PROBLEM 1.1

FIND: Explain the hierarchy of standards. Explain the term *standard*. Cite example.

SOLUTION

The term *standard* refers to an object or instrument, a method or a procedure that provides a value of an acceptable accuracy for comparison.

A *primary standard* defines the value of the unit to which it is associated. *Secondary standards*, while based on the primary standard, are more readily accessible and amenable for use in a calibration. There is a hierarchy of secondary standards: A *transfer standard* might be maintained by a national standards lab (such as NIST in the United States) to calibrate industrial "laboratory standards". It is costly and time-consuming to certify a laboratory standard, so they are treated carefully and not used too regularly. A *laboratory standard* would be maintained by a company to be used to certify a more common in-house reference called the *working standard*. A working standard would be calibrated against the laboratory standard. The working standard is used on a more regular basis to calibrate everyday measurement devices or products being manufactured. Working standards are more the norm for most of us. A working standard is simply the value or instrument that we assume is correct in checking the output operation of another instrument.

Example: A government lab maintains the primary standard for pressure. It calibrates a an instrument called a "deadweight tester" (see C9 discussion) for high pressure calibrations. These form its transfer standard for high pressure. A company that makes pressure transducers needs an in-house standard to certify their products. They purchase two deadweight testers. They send one tester to the national lab to be calibrated; this becomes their laboratory standard. On return, they use it to calibrate the other; this becomes their working standard. They test their manufactured transducers using the working standard – usually at one or two points over the transducer range to assure that it is working. Because the working standard is being used regularly, it can go out of calibration. Periodically, they check the working standard calibration against the laboratory standard.

See ASME PTC 19.2 Pressure Measurements for a further discussion.

A *test standard* defines a specific procedure that is to be followed.

PROBLEM 1.2

FIND: Why calibrate? What does calibrated mean?

SOLUTION:

The purpose of a calibration is to evaluate and document the accuracy of a measuring device. A *calibration* should be performed whenever the accuracy level of a measured value must be ascertained.

An instrument that has been calibrated provides the engineer a basis for interpreting the device's output indication. It provides assurance in the measurement. Besides this purpose, a calibration assures the engineer that the device is working as expected.

A periodic calibration of measuring instruments serves as a performance check on those instruments and provides a level of confidence in their indicated values. A good rule is to calibrate measuring systems annually or more often as needed.

ISO 9000 certifications have strict rules on calibration results and the frequency of calibration.

PROBLEM 1.3

FIND: Suggest methods to estimate the values of the random and systematic errors of a dial thermometer.

SOLUTION:

Instrument random error is related to repeatability: how closely an instrument indicates the same value on repeated measurements. So a method that repeatedly exposes the instrument to one or more known temperatures could be developed to estimate the range of the random error, a value called the random uncertainty. This is usually stated as a statistical estimate of the variation of the readings. An important aspect of such a test is to include some mechanism to allow the instrument to change its indicated value following each reading so that it must readjust itself.

For example, we could place the instrument in an environment of constant temperature and note its indicated value and then move the instrument to another constant temperature environment and note its value there. The two chosen temperatures could be representative of the range of intended use of the instrument. By alternating between the two constant temperature environments, differences in indicated values within each environment would be indicative of the precision error to be expected of the instrument at that temperature. Of course, this assumes that the constant temperatures do indeed remain constant throughout the test and the instrument is used in an identical manner for each measurement.

Instrument systematic error is a fixed offset. In the absence of random error, the amount of systematic error would be how closely the instrument indicates the correct value. This offset would be present in every reading. So an important aspect of this check is to calibrate it against a value that is at least as accurate as you need. This is not a trivial undertaking. The systematic uncertainty would be the value that estimates the possible range of the offset.

For example, you could use the ice point (0^{o}C) as a check for systematic error. The ice point is formed from a carefully prepared bath of solid ice and liquid water. As another check, the melting point of a pure substance, such as silver, could be used. Or easier, the steam point.

PROBLEM 1.4

FIND: Discuss interference in the test of Figure 1.6

SOLUTION :

In the example described by Figure 1.6, tests were run on different days on which the local barometric pressure had changed. Between any two days of different barometric pressure, the boiling point measured would be different – this offset is due to the interference effect of the pressure.

Consider a test run over several days coincident with the motion of a major weather front through the area. Clearly, this would impose a trend on the dataset. For example, the measured boiling point may be seen as increasing from day to day.

By running tests over random days separated by a sufficient period of days, so as not to allow any one atmospheric front to impose a trend on the data, the effects of atmospheric pressure can be broken up into noise. The measured boiling point might then be high one test but then low on the next, in effect, making it look like random data scatter, i.e. noise.

PROBLEM 1.5

FIND: How does resolution affect uncertainty?

SOLUTION

The resolution of a scale is defined by the least significant increment or division on the output display. Resolution affects a user's ability to resolve the output display of an instrument or measuring system, and thereby can introduce error, in this case a resolution error. Thus, there is a source of uncertainty associated with its use. The uncertainty value is the range of possible resolution error.

Consider a simple experiment to show the effects of resolution. Under some fixed condition, ask several competent, independent observers to record the indicated value from a measurement system. Collect the results – this becomes your dataset. Each observer is likely to record a slightly different value based on the resolution of the instrument. Because the indicated value is really the same for each observer, the scatter in your dataset will be close to the value of the resolution of the measurement system.

Data scatter contributes to the random uncertainty. As such, the output resolution of a measurement system forms a lower limit as to the uncertainty in the random error expected.

On multiple readings, the resolution would not contribute to systematic error, as systematic error is a fixed offset.

PROBLEM 1.6

FIND: How does hysteresis affect uncertainty?

SOLUTION

Hysteresis error is the difference between the values indicated by a measurement system when the value measured is increasing in value as opposed to when it is decreasing in value; this despite the fact that the value being measured is actually the same for either case. So this difference in possible outcomes is an uncertainty in the measured value.

A common cause of hysteresis in analog instruments is friction in the moving parts. This can cause the output indicator to 'stick'. In digital instruments, hysteresis can be caused by the discretization, the ability to resolve a value into a digital scale.

The effect of hysteresis on any single measurement can be viewed as a systematic error; its uncertainty is the possible offset from the correct value.

The use of randomization methods can break up the trends implied by hysteresis effects and lead to data scatter. A random uncertainty can be assigned based on some measure of the data scatter, such as a statistical standard deviation.

PROBLEM 1.7

SOLUTION

This problem is open-ended and has no unique solution. We suggest that the instructor use this Problem as the basis for an in-class or small group discussion.

PROBLEM 1.8

FIND: Identify measurement stages for each device.

SOLUTION

a) *thermostat*

Sensor/transducer: bimetallic thermometer
Output: displacement of thermometer tip
Controller: mercury contact switch (open:furnace off; closed:furnace on)

b) *speedometer*

Method 1:

Sensor: usually a mechanically coupled cable

Transducer: typically a dc generator that is turned by the cable producing an electrical signal

Output: typically a pointer/scale (note: often a galvanometer is used to convert the electrical signal in a mechanical rotation of the pointer)

Method 2:

Sensor: A magnet attached to the rotating shaft
Transducer: A Hall Effect device that is stationary but detects each sensor passage by creating voltage pulse
Signal Conditioning: A pulse counting circuit; maybe also digital-analog converter (if analog readout is used)
Output: An analog or digital readout calibrated to convert pulses per minute to kph or mph.

c) *Portable CD Stereo Player*

Sensor: laser with optical reader (reflected light signal differentiates between a "1" and "0")
Transducer: digital register (stores digital information for signal conditioning)
Signal conditioning: digital-to-analog converter and amplifier (converts digital numbers to voltages and amplifies the voltage)
Output: headset/speaker (note: the headeset/speaker is a second transducer in this system converting an electrical signal back to a mechanical displacement)

d) *anti-lock braking system*

Sensor: brake activation switch senses brakes 'on'; encoder counts wheels revolutions per unit time
Signal conditioning: timing circuit
Output: a feedback signal that pulsates brake action overriding the driver's constant pedal pressure

e) *automotive cruise control*

Sensor: vehicle speed sensor, basic principle is the same as the speedometer.

Actuator: Typically a vacuum driven actuator that is controlled by an Electronic Control Unit (ECU) that most likely implements a PID control scheme

f) Throttle position sensor

Sensor: LVDT
Output: Telemetry to pits, throttle position is one of several variables monitored on Formula 1 series race cars

PROBLEM 1.9

KNOWN: Data of Table 1.5

FIND: input range, output range

SOLUTION

By inspection

$0.5 \le x \le 100$ cm

$0.4 \le y \le 253.2$ V

The input range (x) is from 0.5 to 100 cm. The output range (y) is from 0.4 to 253.2V. The corresponding spans are given by

$r_i = 99.5$ cm

$r_o = 252.8$ V

COMMENT

Note that each answer has units shown. By themselves, numerical answers are meaningless. Always show units for data, for each step of data reduction and in all reported results.

