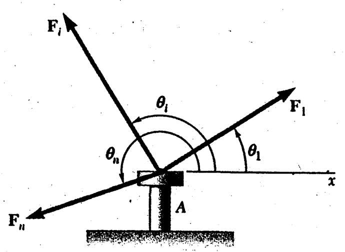
2.C1



GIVEN:
$F_1, F_2, \dots, F_n$
AND $\theta_1, \theta_2, \dots \theta_n$

FIND:
MAGNITUDE AND DIRECTION OF RESULTANT, USING DATA OF PROBS. 2.32, 2.33, 2.35, AND 2.38.

ANALYSIS

$$R_x = \sum_{i=1}^{n} F_i \cos\theta_i, \quad R_y = \sum_{i=1}^{n} F_i \sin\theta \quad (1)$$

WE HAVE $$R = \sqrt{R_x^2 + R_y^2} \quad (2)$$

AND $$\theta_R = \tan^{-1}\frac{R_y}{R_x} \quad (3)$$

LET θ_R^* BE THE VALUE DEFINED BY EQ. (3) AND SUCH THAT $-90° \le \theta_R^* \le +90°$. THEN, THE CORRECT ANSWER WILL BE

IF $R_x \ge 0$ AND $R_y \ge 0$: $\quad \theta_R = \theta_R^* \quad (4)$

IF $R_x \ge 0$ AND $R_y < 0$: $\quad \theta_R = 360° + \theta_R^* \quad (4')$

IF $R_x < 0$: $\quad \theta_R = 180° + \theta_R^* \quad (4'')$

OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER AND NUMBER OF FORCES n.
COMPUTE R_x, R_y, R, AND θ_R^* FROM EQS. ABOVE
DETERMINE θ_R FROM EQ. (4), (4'), OR (4'')

PROGRAM OUTPUT

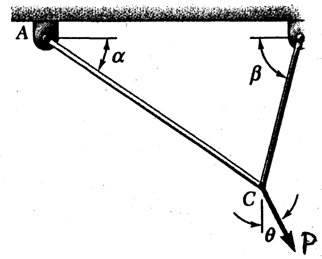
```
Data of Prob. 2.32
Number of forces : N = 3
Force and angle (F, TH)? 80,40
Force and angle (F, TH)? 120,70
Force and angle (F, TH)? 150,145
Resultant =    251.065
Angle between resultant and x axis =   94.69 degrees

Data of Prob. 2.33
Number of forces : N = 3
Force and angle (F, TH)? 60,25
Force and angle (F, TH)? 50,220
Force and angle (F, TH)? 40,300
Resultant =     54.931
Angle between resultant and x axis = 311.05 degrees

Data of Prob. 2.35
Number of forces : N = 3
Force and angle (F, TH)? 200,215
Force and angle (F, TH)? 150,295
Force and angle (F, TH)? 100,325
Resultant =    308.576
Angle between resultant and x axis = 266.56 degrees

Data of Prob. 2.38
Number of forces : N = 3
Force and angle (F, TH)? 60,20
Force and angle (F, TH)? 80,95
Force and angle (F, TH)? 120,5
Resultant =    201.975
Angle between resultant and x axis =   33.23 degrees
```

2.C2



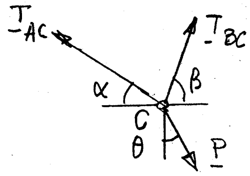
GIVEN:
LOAD P SUPPORTED BY TWO CABLES AS SHOWN

FIND FOR EACH SET OF VALUES SHOWN AND FOR VALUES OF θ FROM $\theta_1 = \beta - 90°$ TO $\theta_2 = 90° - \alpha$ USING INCREMENTS $\Delta\theta$:
(a) TENSION IN EACH CABLE
(b) VALUE OF θ FOR WHICH THE TENSION IN THE CABLES IS AS SMALL AS POSSIBLE.
(c) CORRESPONDING TENSION

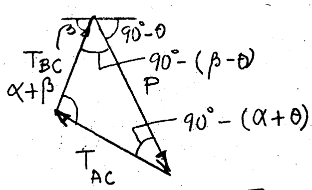
(1) $\alpha = 35°$, $\beta = 75°$, $P = 400$ lb, $\Delta\theta = 5°$
(2) $\alpha = 50°$, $\beta = 30°$, $P = 600$ lb, $\Delta\theta = 10°$
(3) $\alpha = 40°$, $\beta = 60°$, $P = 250$ lb, $\Delta\theta = 5°$

