Chapter 2—An Introduction to Linear Programming

MULTIPLE CHOICE

1. The maximization or minimization of a quantity is the
   a. goal of management science.
   b. decision for decision analysis.
   c. constraint of operations research.
   d. objective of linear programming.

   ANS: D PTS: 1 TOP: Introduction

2. Decision variables
   a. tell how much or how many of something to produce, invest, purchase, hire, etc.
   b. represent the values of the constraints.
   c. measure the objective function.
   d. must exist for each constraint.

   ANS: A PTS: 1 TOP: Objective function

3. Which of the following is a valid objective function for a linear programming problem?
   a. Max 5xy
   b. Min 4x + 3y + (2/3)z
   c. Max $5x^2 + 6y^2$
   d. Min $(x_1 + x_2)/x_3$

   ANS: B PTS: 1 TOP: Objective function

4. Which of the following statements is NOT true?
   a. A feasible solution satisfies all constraints.
   b. An optimal solution satisfies all constraints.
   c. An infeasible solution violates all constraints.
   d. A feasible solution point does not have to lie on the boundary of the feasible region.

   ANS: C PTS: 1 TOP: Graphical solution

5. A solution that satisfies all the constraints of a linear programming problem except the nonnegativity constraints is called
   a. optimal.
   b. feasible.
   c. infeasible.
   d. semi-feasible.

   ANS: C PTS: 1 TOP: Graphical solution

6. Slack
   a. is the difference between the left and right sides of a constraint.
   b. is the amount by which the left side of a ≤ constraint is smaller than the right side.
   c. is the amount by which the left side of a ≥ constraint is larger than the right side.
   d. exists for each variable in a linear programming problem.

   ANS: B PTS: 1 TOP: Slack variables

7. To find the optimal solution to a linear programming problem using the graphical method
   a. find the feasible point that is the farthest away from the origin.
   b. find the feasible point that is at the highest location.
   c. find the feasible point that is closest to the origin.
   d. None of the alternatives is correct.

   ANS: D PTS: 1 TOP: Extreme points

8. Which of the following special cases does not require reformulation of the problem in order to obtain a solution?
   a. alternate optimality
   b. infeasibility
   c. unboundedness
   d. each case requires a reformulation.

   ANS: A PTS: 1 TOP: Special cases

9. The improvement in the value of the objective function per unit increase in a right-hand side is the
   a. sensitivity value.
   b. dual price.
   c. constraint coefficient.
   d. slack value.

   ANS: B PTS: 1 TOP: Right-hand sides

10. As long as the slope of the objective function stays between the slopes of the binding constraints
    a. the value of the objective function won't change.
    b. there will be alternative optimal solutions.
    c. the values of the dual variables won't change.
    d. there will be no slack in the solution.

    ANS: C PTS: 1 TOP: Objective function

11. Infeasibility means that the number of solutions to the linear programming models that satisfies all constraints is
    a. at least 1.
    b. 0.
    c. an infinite number.
    d. at least 2.

    ANS: B PTS: 1 TOP: Alternative optimal solutions

12. A constraint that does not affect the feasible region is a
    a. non-negativity constraint.
    b. redundant constraint.
    c. standard constraint.
    d. slack constraint.

    ANS: B PTS: 1 TOP: Feasible regions

13. Whenever all the constraints in a linear program are expressed as equalities, the linear program is said to be written in
    a. standard form.
    b. bounded form.
    c. feasible form.
    d. alternative form.

    ANS: A PTS: 1 TOP: Slack variables

14. All of the following statements about a redundant constraint are correct EXCEPT
    a. A redundant constraint does not affect the optimal solution.
    b. A redundant constraint does not affect the feasible region.
    c. Recognizing a redundant constraint is easy with the graphical solution method.
    d. At the optimal solution, a redundant constraint will have zero slack.

    ANS: D PTS: 1 TOP: Slack variables

15. All linear programming problems have all of the following properties EXCEPT
    a. a linear objective function that is to be maximized or minimized.
    b. a set of linear constraints.
    c. alternative optimal solutions.
    d. variables that are all restricted to nonnegative values.