PROBLEM 1.10

KNOWN: Data set of Table 1.5

FIND: Discuss advantages of different plot formats for this data

SOLUTION:

Both rectangular and log-log plots are shown below.

Rectangular grid (left plot below):

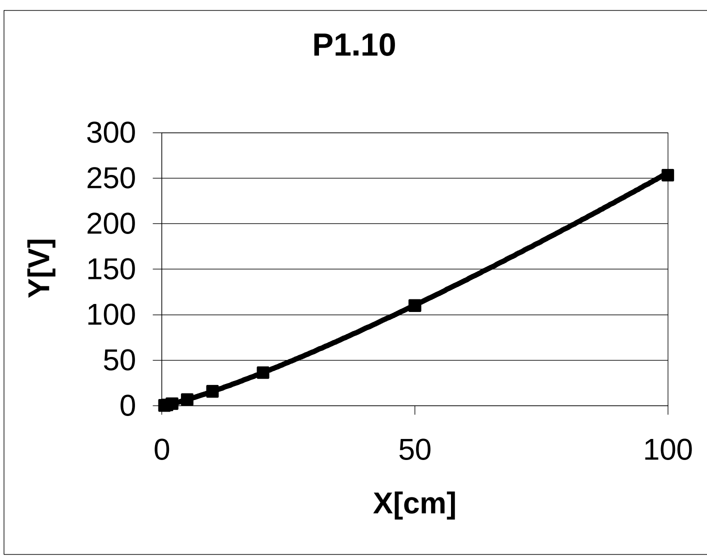
An advantage of this format is that is displays the data clearly as having a non-linear relationship. The data trend, while not immediately quantifiable, is established.

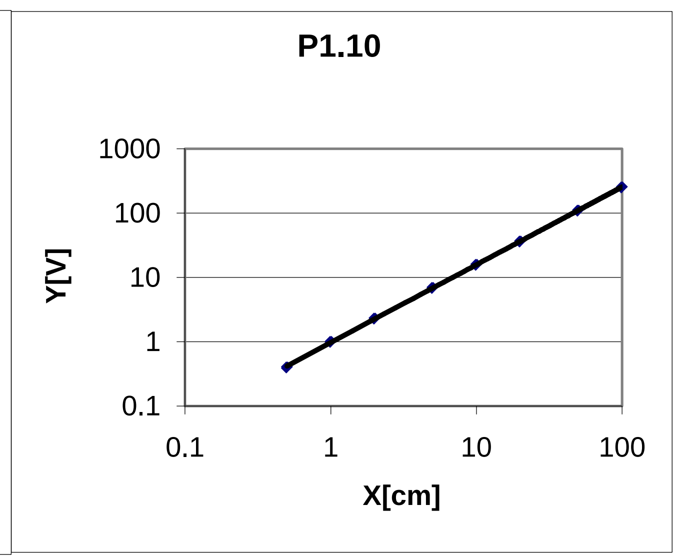
A disadvantage with this data set is that the poor resolution at low x values makes quantification at low values difficult.

Log-log grid (right plot below):

An advantage of this format with this particular data set is that the data display a linear relationship of the form: log y = m log x + log b. This tells us that the data have the relationship, $y = bx^m$. Because of these facts, resolution is equally good over the whole scale.

A disadvantage with this format is that one must remember the data has been conditioned to look linear. We are no longer plotting x versus y. This is particularly important to remember when attempting to find the slope of y against x.





PROBLEM 1.11

KNOWN: Calibration data of Table 1.5

FIND: K at x = 5, 10, 20 cm

SOLUTION:

The data reveal a linear relation on a log-log plot suggesting $y = bx^m$. That is:

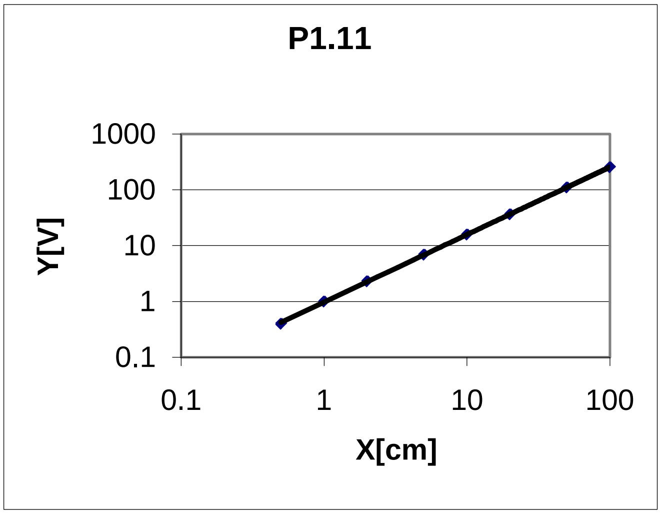
$$\log y = \log (bx^m) = \log b + m \log x \qquad \text{or}$$
$$Y = B + mX$$

From the plot, B = 0, so that b = 1, and m = 1.2. Thus, we find from the calibration the relationship

$$y = x^{1.2}$$

Because $K = [dy/dx]_x = 1.2x^{0.2}$, we obtain

x [cm]	K [V/cm]
5	1.66
10	1.90
20	2.18



We should expect that errors would propagate with the same sensitivity as the data. Hence for y=f(x), as sensitivity increases, the influence of the errors on y due to errors in x between would increase.

COMMENT

A common shortcut is to use the approximation that

$$dy/dx = \lim_{x \to 0} \Delta y/\Delta x$$

This approximation is valid only for very small changes in x, otherwise errors result. This is a common mistake. An important aspect of this problem is to draw attention to the fact that many measurement systems may have a static sensitivity that is dependent on input value.

PROBLEM 1.12

KNOWN: Sequence calibration data set of Table 1.6

$r_i = 5$ mV
$r_o = 5$ mV

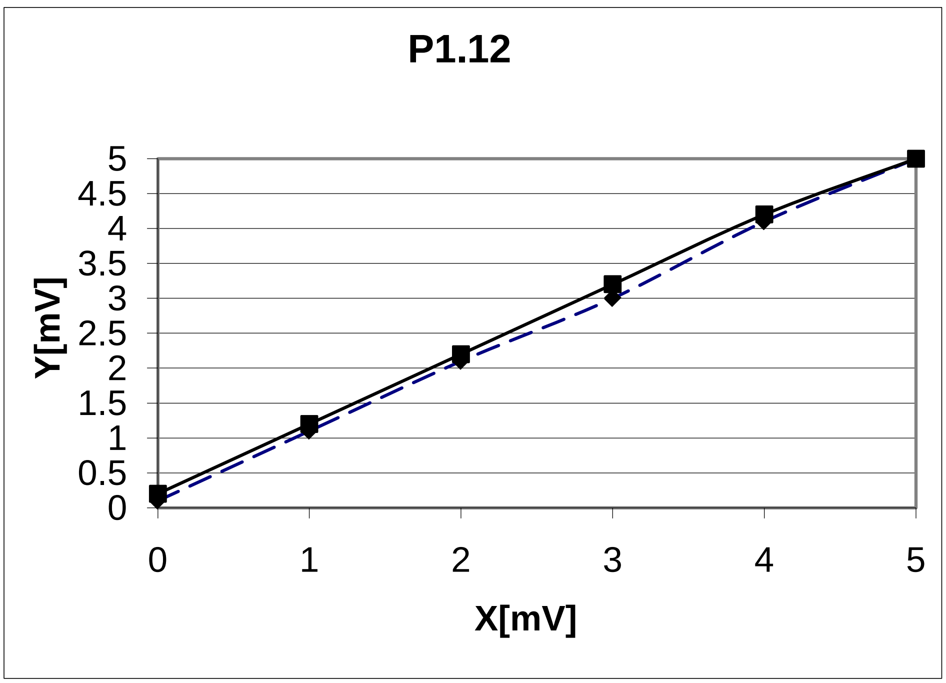
FIND: $\%(u_h)_{max}$

SOLUTION

By inspection of the data, the maximum hysteresis occurs at x = 3.0. For this case,

$u_h = y_{up} - y_{down}$
$= 0.2$ mV or

$\%(u_h)_{max} = 100 \times (0.2 \text{ mV}/5 \text{ mV})$
$= 4\%$



Problem 1.13

KNOWN: Comparison of three clock outputs with standard time

FIND: Discuss estimated accuracy

SOLUTION

Clock A shows a bias error of 2:23 s. The bias would appear to be increasing at a rate of 1 s/hr. However, clock resolution is 1 s which by itself can lead to precision error (data scatter) of ± ½ s; this can create the situation noted here. Another reading would clarify this.