ANALYSIS

F. B. DIAGRAM OF C



FORCE TRIANGLE



LAW OF SINES: $$\frac{P}{\sin(\alpha+\beta)} = \frac{T_{AC}}{\cos(\beta-\theta)} = \frac{T_{BC}}{\cos(\alpha+\theta)}$$

$$T_{AC} = P\,\frac{\cos(\beta-\theta)}{\sin(\alpha+\beta)} \qquad T_{BC} = P\,\frac{\cos(\alpha+\theta)}{\sin(\alpha+\beta)} \quad (1)$$

TENSION IN EACH CABLE IS AS SMALL AS POSSIBLE WHEN $T_{AC} = T_{BC}$, THAT IS, WHEN $\beta - \theta = \alpha + \theta$, $\theta = \frac{\beta-\alpha}{2}$

OUTLINE OF PROGRAM

ENTER SET NUMBER AND VALUES OF $\alpha, \beta, P, \Delta\theta$
COMPUTE $\theta_1 = \beta - 90°$ AND $\theta_2 = 90° - \alpha$
FOR VALUES OF θ FROM θ_1 TO θ_2, USING INCREMENTS $\Delta\theta$,
COMPUTE T_{AC} AND T_{BC} FROM EQS. (1)
PRINT θ, T_{AC}, T_{BC}
CHECK THAT T_{AC} AND T_{BC} ARE EQUAL AND MINIMUM FOR $\theta = \frac{1}{2}(\beta - \alpha)$.

PROGRAM OUTPUT

```
Set No.1
Angle ALPHA = 35
Angle BETA = 75
Magnitude of load P = 400
Increment of THETA = 5

 THETA      TAB      TAC

-15.000   -0.000  400.000
-10.000   37.100  385.789
 -5.000   73.917  368.642
  0.000  110.172  348.689
  5.000  145.588  326.083
 10.000  179.896  300.995
 15.000  212.836  273.616
 20.000  244.155  244.155
 25.000  273.616  212.836
 30.000  300.995  179.896
 35.000  326.083  145.588
 40.000  348.689  110.172
 45.000  368.642   73.917
 50.000  385.789   37.100
 55.000  400.000   -0.000
```

(b) (row 20.000) (c)

continued

2.C2 continued

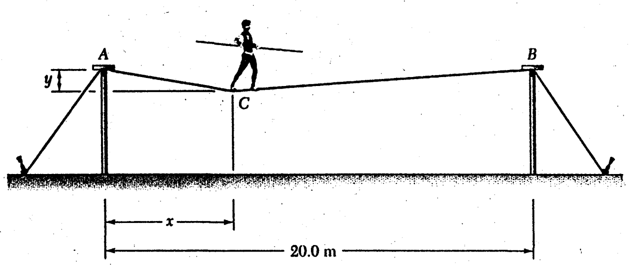
Set No.2
Angle ALPHA = 50
Angle BETA = 30
Magnitude of load P = 600
Increment of THETA = 10

THETA	TAB	TAC
-60.000	-0.000	600.000
-50.000	105.796	609.256
-40.000	208.378	600.000
-30.000	304.628	572.513
-20.000	391.622	527.631
(b) ▶ -10.000	466.717	466.717 ◀ (c)
0.000	527.631	391.622
10.000	572.513	304.628
20.000	600.000	208.378
30.000	609.256	105.796
40.000	600.000	-0.000

Set No.3
Angle ALPHA = 40
Angle BETA = 60
Magnitude of load P = 250
Increment of THETA = 5

THETA	TAB	TAC
-30.000	-0.000	250.000
-25.000	22.125	245.207
-20.000	44.082	238.547
-15.000	65.703	230.072
-10.000	86.824	219.846
-5.000	107.284	207.947
0.000	126.928	194.465
5.000	145.606	179.504
(b) ▶ 10.000	163.176	163.176 ◀ (c)
15.000	179.504	145.606
20.000	194.465	126.928
25.000	207.947	107.284
30.000	219.846	86.824
35.000	230.072	65.703
40.000	238.547	44.082
45.000	245.207	22.125
50.000	250.000	-0.000