    ANS: C PTS: 1 TOP: Problem formulation

TRUE/FALSE

1. Increasing the right-hand side of a nonbinding constraint will not cause a change in the optimal solution.

   ANS: F PTS: 1 TOP: Introduction

2. In a linear programming problem, the objective function and the constraints must be linear functions of the decision variables.

   ANS: T PTS: 1 TOP: Mathematical statement of the RMC Problem

3. In a feasible problem, an equal-to constraint cannot be nonbinding.

   ANS: T PTS: 1 TOP: Graphical solution

4. Only binding constraints form the shape (boundaries) of the feasible region.

   ANS: F PTS: 1 TOP: Graphical solution

5. The constraint $5x_1 - 2x_2 \leq 0$ passes through the point (20, 50).

   ANS: T PTS: 1 TOP: Graphing lines

6. A redundant constraint is a binding constraint.

   ANS: F PTS: 1 TOP: Slack variables

7. Because surplus variables represent the amount by which the solution exceeds a minimum target, they are given positive coefficients in the objective function.

   ANS: F PTS: 1 TOP: Slack variables

8. Alternative optimal solutions occur when there is no feasible solution to the problem.

   ANS: F PTS: 1 TOP: Alternative optimal solutions

9. A range of optimality is applicable only if the other coefficient remains at its original value.

   ANS: T PTS: 1 TOP: Simultaneous changes

10. Because the dual price represents the improvement in the value of the optimal solution per unit increase in right-hand side, a dual price cannot be negative.

    ANS: F PTS: 1 TOP: Right-hand sides

11. Decision variables limit the degree to which the objective in a linear programming problem is satisfied.

    ANS: F PTS: 1 TOP: Introduction

12. No matter what value it has, each objective function line is parallel to every other objective function line in a problem.

    ANS: T PTS: 1 TOP: Graphical solution

13. The point (3, 2) is feasible for the constraint $2x_1 + 6x_2 \leq 30$.

    ANS: T PTS: 1 TOP: Graphical solution

14. The constraint $2x_1 - x_2 = 0$ passes through the point (200, 100).

    ANS: F PTS: 1 TOP: A note on graphing lines

15. The standard form of a linear programming problem will have the same solution as the original problem.

    ANS: T PTS: 1 TOP: Surplus variables

16. An optimal solution to a linear programming problem can be found at an extreme point of the feasible region for the problem.

    ANS: T PTS: 1 TOP: Extreme points

SHORT ANSWER

1. Explain the difference between profit and contribution in an objective function. Why is it important for the decision maker to know which of these the objective function coefficients represent?

ANS:
Answer not provided.

PTS: 1 TOP: Objective function

2. Explain how to graph the line $x_1 - 2x_2 \geq 0$.

   ANS:
   Answer not provided.

   PTS: 1 TOP: Graphing lines

3. Create a linear programming problem with two decision variables and three constraints that will include both a slack and a surplus variable in standard form. Write your problem in standard form.

   ANS:
   Answer not provided.

   PTS: 1 TOP: Standard form

4. Explain what to look for in problems that are infeasible or unbounded.

   ANS:
   Answer not provided.

   PTS: 1 TOP: Special cases

5. Use a graph to illustrate why a change in an objective function coefficient does not necessarily lead to a change in the optimal values of the decision variables, but a change in the right-hand sides of a binding constraint does lead to new values.

   ANS:
   Answer not provided.

   PTS: 1 TOP: Graphical sensitivity analysis

6. Explain the concepts of proportionality, additivity, and divisibility.

   ANS:
   Answer not provided.

   PTS: 1 TOP: Notes and comments

PROBLEM

1. Solve the following system of simultaneous equations.

   $6X + 2Y = 50$
   $2X + 4Y = 20$

ANS:

X = 8, Y = 1

PTS: 1 TOP: Simultaneous equations

2. Solve the following system of simultaneous equations.

$6X + 4Y = 40$
$2X + 3Y = 20$

ANS:
X = 4, Y = 4

PTS: 1 TOP: Simultaneous equations

3. Consider the following linear programming problem

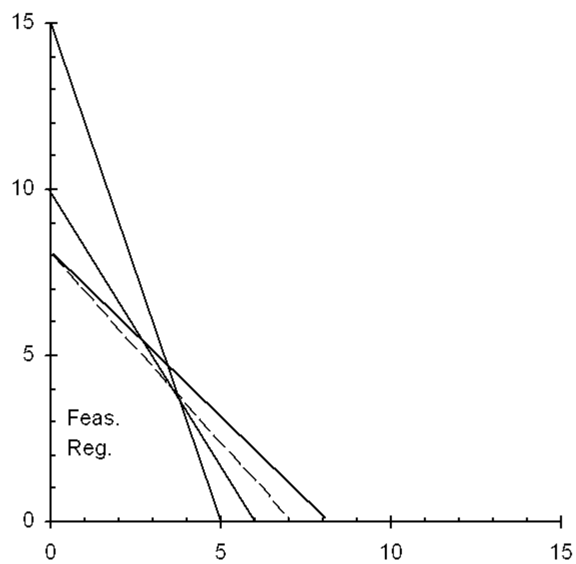
Max $8X + 7Y$

s.t.
$15X + 5Y \leq 75$
$10X + 6Y \leq 60$
$X + Y \leq 8$
$X, Y \geq 0$

a. Use a graph to show each constraint and the feasible region.
b. Identify the optimal solution point on your graph. What are the values of X and Y at the optimal solution?
c. What is the optimal value of the objective function?

ANS:

a.



b. The optimal solution occurs at the intersection of constraints 2 and 3. The point is X = 3, Y = 5.

c. The value of the objective function is 59.

PTS: 1 TOP: Graphical solution

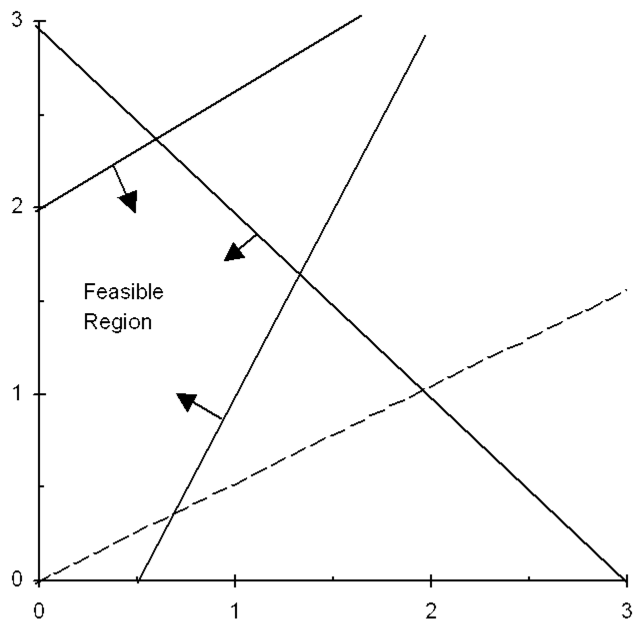
4. For the following linear programming problem, determine the optimal solution by the graphical solution method

Max $-X + 2Y$

s.t.

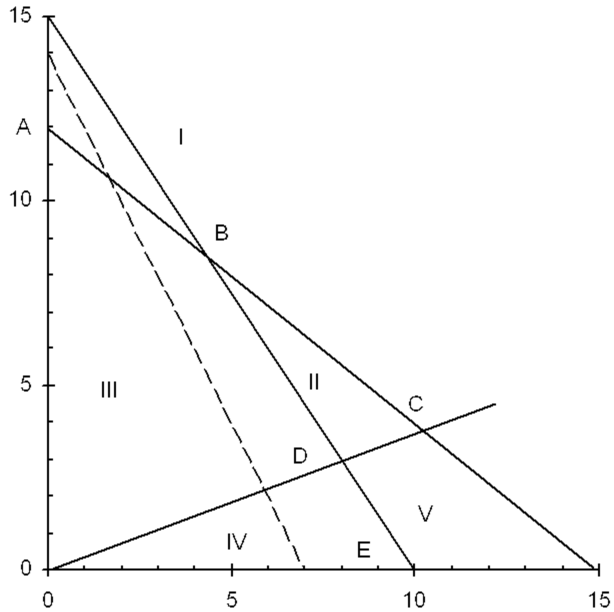
$$\begin{aligned} 6X - 2Y &\leq 3 \\ -2X + 3Y &\leq 6 \\ X + Y &\leq 3 \\ X, Y &\geq 0 \end{aligned}$$