Clock B shows a bias error of 5 s. There does not appear to be any precision error in the output.

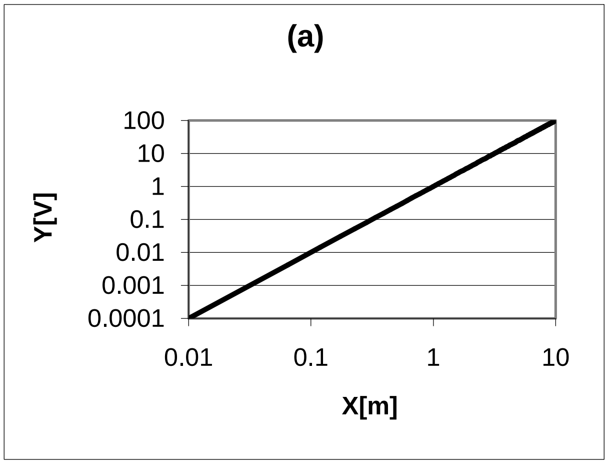
Clock C shows a 0 s bias error and a precision error on the order of ± 2 s.

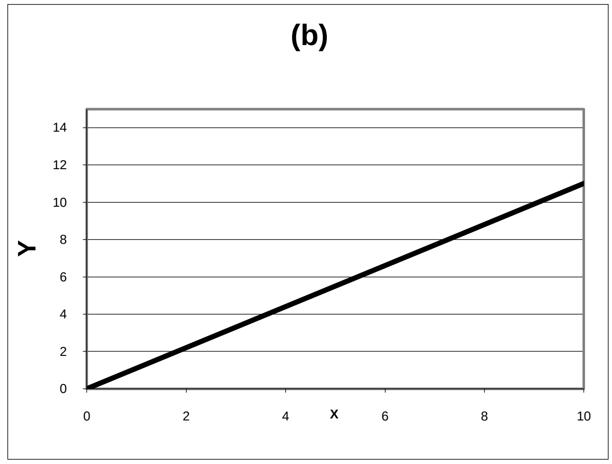
Because of the calibration, we now know the values of bias error for each clock. Correcting for bias error, we can consider Clock B to provide the more accurate time. Over time, the bias error in Clock A could become cumbersome to deal with, that is if the bias is indeed increasing in time. Therefore, it provides the least reliable value of time.

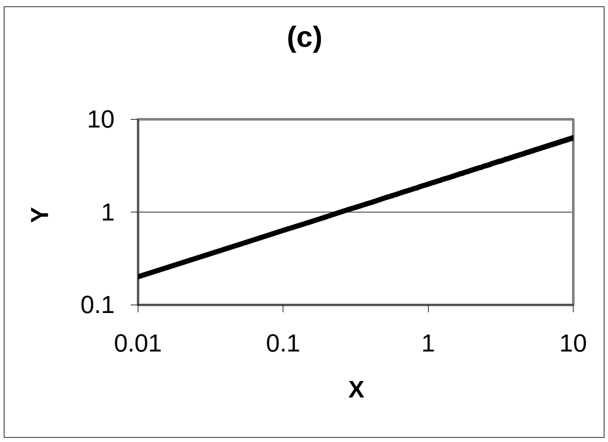
PROBLEM 1.14

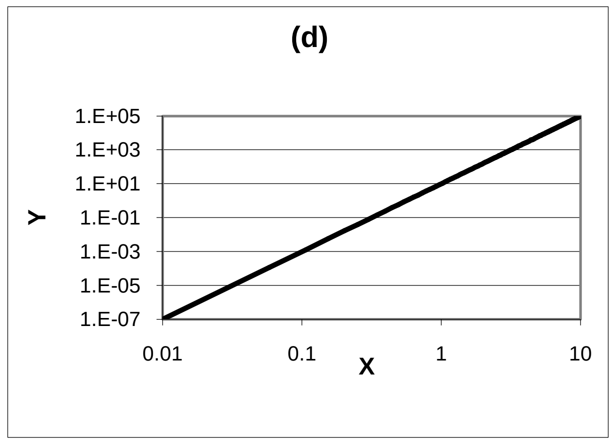
SOLUTION

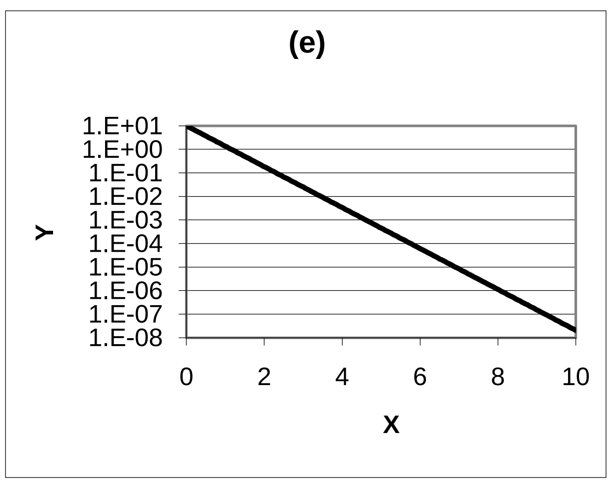
Each curve is plotted below in a suitable format to yield a linear shape.











PROBLEM 1.15

KNOWN: $y = 10e^{-5x}$

FIND: Slope at x = 0, 2 and 20

SOLUTION

The equation has been plotted below. The slope of the equation at any value of x can be found graphically or by the derivative

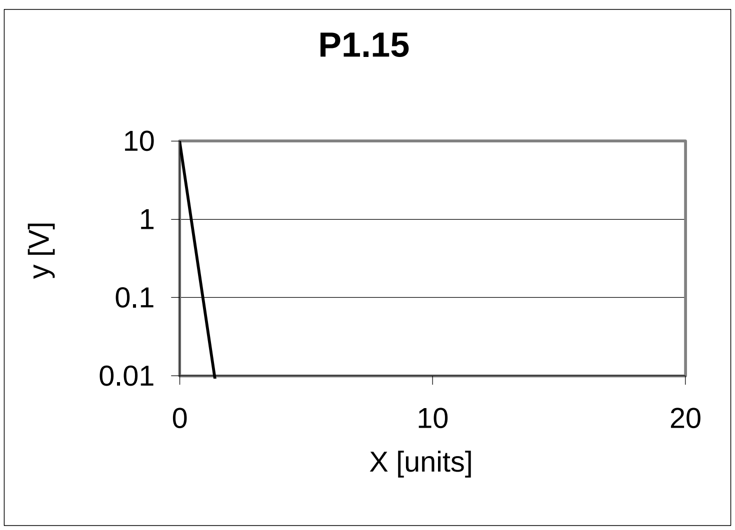
$dy/dx = -50e^{-5x}$

x [V]	dy/dx [V/unit]
0	-50
2	-0.00227
20	0

The sensitivity of y to x decreases with x.

COMMENT

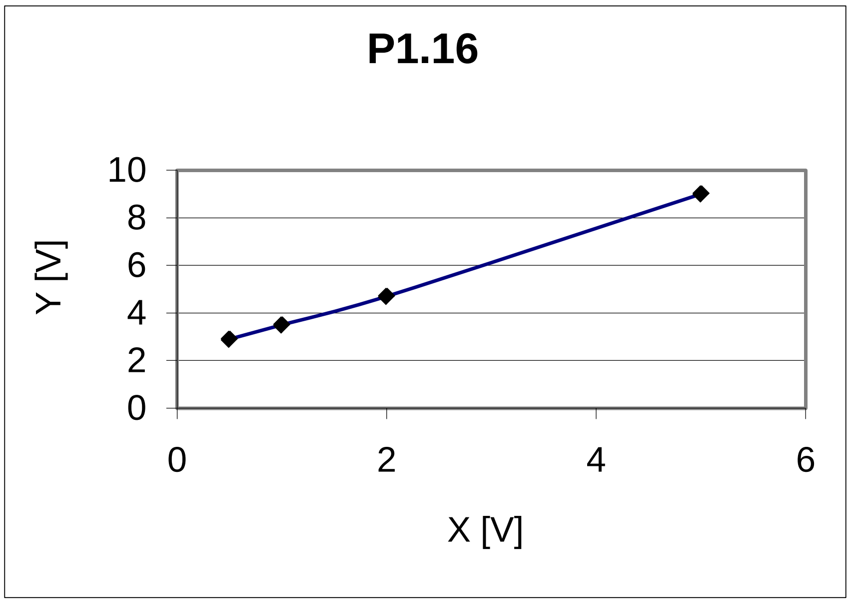
An important aspect of this problem, is to draw attention to the fact that many measurement systems may have a static sensitivity that is dependent on input value. While it is desirable to have a constant K value, the operating principle of many systems will preclude this or incorporate signal conditioning stages to overcome such nonlinearity. In Chapter 3, the concept that system's also have a dynamic sensitivity that is frequency dependent will be introduced.