2.C3



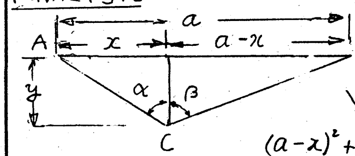
GIVEN:
LENGTH OF ROPE $= L = 20.1\,m$
DISTANCE BETWEEN A AND B $= a = 20\,m$
WEIGHT OF ACROBAT AND BALANCING POLE $= W = 800\,N$
ASSUME NO SLIPPING AND NO ELASTIC DEFORMATION OF ROPE

FIND:
DEFLECTION y AND TENSIONS T_{AC} AND T_{BC} FOR VALUES OF x FROM 0.5 m TO 10.0 m, USING 0.5-m INCREMENTS
FROM DATA OBTAINED, ALSO FIND
(a) MAXIMUM DEFLECTION OF ROPE
(b) MAXIMUM TENSION IN ROPE
(c) SMALLEST VALUES OF T_{AC} AND T_{BC}

continued

2.C3 continued

ANALYSIS



FOR ANY x:

$$AC + CB = L$$

$$\sqrt{x^2+y^2} + \sqrt{(a-x)^2+y^2} = L$$

$$(a-x)^2 + y^2 = \left(L - \sqrt{x^2+y^2}\right)^2$$

$$a^2 - 2ax + \cancel{x^2} + \cancel{y^2} = L^2 - 2L\sqrt{x^2+y^2} + \cancel{x^2} + \cancel{y^2}$$

$$2L\sqrt{x^2+y^2} = L^2 + 2ax - a^2$$

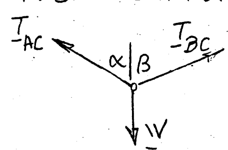
$$4L^2(x^2+y^2) = (L^2+2ax-a^2)^2$$

$$4L^2y^2 = (L^2+2ax-a^2)^2 - 4L^2x^2$$

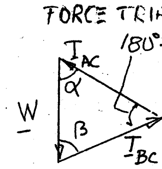
$$y = \frac{1}{2L}\sqrt{(L^2+2ax-a^2)^2 - 4L^2x^2} \qquad (1)$$

ALSO: $\alpha = \tan^{-1}\left(\frac{x}{y}\right) \qquad \beta = \tan^{-1}\left(\frac{a-x}{y}\right) \qquad (2)$

F. B. DIAGRAM OF C:



FORCE TRIANGLE:



LAW OF SINES:

$$\frac{T_{AC}}{\sin\beta} = \frac{T_{BC}}{\sin\alpha} = \frac{W}{\sin(\alpha+\beta)}$$

THUS: $T_{AC} = W\dfrac{\sin\beta}{\sin(\alpha+\beta)}, \qquad T_{BC} = W\dfrac{\sin\alpha}{\sin(\alpha+\beta)} \qquad (3)$

OUTLINE OF PROGRAM

ENTER $a = 20\,m$, $L = 20.1\,m$, $W = 800\,N$
FOR $x = 0.5$ TO 10.1 USING 0.5 STEPS:
COMPUTE y FROM EQ. (1)
COMPUTE α AND β FROM EQS. (2)
COMPUTE T_{AC} AND T_{BC} FROM EQS. (3)

PROGRAM OUTPUT

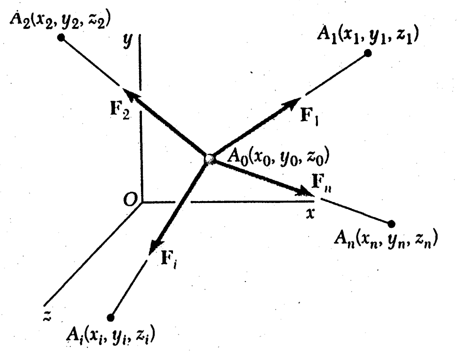
X	Y	TAC	TBC
0.5	0.327	1426.03	1193.97
1.0	0.446	1867.33	1706.14
1.5	0.534	2205.66	2078.69
2.0	0.606	2482.82	2377.48
2.5	0.666	2717.45	2627.65
3.0	0.718	2919.79	2842.03
3.5	0.764	3096.14	3028.22
4.0	0.803	3250.74	3191.13
4.5	0.838	3386.57	3334.19
5.0	0.869	3505.79	3459.85
5.5	0.895	3610.06	3569.95
6.0	0.919	3700.65	3665.90
6.5	0.939	3778.51	3748.76
7.0	0.956	3844.45	3819.40
7.5	0.970	3899.06	3878.49
8.0	0.981	3942.81	3926.55
8.5	0.990	3976.04	3963.95
9.0	0.996	3999.07	3991.06
9.5	1.000	4012.00	4008.01
10.0	1.001	4014.97	4014.97