ANS:
X = 0.6 and Y = 2.4



PTS: 1 TOP: Graphical solution

5. Use this graph to answer the questions.



Max $\quad 20X + 10Y$

s.t. $\quad 12X + 15Y \leq 180$
$15X + 10Y \leq 150$
$3X - 8Y \leq 0$
$X , Y \geq 0$

a. Which area (I, II, III, IV, or V) forms the feasible region?
b. Which point (A, B, C, D, or E) is optimal?
c. Which constraints are binding?
d. Which slack variables are zero?

ANS:

a. Area III is the feasible region
b. Point D is optimal
c. Constraints 2 and 3 are binding
d. S_2 and S_3 are equal to 0

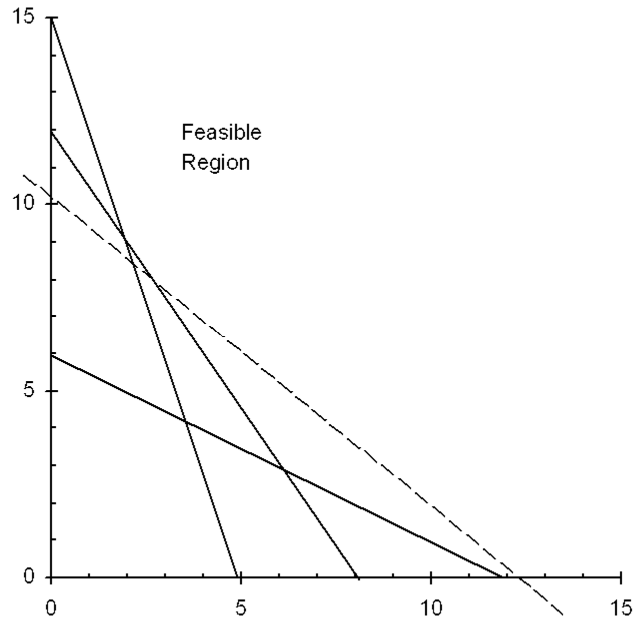
PTS: 1 TOP: Graphical solution

6. Find the complete optimal solution to this linear programming problem.

Min $\quad 5X + 6Y$

s.t. $\quad 3X + Y \geq 15$
$X + 2Y \geq 12$
$3X + 2Y \geq 24$
$X , Y \geq 0$

ANS:



The complete optimal solution is $X = 6, Y = 3, Z = 48, S_1 = 6, S_2 = 0, S_3 = 0$

PTS: 1 TOP: Graphical solution

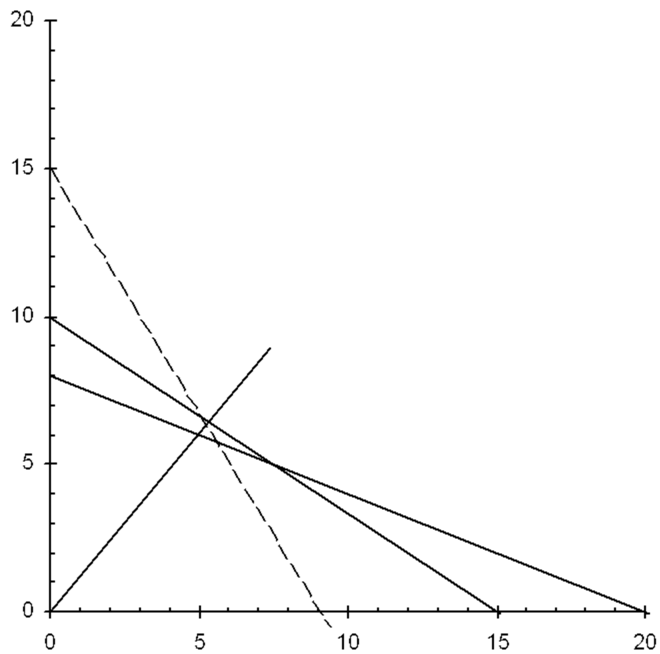
7. And the complete optimal solution to this linear programming problem.