PROBLEM 1.16

SOLUTION

The data are plotted below. The slope of a line passing through the data is 1.365 and the y intercept is 2.12. The data can be fit to the line $y = 1.365x + 2.12$. Therefore, the static sensitivity is $K = 1.365$ for all x.



PROBLEM 1.17

KNOWN: Data of form $y = ax^b$.

FIND: a and b; K

SOLUTION

The data are plotted below. If $y = ax^b$, then in log-log format the data will take the linear form

$$\log y = \log a + b \log x$$

A more or less linear curve results with this data. From the plot, the curve fit found is

$$\log y = -0.23 + 2x$$

This implies that

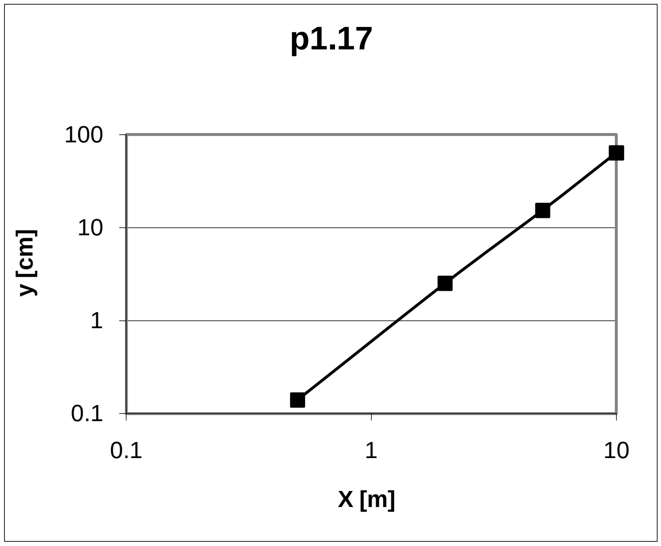
$$y = 0.59x^2$$

so that a = 0.59 and b = 2. The static sensitivity is found by the slope dy/dx at each value of x.

x [m]	$K(x_1) = dy/dx \mid_{x1}$ [cm/m]
0.5	0.54
2.0	2.16
5.0	5.40
10.0	10.80

COMMENT

An aspect of this problem is to draw attention to the fact that many measurement systems may have a static sensitivity that is dependent on input value. The operating principle of many systems will determine how K behaves.


p1.17
y [cm]
100
10
1
0.1
0.1
1
10
X [m]

PROBLEM 1.18

KNOWN: Calibration data

FIND: Plot data. Estimate K.

SOLUTION

The data are plotted below in semi-log format. A linear curve results. This suggests $y = ae^{bx}$. Plotting y vs x in semi-log format is equivalent to plotting

$$\log y = \log a + bx$$

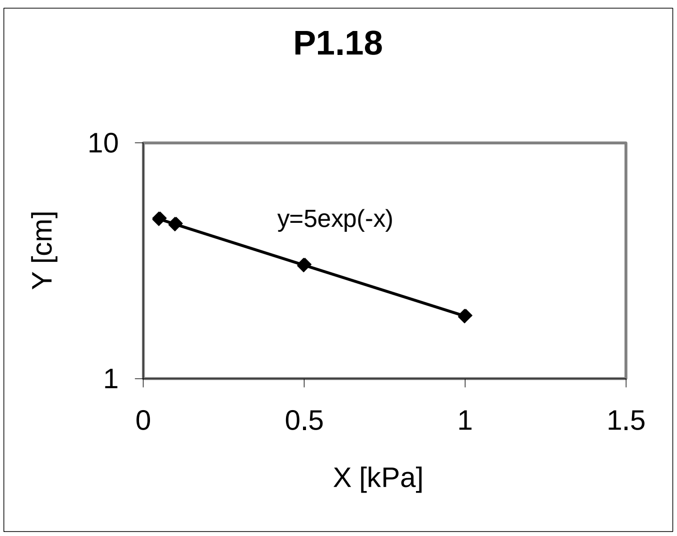
From the plot, a = 5 and b = -1. Hence, the data describe $y = 5e^{-x}$. Now, $K = dy/dx\,\big|_x$, so that

X [psi]	K
0.05	-4.76
0.1	-4.52
0.5	-3.03
1.0	-1.84

The magnitude of the static sensitivity decreases with x. The negative sign indicates that y will decrease as x increases.

COMMENT

An aspect of this problem is to draw attention to the fact that many measurement systems may have a static sensitivity that is dependent on input value. The operating principle of many systems will determine how K behaves.



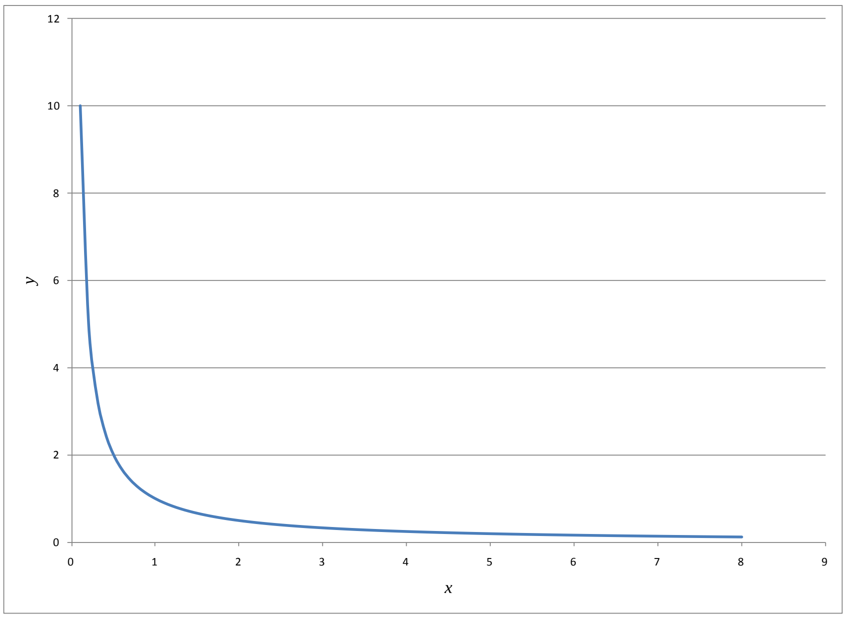
PROBLEM 1.19

KNOWN: $y = \dfrac{1}{x}$

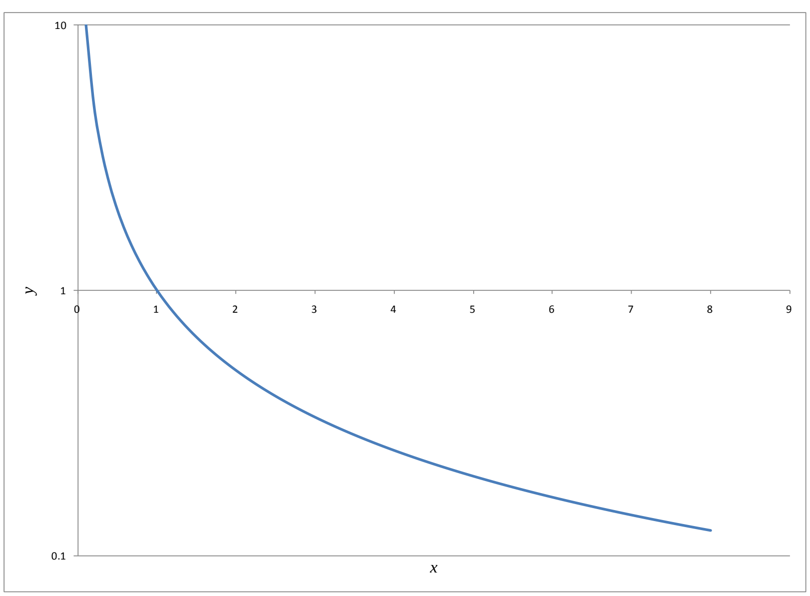
FIND: Plot the function on various coordinates, and discuss the implications.

