

$$a(0) = \quad \quad \quad c = 1 \quad \text{from Section 1.2}$$

$$a(.5) = .25a + .5b + c = 1.025 \quad 5\% \text{ convertible semiannually}$$

$$a(1) = \quad a + b + c = 1.07 \quad 7\% \text{ effective for the year}$$

Solving three equations in three unknowns, we have

$$a = .04 \quad b = .03 \quad c = 1.$$

36. Let the excess be denoted by E_t . We then have

$$E_t = (1 + it) - (1 + i)^t$$

which we want to maximize. Using the standard approach from calculus

$$\frac{d}{dt}E_t = i - (1 + i)^t \ln(1 + i) = i - \delta(1 + i)^t = 0$$

$$(1 + i)^t = \frac{i}{\delta} \quad \text{and} \quad t \ln(1 + i) = t\delta = \ln i - \ln \delta$$

so that

$$t = \frac{\ln i - \ln \delta}{\delta}.$$

37. We need to modify formula (1.39) to reflect rates of discount rather than rates of interest. Then from the definition of equivalency, we have

$$\begin{aligned} a(3) &= (1 + i)^3 = (1 - d_1)^{-1} (1 - d_2)^{-1} (1 - d_3)^{-1} \\ &= (.92)^{-1} (.93)^{-1} (.94)^{-1} = .804261^{-1} \end{aligned}$$

and

$$i = (.804264)^{-1/3} - 1 = .0753, \quad \text{or} \quad 7.53\%.$$

38. (a) From formula (1.39)

$$a(n) = (1 + i_1)(1 + i_2) \dots (1 + i_n) \quad \text{where} \quad 1 + i_k = (1 + r)^k (1 + i)$$

so that

$$a(n) = [(1+r)(1+i)][(1+r)^2(1+i)] \dots [(1+r)^n(1+i)]$$

and using the formula for the sum of the first n positive integers in the exponent, we have

$$a(n) = (1+r)^{n(n+1)/2} (1+i)^n.$$

(b) From part (a)

$$(1+j)^n = (1+r)^{n(n+1)/2} (1+i)^n \quad \text{so that} \quad j = (1+r)^{(n+1)/2} - 1.$$

39. Adapting formula (1.42) for $t=10$, we have

$$a(10) = e^{5(.06)} e^{5\delta} = 2, \quad \text{so that} \quad e^{5\delta} = 2e^{-.3}$$

and

$$\delta = \frac{1}{5} \ln(2e^{-.3}) = .0786, \quad \text{or} \quad 7.86\%.$$

40. Fund X: $a^X(20) = e^{\int_0^{20} (.01t+1)dt} = e^4$ performing the integration in the exponent.

Fund Y: $a^Y(20) = (1+i)^{20} = e^4$ equating the fund balances at time $t=20$.

The answer is

$$a^Y(1.5) = (1+i)^{1.5} = [(1+i)^{20}]^{.075} = (e^4)^{.075} = e^{.3}.$$

41. Compound discount:

$$a(3) = (1-d_1)^{-1} (1-d_2)^{-1} (1-d_3)^{-1} = (.93)^{-1} (.92)^{-1} (.91)^{-1} = 1.284363$$

using the approach taken in Exercise 37.

Simple interest: $a(3) = 1 + 3i$.

Equating the two and solving for i , we have

$$1 + 3i = 1.284363 \quad \text{and} \quad i = .0948, \quad \text{or} \quad 9.48\%.$$

42. Similar to Exercise 35 we need to solve three equations in three unknowns. We have

$$A(t) = At^2 + Bt + C$$

and using the values of $A(t)$ provided

$$A(0) = \quad \quad \quad C = 100$$

$$A(1) = \quad A + B + C = 110$$

$$A(2) = \quad 4A + 2B + C = 136$$

which has the solution $A=8 \quad B=2 \quad C=100$.

$$(a) \quad i_2 = \frac{A(2) - A(1)}{A(1)} = \frac{136 - 110}{110} = \frac{26}{110} = .236, \text{ or } 23.6\%.$$

$$(b) \quad \frac{A(1.5) - A(.5)}{A(1.5)} = \frac{121 - 103}{121} = \frac{18}{121} = .149, \text{ or } 14.9\%.$$

$$(c) \quad \delta_t = \frac{A'(t)}{A(t)} = \frac{16t + 2}{8t^2 + 2t + 100} \text{ so that } \delta_{1.2} = \frac{21.2}{113.92} = .186, \text{ or } 18.6\%.$$

$$(d) \quad \frac{A(.75)}{A(1.25)} = \frac{106}{115} = .922.$$

43. The equation for the force of interest which increases linearly from 5% at time $t = 0$ to 8% at time $t = 6$ is given by

$$\delta_t = .05 + .005t \quad \text{for } 0 \leq t \leq 6.$$

Now applying formula (1.27) the present value is

$$1,000,000a^{-1}(6) = 1,000,000e^{-\int_0^6 (.05 + .005t) dt} = 1,000,000e^{-.39} = \$677,057.$$