Max $5X + 3Y$

s.t.

$$\begin{aligned} 2X + 3Y &\leq 30 \\ 2X + 5Y &\leq 40 \\ 6X - 5Y &\leq 0 \\ X, Y &\geq 0 \end{aligned}$$

ANS:



The complete optimal solution is X = 15, Y = 0, Z = 75, S_1 = 0, S_2 = 10, S_3 = 90

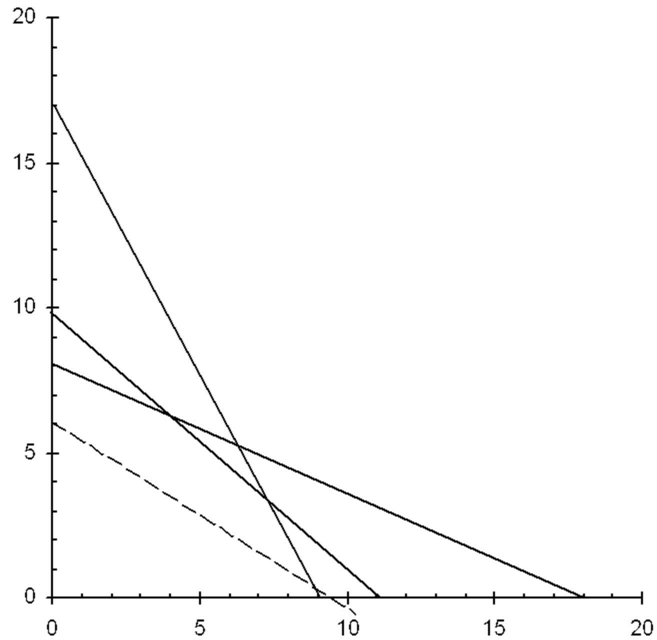
PTS: 1 TOP: Graphical solution

8. Find the complete optimal solution to this linear programming problem.

Max $2X + 3Y$

s.t.
$4X + 9Y \leq 72$
$10X + 11Y \leq 110$
$17X + 9Y \leq 153$
$X, Y \geq 0$

ANS:



The complete optimal solution is $X = 4.304$, $Y = 6.087$, $Z = 26.87$, $S_1 = 0$, $S_2 = 0$, $S_3 = 25.043$

PTS: 1 TOP: Graphical solution

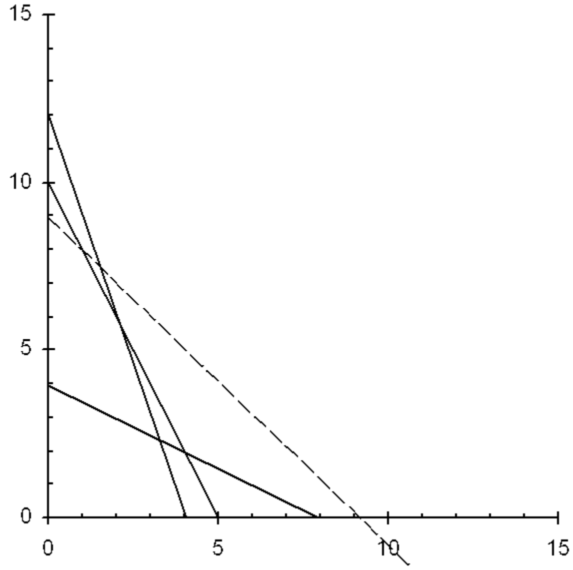
9. Find the complete optimal solution to this linear programming problem.