SOLUTION

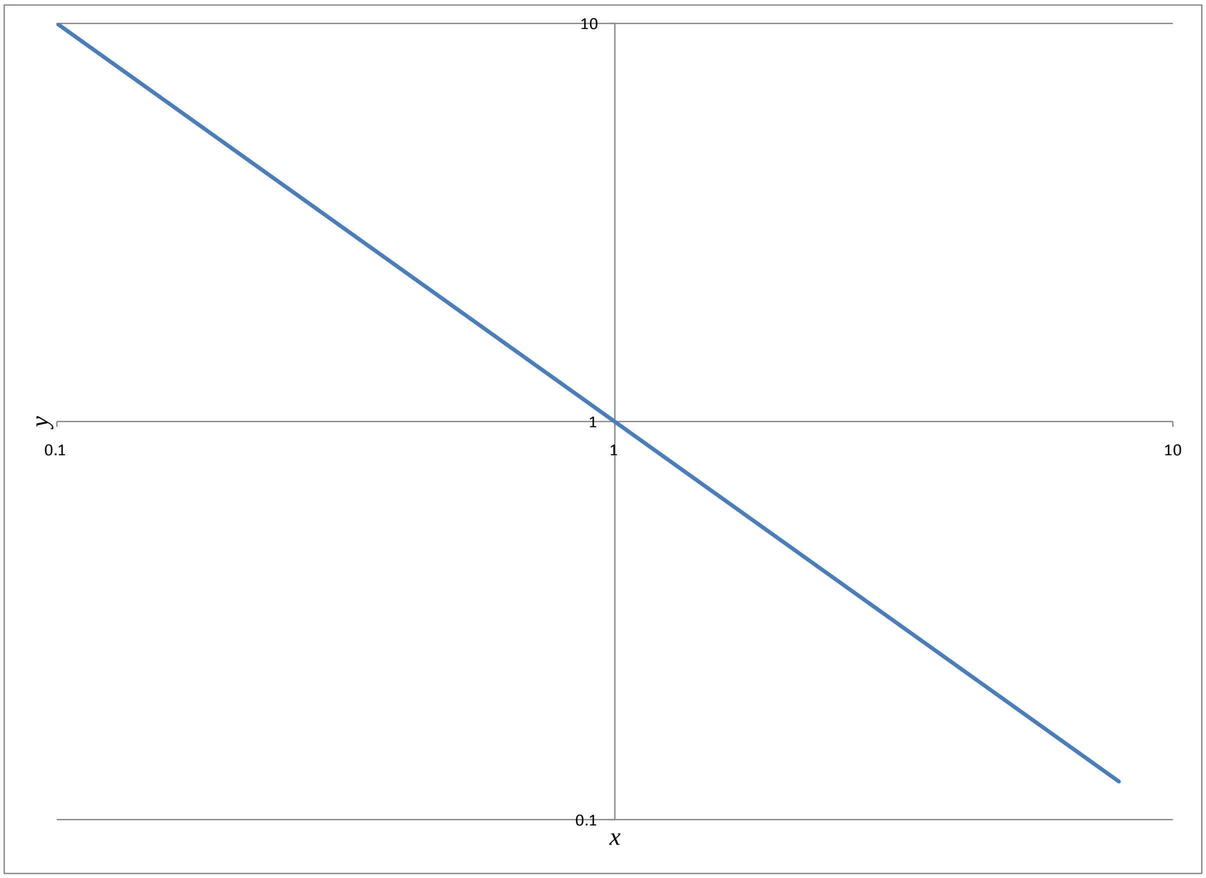
a)



b)



c)



COMMMENT: Although each plot is useful in understanding the behavior of the function, the log-log plot would have the most usefulness if the function represented experimental data.

PROBLEM 1.20

KNOWN: A bulb thermometer is used to measure outside temperature.

FIND: Extraneous variables that might influence thermometer output.

SOLUTION

A thermometer's indicated temperature will be influenced by the temperature of solid objects to which it is in contact, and radiation exchange with bodies at different temperatures (including the sky or sun, buildings, people and ground) within its line of sight. Hence, location should be carefully selected and even randomized. We know that a bulb thermometer does not respond quickly to temperature changes, so that a sufficient period of time needs to be allowed for the instrument to adjust to new temperatures. By replication of the measurement, effects due to instrument hysteresis and instrument and procedural repeatability can be randomized.

Because of limited resolution in such an instrument, different competent temperature observers might record different indicated temperatures even if the instrument output were fixed. Either observers should be randomized or, if not, the test replicated. It is interesting to note that such a randomization will bring about a predictable scatter in recorded data of about ½ the resolution of the instrument scale.

PROBLEM 1.21

KNOWN: Input voltage, (E_i) and Load (τ_L) can be controlled and varied.

Efficiency (η), Winding temperature (T_w), and Current (I) are measured.

FIND: Specify the dependent, independent in the test and suggest any extraneous variables.

SOLUTION

The measured variables are the dependent variables in the test and they depend on the independent variables of input voltage and load. Several influencing extraneous variables include: ambient temperature (T_a) and relative humidity R; Line voltage fluctuations (e); and each of the individual measuring instruments (m_i). The variation of the independent variables should be performed separately maintaining one independent variable fixed while the other is systematically varied over the test range. A random test procedure for the independent variable will randomize the effects of T_a, R and e. Replication methods using different test instruments would be one way to randomize the effects of the m_i; alternatively, calibration of all measuring instruments would provide a good degree of control over these variables.

$\eta = \eta(E_{I,} \tau_L; T_a, R, e, m_i)$
$T_w = T_w(E_i, \tau_L ; T_a, R, e, m_i)$
$I = I(E_i, \tau_L ; T_a, R, e, m_i)$

PROBLEM 1.22

KNOWN: Specifications Table 1.1
Nominal pressure of 500 cm H_2O to be measured.
Ambient temperature drift between 18 to 25 ^oC

FIND: Magnitude of each elemental error listed.

SOLUTION

Based on the specifications:

r_i = 1000 cm H_2O
r_o = 5 V

Hence, K = 5 V/ 1000 cm H_2O = 5 mV/cm H_2O = 0.005 V/cm H_2O. This gives a nominal output at 500 cm H_2O input of 2.5 V. This assumes that the input/output relation is linear over range but we are told that it is linear to within some linearity error.

linearity error = u_L = (±0.005) (1000 cm H_2O)

= ± 5.0 cm H_2O
= ± 0.025 V

hysteresis error = u_h = (±0.0015)(1000 cm H_2O)

= ±1.5 cm H_2O
= ±0.0075 V

sensitivity error = u_K = (±0.0025)(500 cm H_2O)

= ± 1.25 cm H_2O = ± 0.00625 V

thermal sensitivity error = (±0.0002)(7^{o}C)(500 cm H_2O)

= ±0.7 cm H_2O
= ± 0.0035 V

thermal drift error = (0.0002)(7^{o}C)(1000 cm H_2O)

= 1.4 cm H_2O
= 0.007 V

overall instrument error = $(5^2+1.5^2+1.25^2+0.7^2+1.4^2)^{1/2}$ = 5.6 cm H_2O

PROBLEM 1.23

KNOWN: FSO = 1000 N

FIND: u_c

SOLUTION

From the given specifications, the elemental errors are estimated by:

$u_L = 0.001 \times 1000N = 1N$
$u_H = 0.001 \times 1000N = 1N$

$u_K = 0.0015 \times 1000N = 1.5N$
$u_z = 0.002 \times 1000N = 2N$

The overall instrument error is estimated as:

$u_c = (1^2 + 1^2 + 1.5^2 + 2^2)^{1/2} = 2.9\ N$

COMMENT

This root-sum-square (RSS) method provides a "probable" estimate (i.e. the most likely estimate) of the uncertainty in the instrument error possible in any given measurement.

"Possible" is a key concept here as error values will likely change between individual measurements.

PROBLEM 1.24

SOLUTION

Repetition through repeated measurements made under a fixed set of operating conditions provides a measure of the time (or spatial) variation of a measured variable.

Replication through the duplication of tests conducted under similar operating conditions provides a measure of the effect of control of the operating conditions on the measured variable.

Repetition refers to repeating the measurement during a test.

Replication refers to repeating the test (to repeat the measurements).

PROBLEM 1.25

SOLUTION

Replication is used to assess the ability to control any aspect of a test or its operating condition. Repeat the test resetting the operating conditions to their original set points.

PROBLEM 1.26

SOLUTION

Randomization is used to break-up the effects of interference from either continuous or discrete extraneous (i.e. uncontrolled) variables.

PROBLEM 1.27

SOLUTION

This problem does not have a unique solution. We suggest that the instructor use this problem as a basis for an in-class or small group discussion.