44. The interest earned amounts are given by

$$A: \quad X \left[\left(1 + \frac{i}{2} \right)^{16} - \left(1 + \frac{i}{2} \right)^{15} \right] = X \left(1 + \frac{i}{2} \right)^{15} \left(\frac{i}{2} \right)$$

$$B: \quad 2X \cdot \frac{i}{2}.$$

Equating two expressions and solving for i

$$X \left(1 + \frac{i}{2} \right)^{15} \left(\frac{i}{2} \right) = 2X \cdot \frac{i}{2} \quad \left(1 + \frac{i}{2} \right)^{15} = 2 \quad i = 2(2^{1/15} - 1) = .0946, \text{ or } 9.46\%.$$

45. Following a similar approach to that taken in Exercise 44, but using rates of discount rather than rates of interest, we have

$$A: \quad X = 100 \left[(1-d)^{-11} - (1-d)^{-10} \right] = 100(1-d)^{-10} \left[(1-d)^{-1} - 1 \right]$$

$$B: \quad X = 50 \left[(1-d)^{-17} - (1-d)^{-16} \right] = 50(1-d)^{-16} \left[(1-d)^{-1} - 1 \right].$$

Equating the two expressions and solving for d

$$100(1-d)^{-10} = 50(1-d)^{-16} \quad (1-d)^{-6} = 2 \quad (1-d)^{-1} = 2^{1/6}.$$

Finally, we need to solve for X . Using A we have

$$X = 100 \cdot 2^{10/6} (2^{1/6} - 1) = 38.88.$$

46. For an investment of one unit at $t = 2$ the value at $t = n$ is

$$a(n) = e^{\int_2^n \delta_t dt} = e^{2 \int_2^n (t-1)^{-1} dt} = e^{2 \ln(t-1) \Big|_2^n} = \frac{(n-1)^2}{(2-1)^2} = (n-1)^2.$$

Now applying formula (1.13)

$$d_n = \frac{a(n+1) - a(n)}{a(n+1)} = \frac{n^2 - (n-1)^2}{n^2}$$

and

$$1 - dn = \left(\frac{n-1}{n} \right)^2.$$

Finally, the equivalent $d_n^{(2)}$ is

$$d_n^{(2)} = 2 \left[1 - (1 - d_n)^{1/2} \right] = 2 \left[1 - \frac{n-1}{n} \right] = \frac{2}{n}.$$

47. We are given $i = .20 = \frac{1}{5}$, so that

$$d = \frac{i}{1+i} = \frac{1/5}{1+1/5} = \frac{1}{6}.$$

We then have

$$PV_A = (1.20)^{-1} \left[1 + \frac{1}{2} \cdot \frac{1}{5} \right]^{-1}$$

$$PV_B = (1.20)^{-1} \left[1 - \frac{1}{2} \cdot \frac{1}{6} \right]$$

and the required ratio is

$$\frac{PV_A}{PV_B} = \frac{(1 + 1/10)^{-1}}{1 - 1/12} = \frac{10}{11} \cdot \frac{12}{11} = \frac{120}{121}.$$

48. (a) $i = e^\delta - 1 = \delta + \frac{\delta^2}{2!} + \frac{\delta^3}{3!} + \frac{\delta^4}{4!} + \dots$

using the standard power series expansion for e^δ .

(b) $\delta = \ln(1+i) = i - \frac{i^2}{2} + \frac{i^3}{3} - \frac{i^4}{4} + \dots$

using a Taylor series expansion.

$$(c) \quad d = \frac{i}{1+i} = i(1+i)^{-1} = i(1-i^2+i^2-i^3+\dots) = i-i^2+i^3-i^4+\dots$$

using the sum of an infinite geometric progression.

$$(d) \quad \delta = -\ln(1-d) = -\left(-d - \frac{d^2}{2} - \frac{d^3}{3} - \frac{d^4}{4} - \dots\right)$$

adapting the series expansion in part (b).

$$49. (a) \quad \frac{dd}{di} = \frac{d}{di} \left(\frac{i}{1+i} \right) = \frac{(1+i) - i}{(1+i)^2} = (1+i)^{-2}.$$

$$(b) \quad \frac{d\delta}{di} = \frac{d}{di} \ln(1+i) = \frac{1}{1+i} = (1+i)^{-1}.$$

$$(c) \quad \frac{d\delta}{di} = \frac{d}{dv} (-\ln v) = -\frac{1}{v} = -v^{-1}.$$

$$(d) \quad \frac{dd}{d\delta} = \frac{d}{d\delta} (1 - e^{-\delta}) = -e^{-\delta} (-1) = e^{-\delta}.$$

$$50. (a) (1) \quad a(t) = e^{\int_0^t (a+br) dr} = e^{at+bt^2/2}.$$

$$(2) \quad 1+i_n = \frac{a(n)}{a(n-1)} = \frac{e^{an+5bn^2}}{e^{a(n-1)+5b(n-1)^2}} = e^{an+5bn^2-an+a-.5bn^2+bn-.5b} = e^{(a-b/2)+bn}.$$

$$(b) (1) \quad a(t) = e^{\int_0^t ab^r dr} = e^{a(b^t-1)/\ln b}.$$

$$(2) \quad 1+i_n = \frac{a(n)}{a(n-1)} = e^{\frac{a}{\ln b} [(b^n-1)-(b^{n-1}-1)]} = e^{a(b-1)b^{n-1}/\ln b}.$$