(a) MAXIMUM DEFLECTION: $y_m = 1.001\,m$ ◀

(b) MAXIMUM TENSION:
FOR $x = 10.0\,m$: $T_{AC} = T_{BC} = 4.01\,kN$ ◀

(c) SMALLEST VALUES OF T_{AC} AND T_{BC}:
FOR $x = 0.5\,m$: $T_{AC} = 1.426\,kN$ ◀
$T_{BC} = 1.194\,kN$ ◀

2.C4



GIVEN: FORCES SHOWN, ACTING ON A_0.
WRITE PROGRAM TO DETERMINE THE MAGNITUDE AND DIRECTION OF THEIR RESULTANT.
APPLY PROGRAM TO SOLVE PROBS. 2.93, 2.94, 2.95, AND 2.135.

ANALYSIS

FIRST, FOR EACH FORCE $\underline{F}_i$ WE DETERMINE THE DISTANCE d_i FROM A_0 TO A_i:

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \quad (1)$$

THE COMPONENTS OF $\underline{F}_i$ ARE

$$(F_x)_i = F_i \frac{x_i - x_0}{d_i}, \ (F_y)_i = F_i \frac{y_i - y_0}{d_i}, \ (F_z)_i = F_i \frac{z_i - z_0}{d_i} \quad (2)$$

THE COMPONENTS OF THE RESULTANT $\underline{R}$ ARE

$$R_x = \sum_{i=1}^{n} (F_x)_i, \ R_y = \sum_{i=1}^{n} (F_y)_i, \ R_z = \sum_{i=1}^{n} (F_z)_i \quad (3)$$

THE MAGNITUDE OF THE RESULTANT $\underline{R}$ IS

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \quad (4)$$

AND ITS DIRECTION COSINES ARE

$$\lambda_x = R_x/R, \ \lambda_y = R_y/R, \ \lambda_z = R_z/R \quad (5)$$

THE ANGLES THAT $\underline{R}$ FORMS WITH THE AXES ARE

$$\theta_x = \cos^{-1}\lambda_x, \ \theta_y = \cos^{-1}\lambda_y, \ \theta_z = \cos^{-1}\lambda_z \quad (6)$$

WHERE VALUES BETWEEN 0 AND 180° SHOULD BE SELECTED

OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER
ENTER COORDINATES x_0, y_0, z_0 OF POINT A_0
ENTER NUMBER OF FORCES n
FOR EACH FORCE $\underline{F}_i$:
- ENTER MAGNITUDE F_i
- ENTER COORDINATES x_i, y_i, z_i OF POINT A_i
- COMPUTE d_i FROM EQ. (1)
- COMPUTE $(F_x)_i, (F_y)_i, (F_z)_i$ FROM EQS. (2)

COMPUTE R_x, R_y, R_z FROM EQS. (3)
COMPUTE R FROM EQ. (4)
COMPUTE $\theta_x, \theta_y, \theta_z$ FROM EQS. (5) AND (6)
IF YOU OBTAIN A NEGATIVE VALUE FOR ANY OF THE ANGLES, ADD 180° TO THAT VALUE.