PROBLEM 1.28

FIND: Test matrix to correlate thermostat setting with average room setting

SOLUTION

Although there is no single test matrix, one method of solution follows.

Assume that average room temperature, T, is a function of actual thermostat setting, spatial distribution of temperature, temporal temperature distribution, and thermostat location. We might imagine that for a controlled (fixed) thermostat location, a direct correlation between setting and T could be achieved. However, factors could influence the temperature measured by the thermostat such as sunlight directly hitting the thermostat or the wall on which it is attached or a location directly exposed to furnace forced convection, a condition aggrevated by air conditioners or heat pumps in which delivered air temperature is a strong function of outside temperature. Assume a proper location is selected and controlled.

Further, the average room temperature must be defined because local room temperature will vary will position within the room and with time. For the test matrix, the room should be divided into equal areas with temperature sensing devices placed at the center of each area. The output from each sensor will be averaged over a time period that is long compared to the typical furnace on/off cycle.

Select four temperature sensors: A, B, C, D. Select four thermostat settings: s_1, s_2, s_3, s_4, where $s_1 < s_2 < s_3 < s_4$. Temperatures are to be measured under each setting after the room has adjusted to the new setting. One matrix might be:

BLOCK

1 s_1: A, B, C, D
2 s_4: A, B, C, D
3 s_3: A, B, C, D
4 s_2: A, B, C, D

Note that the order of set temperature has been shuffled to attempt to randomize the test matrix (hysteresis is a common problem in thermostats). The four blocks will yield the average temperatures, T_1, T_4, T_3, T_2. The data can be presented in a form of T versus s.

PROBLEM 1.29

FIND: Test matrix to evaluate fuel efficiency of a production model of automobile

ASSUMPTIONS: Automobile model design is fixed (i.e. neglect options). Require representative estimate of efficiency.

SOLUTION

Although there is no single test matrix, one method of solution follows. Many variables can affect auto model efficiency: e.g. individual car, driver, terrain, speed, ambient conditions, engine model, fuel, tires, options. Whether these are treated as controlled variables or as extraneous variables depends on the test matrix. Suppose we "control" the options, fuel, tires, and engine model, that is fix these for the test duration. Furthermore, we can fix the terrain and the ambient conditions by using a mechanical chassis dynamometer (a device which drives the wheels with a prescribed mechanical load) in an enclosed, controlled environment. In fact, such a machine and its test conditions have been specified within the U.S.A. by government test standards. By programming the dynamometer to start, accelerate and stop using a preprogrammed routine, we can eliminate the effects of different drivers on different cars. However, this test will fail to randomize the effects of different drivers and terrain as noted in the government statement "... these figures may vary depending on how and where you drive ... ." This leaves the car itself and the test speed as independent variables, x_a and x_b, respectively. We defer considering the effects of the instruments and methods used to compute fuel efficiency until a later chapter, but assume here that this can be done with sufficient accuracy.

With this in mind, we could choose three representative cars and three speeds with the test matrix:

BLOCK

1 x_{a1}: x_{b1}, x_{b2}, x_{b3}
2 x_{a2}: x_{b1}, x_{b2}, x_{b3}
3 x_{a3}: x_{b1}, x_{b2}, x_{b3}

Note that since slight differences will exist between cars that can not be controlled, the autos are treated as extraneous variables. This matrix randomizes the effects of differences between cars at three different speeds and yields a curve for fuel efficiency versus speed.

As an alternative, we could introduce a driver into the matrix. We could develop a test track of fixed (controlled) terrain. And we could have three drivers drive three cars at three different speeds. This introduces the driver as an extraneous variable, noted as A_1, A_2 and A_3 for each driver.

Assuming that the tests are run under similar ambient conditions, one test matrix may be

$$
\begin{array}{c|ccc}
 & x_{a1} & x_{a2} & x_{a3} \\
\hline
A_1 & x_{b1} & x_{b2} & x_{b3} \\
A_2 & x_{b2} & x_{b3} & x_{b1} \\
A_3 & x_{b3} & x_{b1} & x_{b2}
\end{array}
$$

PROBLEM 1.30

SOLUTION:

Test stand:

Here one would operate the engine under simulated conditions similar to those encountered at the track – such as anticipated engine RPM and engine load (load: estimated mechanical loads on the engine due to mechanical losses, tire rolling resistance, aerodynamic resistance, etc).

Measure:

- fuel and air consumption
- torque and power output
- exhaust gas temperatures to set air:fuel ratio

Track:

Here one would operate the car at conditions similar to those anticipated during the race.

Measure:

- lap time
- wind and temperature conditions (to normalize lap time)
- depending on team other factors can be measured to estimate loads on the car and car behavior. *Clemson Motorsports Engineering has been active in test method development for professional race teams.*

Obvious major differences:

- Environmental conditions, which effect engine performance, car behavior and tire behavior.
- Engine load on a test stand is well-controlled. On track, the driver does not execute exactly on each lap, hence varies load such as due to differences in drive path 'line' and this affects principally aerodynamic loads and tire rolling resistance. Incidentally, all of these are coupled effects in that a change in one affects the values of the others.
- Ram air effect of moving car can be simulated but difficult to get exactly
- Each engine is an individual. Even slight differences affect handling and therefore, how a driver drives the car (thus changing the engine load).

PROBLEM 1.31

KNOWN: Four lathes, 12 machinists are available to produce batches of machine shafts.

FIND: Test matrix to estimate the tolerances held within a batch

SOLUTION

If we assume that batch precision, P, is only a function of lathe and machinist, then

P = f(lathe, machinist)

We can set up a test matrix using all four lathes, L_1, L_2, L_3, L_4, and all 12 machinists, A, B, ..., L. The machinists are randomly assigned.

BLOCK	
1	L_1: A, B, C
2	L_2: D, E, F
3	L_3: G, H, I
4	L_4: J, K, L

Data from each lathe should be indicative of the precision associated with each lathe and the total ensemble of data indicative of batch precision. However, this test matrix neglects the effects of shift and day of the week.

One method which treats machinist and lathe as extraneous variables and reduces test size selects 4 machinists at random. Suppose more than one shaft size is produced at the plant. We could select 4 shaft diameters, D1, D2, D3, D4 and set up a Latin square matrix:

	L_1	L_2	L_3	L_4
B	D1	D2	D3	D4
E	D2	D3	D4	D1
G	D3	D4	D1	D2
L	D4	D1	D2	D3

Note that neither matrix includes shift or day of the week effects and these could be incorporated in an expanded test matrix.

PROBLEM 1.32

SOLUTION

Linearity error

A random static calibration over a specified range will provide the input-output relationship between y and x (i.e. $y = f(x)$). A first-order curve fit to this data, for example using a least squares regression analysis, will provide the fit $y_L(x)$. The linearity error is simply the difference between the measured value of y at any value of x and the value of y_L predicted by the fit at that x. The uncertainty value assigned as linearity error is defined by the range of these error values over span..This might be the maximum deviation or some statistical measure.

A manufacturer may wish to keep the linearity error below some target value and, hence, may limit the recommended operating range for the system for this purpose. In your experience, you may notice that some systems can be operated outside of their specification range but be aware their elemental errors may exceed the manufacturer's stated values.

Hysteresis error

A sequential static calibration over a specified range will provide the input-output behavior between y and x during upscale-only and downscale-only operations. This will tend to maximize any hysteresis in the system. The hysteresis error is the difference between the upscale value and the downscale value of y at any given x. The uncertainty value assigned as hysteresis error is defined by the range of these errors over the span.

PROBLEM 1.33

KNOWN: 4 brands of tires

8 cars of the same make

FIND: Test matrix to evaluate performance

SOLUTION

Tire performance can mean different things but for passenger tires usually refers to braking and lateral load adhesion during wet and dry operations. For a given series of performance tests, performance will depend on tire and car (a tire will perform differently on different makes of cars). For the same make, subtle differences in production models can affect test results so we treat the car as an individual and extraneous variable.

We could select 4 cars at random (1,2,3,4) to test four tire brands (A,B,C,D)

1: A, B, C, D
2: A, B, C, D
3: A, B, C, D
4: A, B, C, D

This provides a data pool for evaluating tire performance for a make of car. Note we ignore the variable of the test driver but this method will incorporate driver variation by testing four cars. Other strategies could be created.

PROBLEM 1.34

KNOWN: Water at 20°C

$Q = f(C, A, \Delta p, \rho)$
$C = 0.75$; D = 1 m
$2 < Q < 10$ m^3/min

FIND: Expected calibration curve

SOLUTION

Part of a test matrix is to specify the range of the independent variable and to anticipate the range resulting in the dependent variable. In this case, the pressure drop will be measured so that it is the dependent variable during a static calibration. To anticipate the output range of the calibration then:

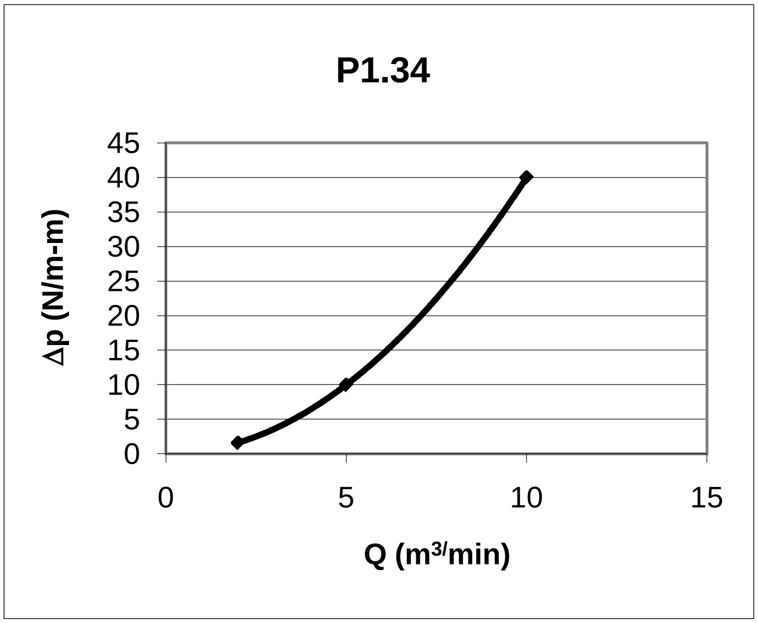
Rearranging the known relation,

$$\Delta p = (Q/CA)^2 (\rho/2)$$

For ρ= 998 kg/m^3 (Appendix C), and A = $\pi D^2/4$, we find:

Q (cmm)	Δp (N/m^2)
2	1.6
5	10
10	40

This is plotted below. It is clear that K will not be a constant as K = f(Q).



PROBLEM 1.35

KNOWN: Address issues associated with metering gasoline at the point of delivery to consumers

FIND: Federal standard and cost of errors

SOLUTION

The federal standard requires that a gas pump be accurate to 6 in^3 in the delivery of 5 gallons of gas, or about 0.52%. In 25 gallons this is 0.13 gallons. With a fleet average for passenger cars of 30.2 MPG, the value of the error in driving 150,000 miles is $65 if gas costs $2.50.

PROBLEM 1.36

SOLUTION

Because $Q \propto \Delta p^{1/2}$ and is not linear, the calibration will not be linear. The term 'linearity' should not be applied directly. The nonlinear calibration result is just a normal consequence of the physics.

However, a signal conditioning stage could be inserted within the signal path to produce a linear output. This is done using logarithmic amplifiers. To illustrate this, plot the calibration curve in Problem 1.34 on a log-log scale (see C1.6). The result will be a linear curve. Alternately, you could take the log of each column and plot them on a rectangular scale to get that same result. A logarithmic amplifier (Chapter 6) performs this same function (as the plot scale or log key) directly on the signal. A linearity measure can then be extracted with some meaning.

As flow rate is the variable varied and pressure drop is the variable measured in this calibration, pressure drop is the dependent variable. The flow rate and the fixed values of area and density are independent variables.

PROBLEM 1.37

KNOWN: pistons are sent out for plating

four subcontractors

FIND: Test matrix for quality control

SOLUTION

Consider four subcontractors as A, B, C, D. One approach is to number the pistons and allocate them to the four subcontractors with subsequent analysis of the plating results. For example, send 24 pistons each to the four subcontractors and analyze the resulting products separately. The variation for each subcontractor can be estimated and can be statistically tested for significant differences.

PROBLEM 1.38

SOLUTION

Controlled variables

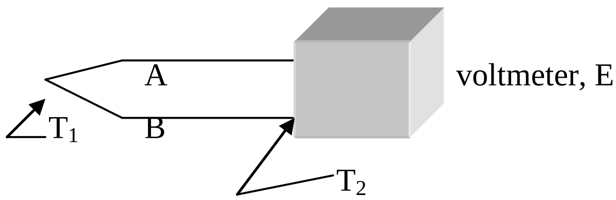
A and B (i.e. control the materials of two alloys)
T_2 (reference junction temperature)

Independent variable

T_1 (measured temperature)

Dependent variable

E (output voltage measured)



PROBLEM 1.39

SOLUTION

Independent variables

micrometer setting (i.e. the applied displacement)

Controlled variable

power supply input

Dependent variable

output voltage measured

Extraneous variables

operator set-up, zeroing of system, and reading of micrometer
ability to set control variables

COMMENT

If you try this you will find that the power supply excitation voltage can have a significant influence on the results. The ability to provide the exact voltage on replication is important in obtaining consistent results in many transducers. Even if you use a regulated laboratory variable power supply, this effect can be seen in your data variation on replication as a random variation. If you use an unregulated source, be prepared to trace these effects as they change from hour to hour or from day to day.

Many LVDT units allow for use of dc power, which is then transformed to ac form before being applied to the coil. It is easiest to see the effect of power setting on the results when using this type of transducer.

PROBLEM 1.40

SOLUTION

To test for repeatability in the LVDT, we might displace the core to various random values over a selected range, such as its expected range, and develop a data base. Data scatter about a curve fit will provide a measure of repeatability for this instrument (methods are discussed in Chapter 4).

Reproducibility involves re-testing the system at a different facility or equivalent (such as different instruments and test fixtures). Think of this as a duplication. Even though a similar procedure and test matrix will be used to test for reproducibility, the duplication involves different individual instruments and test fixtures. A reproducibility test is a special type of replication – by using the different facility constraint added. The combined results allow for interference effects to be randomized.

Bottom Line: The results leading to a reproducibility specification are more representative of what can be expected by the end user (YOU!).

PROBLEM 1.41

SOLUTION:

(i) Running the car on a chassis dynamometer, which applies a desired load to the wheels as the car is operated at a desired speed so as to simulate the car being driven, provides a controlled test environment for estimating fuel consumption. The operating loads form a 'load profile' to simulate the road course.

Allowing a driver to operate a car over a predetermined course provides a realistic simulation of expected consumption. No matter how well controlled the dynamometer test, it is not possible to completely recreate the driving situation that a real driver provides. However, each driver will drive the course a bit differently.

Extraneous variables include: individual entities of driver and of car that affect consumption in either method; road variations and differences between the test methods; road or weather conditions (which are both variable) that change the simulation.

(ii) The dynamometer test is well controlled. In the hands of a good test engineer, valuable information can be ascertained and realistic mileage values obtained. Most important, testing different car models using a predetermined load profile forms an excellent basis for comparison between car makes – this is the basis of a 'standarized test.'.

The variables in a test affect the accuracy of the simulation. Actual values obtained by a particular driver and car are not tested in a standarized test.

(iii) If the two methods are conducted to represent each other, than these are concomitant methods. Even if not exact representations, information obtained in one can be used to get realistic estimates to be expected in the other. For example, a car that gets 10 mpg on the chassis dynamometer should not be expected to get 20 mpg on the road course.

PROBLEM 1.42

SOLUTION:

Spikes in volleyball at the collegiate level may have velocities of 30 m/sec. However, the terminal velocity of a volleyball may be computed from

$$V_t = \sqrt{\frac{2mg}{\rho_{air} A C_d}}$$

where we take the mass of the volleyball to be 0.28 kg, the density of air to be 1 kg/m^3 and the drag coefficient to be 0.5. This yields a terminal velocity below 20 m/sec.

Therefore a reasonable approach would be to drop the volleyball from heights of 5, 10 and 15 m and determine C_R. Examining the data would show a trend from which we could estimate the limiting value of C_R with increasing speed.