continued

2.C4 continued

PROGRAM OUTPUT

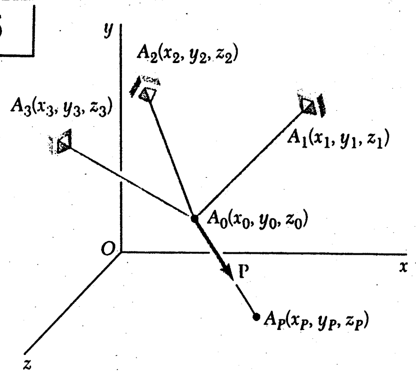
```
Data of Prob. 2.93
Coordinates of A0 ? -40,45,0
Number of given forces: N = 2
Magnitude of F(1)? 425
Coordinates of A(1)? 0,0,60
Magnitude of F(2)? 510
Coordinates of A(2)? 60,0,60
R = 912.92
THX =  48.2, THY = 116.6, THZ =  53.4
```

```
Data of Prob. 2.94
Coordinates of A0 ? -40,45,0
Number of given forces: N = 2
Magnitude of F(1)? 510
Coordinates of A(1)? 0,0,60
Magnitude of F(2)? 425
Coordinates of A(2)? 60,0,60
R = 912.92
THX =  50.6, THY = 117.6, THZ =  51.8
```

```
Data of Prob. 2.95
Coordinates of A0 ? 480,0,600
Number of given forces: N = 2
Magnitude of F(1)? 385
Coordinates of A(1)? 0,510,280
Magnitude of F(2)? 385
Coordinates of A(2)? 210,400,0
R = 747.83
THX = 120.1, THY =  52.5, THZ = 128.0
```

```
Data of Prob. 2.135
Coordinates of A0 ? 0,0,0
Number of given forces: N = 2
Magnitude of F(1)? 10
Coordinates of A(1)? -15.5885,15,12
Magnitude of F(2)? 7.5
Coordinates of A(2)? -15.5885,18.6,-15
R =  15.13
THX = 133.4, THY =  43.6, THZ =  86.6
```

2.C5



GIVEN:

THREE CABLES ATTACHED AT POINT A_0 AND FORCE $\underline{P}$ APPLIED AT A_0 AS SHOWN.

WRITE PROGRAM TO DETERMINE TENSION F_i IN EACH CABLE $(i = 1, 2, 3)$.

APPLY PROGRAM TO SOLVE PROBS. 2.102, 2.106, 2.107, 2.113 AND 2.115.

ANALYSIS

FIRST DETERMINE THE DISTANCE d FROM A_0 TO A_P:

$$d = \sqrt{(z_P - x_0)^2 + (y_P - y_0)^2 + (z_P - z_0)^2} \qquad (1)$$

THE COMPONENTS OF $\underline{P}$ ARE

$$P_x = \frac{P}{d}(x_P - x_0), \quad P_y = \frac{P}{d}(y_P - y_0), \quad P_z = \frac{P}{d}(z_P - z_0) \qquad (2)$$

NEXT DETERMINE FOR EACH CABLE A_0A_i $(i = 1, 2, 3)$ THE DIRECTION THE DISTANCE $d_i = A_0A_i$ AND THE DIRECTION COSINES $(\lambda_x)_i, (\lambda_y)_i, (\lambda_z)_i$:

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \qquad (3)$$

$$(\lambda_x)_i = \frac{x_i - x_0}{d_i}, \quad (\lambda_y)_i = \frac{y_i - y_0}{d_i}, \quad (\lambda_z)_i = \frac{z_i - z_0}{d_i} \qquad (4)$$

THE COMPONENTS OF THE FORCE $\underline{F}_i$ EXERTED BY CABLE A_0A_i ON POINT A_0 ARE

$$(F_x)_i = F_i(\lambda_x)_i, \quad (F_y)_i = F_i(\lambda_y)_i, \quad (F_z)_i = F_i(\lambda_z)_i \qquad (5)$$

WE NOW WRITE THE EQUILIBRIUM EQUATIONS FOR A_0:

$$\sum_{i=1}^{3}(F_x)_i + P_x = 0, \quad \sum_{i=1}^{3}(F_y)_i + P_y = 0, \quad \sum_{i=1}^{3}(F_z)_i + P_z = 0$$

SUBSTITUTING FOR $(F_x)_i, (F_y)_i, (F_z)_i$ FROM (5) AND TRANSFERRING P_x, P_y, P_z:

$$(\lambda_x)_1 F_1 + (\lambda_x)_2 F_2 + (\lambda_x)_3 F_3 = -P_x$$
$$(\lambda_y)_1 F_1 + (\lambda_y)_2 F_2 + (\lambda_y)_3 F_3 = -P_y$$
$$(\lambda_z)_1 F_1 + (\lambda_z)_2 F_2 + (\lambda_z)_3 F_3 = -P_z$$