PROBLEM 1.43

SOLUTION:

The uncertainty in the velocity measured by light gates may be expressed

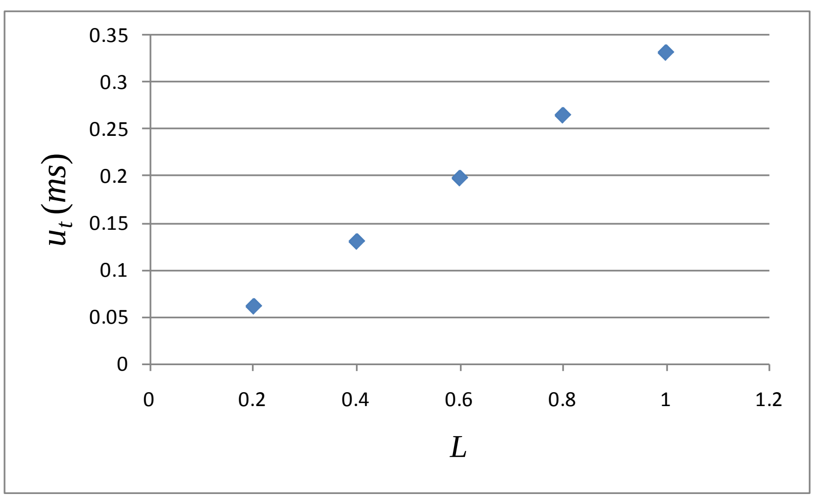
$$\frac{u_V}{V} = \left[\left(\frac{u_L}{L}\right)^2 + \left(\frac{u_{\Delta t}}{\Delta t}\right)\right]^{1/2}$$

Where V is the velocity of the arrow, L is the distance between the light gates, and Δt is the time of flight.

Assume a reasonable uncertainty in L, say 1.5 mm. And assume that we require an uncertainty of 2% in the velocity.

L m	Δt ms
0.2	3.33
0.4	6.66
0.6	10
0.8	13.33
1	16.66

The figure below shows the maximum allowable uncertainty in the time for the projectile to pass through the two light gates to achieve a 2% uncertainty in the velocity, with a nominal velocity of 60 m/sec.



PROBLEM 1.44

SOLUTION:

This is not an uncommon situation when siblings own similar model cars.

The drivers, the cars, and the routes driven are all extraneous variables in this direct comparison. Simply put, you and your brother may drive very differently. You both drive different cars. You likely drive over different routes, maybe very different types of driving routes. You might live in very different geographic locations (altitude, weather). The maintenance of the car would play a role, as well.

An arbitrator might suggest that the two of you swap cars for a few weeks to compare. If the consumption of each car remains the same under different drivers (and associated different routes, location, etc), then the car is the culprit. If not, then driver and other variables remain involved.

PROBLEM 1.45

SOLUTION:

The measure of 'diameter' represents an average or nominal value of the object. Differences along and around the rod affect the value of 'diameter.' Try this with a rod and a micrometer.

Measurements made at different positions along the rod show 'noise', that is data scatter, which is handled statistically (that is, we average the values to obtain a single diameter). Using just a single measurement introduces interference, since that one value may not be the same as the average value.

Tabulated values of material properties represent average or nominal values. These should not be confused as being exact values, regardless of the number of decimal places found in the tables (although the values can be assumed to be reasonably representative to within a decimal place). Properties of a material will vary with individual specimens – as such, differences between a nominal value and the actual specimen value will behave as an interference.

PROBLEM 1.46

SOLUTION

Independent variable:

Applied tensile load

Controlled variable:

Bridge excitation voltage

Dependent variable:

Bridge output voltage (which is related to gauge resistance changes due to the applied load)

Extraneous variables:

Specimen and ambient temperature will affect gauge resistance

A replication will involve resetting the control variable and specimen and duplicating the test.

PROBLEM 1.47

SOLUTION

To test repeatability, apply various tensile loads at random over the useful operating range of the system to build a data base. Be sure to operate within the elastic limit of the specimen. Direct comparison and data scatter about a curve fit will provide a measure of repeatability (specific methods to evaluate this are discussed in C4).

Reproducibility involves re-testing the system at a different facility or equivalent (such as different instruments and test fixtures). Think of this as a duplication. Even though a similar procedure and test matrix will be used to test for reproducibility, the duplication involves different individual instruments and test fixtures. Note that the reproducibility test is also a replication but with the different facility constraint added. The combined results allow for interference effects to be randomized.

Bottom Line: The results leading to a reproducibility specification are more representative of what can be expected by the end user (YOU!).

PROBLEM 1.48

This problem is open-ended and does not have a unique solution. This forms a good opportunity for class discussion.

PROBLEM 1.49

SOLUTION

A car rolling down the hill whose speed is determined by two sensors separated by a distance s. Car speed could be determined as: speed = (distance traveled)/(elapsed time) = $s/(t_2 - t_1)$. The following is a list of the minimum variables that are important in this test:

L: length of car
s: distance between measurements (distance traveled)
θ: angle of inclination
$(t_2 - t_1)$: elapsed time
where
    t_1: instance car passes sensor 1
    t_2: instance car passes sensor 2

Intrinsic assumptions in this test that affect the accuracy of the result:

(1) the speed of the car is actually an average between the speed of the car as it passes sensor 1 and then as it passes sensor 2. The assumption is that any speed change is small in regards to the measured value. This assumption imposes a systematic error on the measured result.

(2) The length of the car could be a factor if it affects how the sensors are triggered. The car is assumed to be a point. This assumption may introduce a systematic error into the results.

As for a concomitant approach:
If we consider the gravitational pull as constant (reasonable over a sensible distance), then the car's acceleration is simply, $a = g\sin\theta$. So its acceleration is easily anticipated and the ideal velocity at any point along the path can be calculated directly from simple physics. The actual velocity will be the ideal velocity reduced by resistance effects, including frictional effects, such as between the car's wheels and the track and within the wheel axles, and aerodynamic effects. The actual velocity will be a bit smaller than the ideal velocity, a consequence of the systematic error in the assumptions. But what it does give us is a value of comparison for our measurement. If the measured value is markedly different, then we will know we have some problems in the test.

PROBLEM 1.50

SOLUTION

The power (P) to move a car at any speed (U) equals the aerodynamic drag (D) plus mechanical drag (M) times the speed plus the parasitic power (Pp) required to turn the compressor and other mechanical components in the car: i.e.

$$P = (D+M)U + Pp$$

With the air conditioning (A/C) off, the parasitic power due to the compressor goes down but because the windows would then be rolled open, the aerodynamic drag goes up. The aerodynamic drag increases with speed while the compressor power remains fairly constant with speed.

To test this question, you might develop a test plan as follows:

Operate the car at several fixed, but well separated, speeds U_1, U_2, U_3 in each of two configurations, A and B. Configuration A uses the compressor and all windows are rolled up closed. Configuration B turns the compressor off but driver window is rolled down (open). Obviously, there can be alternate configurations by rolling down differing windows, but the idea is the same.

A: U_1, U_2, U_3
B: U_1, U_2, U_3

Concomitant approach: An analytical approach to this problem would tradeoff the power required to operate the vehicle at different speeds under the two configurations based on some reasonable published or handbook values (for example, most modern full-sized sedans have a drag coefficient of about 0.33 based on a frontal area of about 2.1 m^2 (these exact values are for a Toyota Camry) for windows closed, increasing to 0.36 with driver window open) and maybe about 3HP to run the compressor. But you might research these numbers.

PROBLEM 1.51

This problem is open-ended and does not have a unique solution. Most of these codes can be found in a library with a quality engineering section or at the appropriate website for the professional group cited. The results from these searches form a good opportunity for class discussion.

PROBLEM 1.52

SOLUTION

a) Your body weight: three significant figures would serve the purpose of security
b) A car's fuel usage: typical range would be 10-50 MPG or 23.5 to 4.7 liters/100 km. So three significant figures would be sufficient for almost all purposes
c) Gold bar: Because of the value of 100 oz of gold, efforts to accurately weigh the gold translate directly to accurately assessing the dollar value. Four or five significant figures should be attained.
d) With the data provided, each with one significant figure, we should report using 1 significant figure.

PROBLEM 1.54

SOLUTION

Transform each relation into the linear form $Y = a_1X + a_0$

(a) KNOWN: $y = bx^m$
This function can be rearranged as

$$\log y = \log bx^m = \log b + m \log x$$

so if we let

$$Y = a_1 + a_0 X$$

then

$$X = \log x \ ; \ Y = \log y \ ; \ a_o = \log b \ ; \ a_1 = m$$

(b) KNOWN: $y = be^{mx}$

identity: ln x = 2.3 log x

This function can be rearranged as

$$\ln y = \ln b + \ln e^{mx} = \ln b + mx$$

so if we let

$$Y = a_1 + a_0 X$$

then

$$X = x \ ; \ a_1 = m \ ; \ a_o = \ln b \ ; \ Y = \ln y$$

(c) KNOWN: $y = b + c\sqrt[m]{x}$
This function can be rearranged as

$$y - b = c\sqrt[m]{x}$$

or $\log (y\text{-}b) = \log c + m \log x$
so if we let

$$Y = a_1 + a_0 X$$

then

$$X = \log x \ ; \ a_1 = m \ ; \ a_o = \log c \ ; \ Y = \log (y\text{-}b)$$