INTRODUCING THE DETERMINANT

$$\Delta = \begin{vmatrix} (\lambda_x)_1 & (\lambda_x)_2 & (\lambda_x)_3 \\ (\lambda_y)_1 & (\lambda_y)_2 & (\lambda_y)_3 \\ (\lambda_z)_1 & (\lambda_z)_2 & (\lambda_z)_3 \end{vmatrix} \qquad (6)$$

AND THE DETERMINANTS $\Delta_1, \Delta_2, \Delta_3$ OBTAINED BY SUBSTITUTING $-P_x, -P_y, -P_z$ SUCCESSIVELY FOR THE ELEMENTS OF THE FIRST, SECOND, AND THIRD COLUMN OF Δ, WE OBTAIN

$$F_1 = \frac{\Delta_1}{\Delta}, \quad F_2 = \frac{\Delta_2}{\Delta} \quad F_3 = \frac{\Delta_3}{\Delta} \qquad (7)$$

continued

2.C5 continued

OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER

ENTER COORDINATES OF POINT A_0

ENTER MAGNITUDE OF LOAD P

ENTER COORDINATES OF A_P, A_1, A_2, AND A_3

USE EQS. (1) AND (2) TO COMPUTE COMPONENTS OF $\underline{P}$

USE EQS. (3) AND (4) TO COMPUTE $\lambda_x, \lambda_y, \lambda_z$ FOR EACH CABLE

COMPUTE Δ FROM EQ. (6) AND $\Delta_1, \Delta_2, \Delta_3$

COMPUTE F_1, F_2, F_3 FROM EQS. (7) AND PRINT

REMARKS:

IN PROBS. 2.102, 2.113, AND 2.115, CHOOSE FOR A_P ANY POINT DIRECTLY ABOVE A.

IN PROB. 2.106, CHOOSE FOR A_P ANY POINT DIRECTLY UNDER A.

IN PROB. 2.107 CONSIDER P AS THE TENSION F_1 IN A FICTITIOUS CABLE PARALLEL TO THE x AXIS AND CHOOSE FOR A_1 ANY POINT DIRECTLY TO THE RIGHT OF A. ALSO CONSIDER THE TENSION IN AD AS THE GIVEN LOAD.

PROGRAM OUTPUT

```
Data of Prob. 2.102
Coordinates of point A0 ? 0,5.6,0
Magnitude of load: P = 800
Coordinates of point AP ? 0,10,0
Coordinates of point A1 ? -4.2,0,0
Coordinates of point A2 ? 2.4,0,4.2
Coordinates of point A3 ? 0,0,-3.3
F1 = 200.9  F2 = 371.7  F3 = 415.5

Data of Prob. 2.106
Coordinates of point A0 ? 0,-60,0
Magnitude of load: P = 1600
Coordinates of point AP ? 0,-100,0
Coordinates of point A1 ? -36,0,-27
Coordinates of point A2 ? 0,0,32
Coordinates of point A3 ? 40,0,-27
F1 = 570.9  F2 = 829.8  F3 = 527.5

Data of Prob. 2.107
Coordinates of point A0 ? 960,240,0
Magnitude of load: P = 305
Coordinates of point AP ? 0,960,-220
Coordinates of point A1 ? 1200,240,0
Coordinates of point A2 ? 0,0,380
Coordinates of point A3 ? 0,0,-320
F1 = 960.0 (F2 = 446.7) (F3 = 341.7)

Data of Prob. 2.113
Coordinates of point A0 ? 0,100,0
Magnitude of load: P = 1800
Coordinates of point AP ? 0,200,0
Coordinates of point A1 ? -20,0,25
Coordinates of point A2 ? 60,0,18
Coordinates of point A3 ? -20,0,-74
F1 = 973.6  F2 = 531.0  F3 = 532.6

Data of Prob. 2.115
Coordinates of point A0 ? 0,480,0
Magnitude of load: P = 792
Coordinates of point AP ? 0,600,0
Coordinates of point A1 ? -320,0,360
Coordinates of point A2 ? 450,0,360
Coordinates of point A3 ? 250,0,-360
F1 = 510.0  F2 =  56.2  F3 = 536.3
```