Min $3X + 3Y$

s.t.

$$\begin{aligned} 12X + 4Y &\geq 48 \\ 10X + 5Y &\geq 50 \\ 4X + 8Y &\geq 32 \\ X, Y &\geq 0 \end{aligned}$$

ANS:



The complete optimal solution is $X = 4, Y = 2, Z = 18, S_1 = 8, S_2 = 0, S_3 = 0$

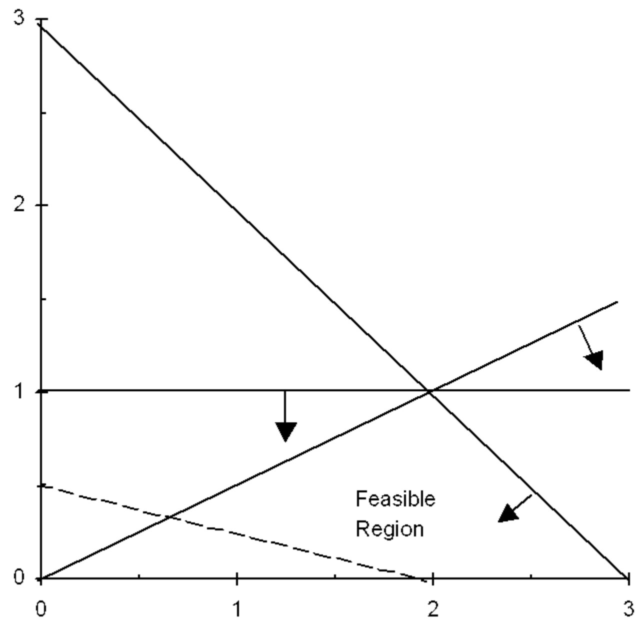
PTS: 1 TOP: Graphical solution

10. For the following linear programming problem, determine the optimal solution by the graphical solution method. Are any of the constraints redundant? If yes, then identify the constraint that is redundant.

Max $X + 2Y$

s.t.
$$\begin{aligned} X + Y &\leq 3 \\ X - 2Y &\geq 0 \\ Y &\leq 1 \\ X, Y &\geq 0 \end{aligned}$$

ANS:
$X = 2$, and $Y = 1$ Yes, there is a redundant constraint; $Y \leq 1$



PTS: 1 TOP: Graphical solution

11. Maxwell Manufacturing makes two models of felt tip marking pens. Requirements for each lot of pens are given below.

	Fliptop Model	Tiptop Model	Available
Plastic	3	4	36
Ink Assembly	5	4	40
Molding Time	5	2	30

The profit for either model is $1000 per lot.

a. What is the linear programming model for this problem?
b. Find the optimal solution.
c. Will there be excess capacity in any resource?

ANS:

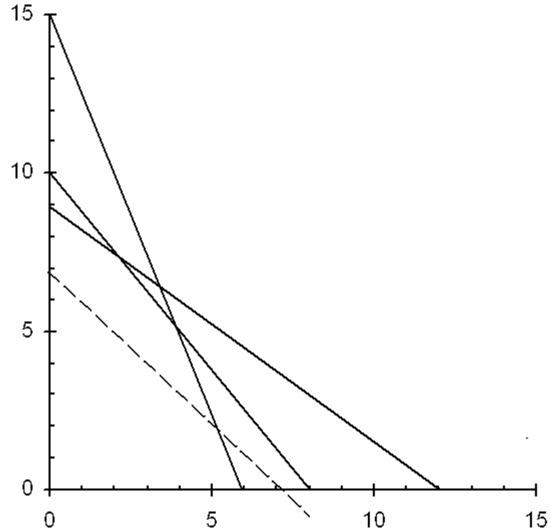
a. Let F = the number of lots of Fliptip pens to produce
Let T = the number of lots of Tiptop pens to produce

Max 1000F + 1000T

s.t.

$$\begin{aligned} 3F + 4T &\leq 36 \\ 5F + 4T &\leq 40 \\ 5F + 2T &\leq 30 \\ F, T &\geq 0 \end{aligned}$$

b.



The complete optimal solution is $F = 2$, $T = 7.5$, $Z = 9500$, $S_1 = 0$, $S_2 = 0$, $S_3 = 5$

c. There is an excess of 5 units of molding time available.

PTS: 1 TOP: Modeling and graphical solution

12. The Sanders Garden Shop mixes two types of grass seed into a blend. Each type of grass has been rated (per pound) according to its shade tolerance, ability to stand up to traffic, and drought resistance, as shown in the table. Type A seed costs \$1 and Type B seed costs \$2. If the blend needs to score at least 300 points for shade tolerance, 400 points for traffic resistance, and 750 points for drought resistance, how many pounds of each seed should be in the blend? Which targets will be exceeded? How much will the blend cost?

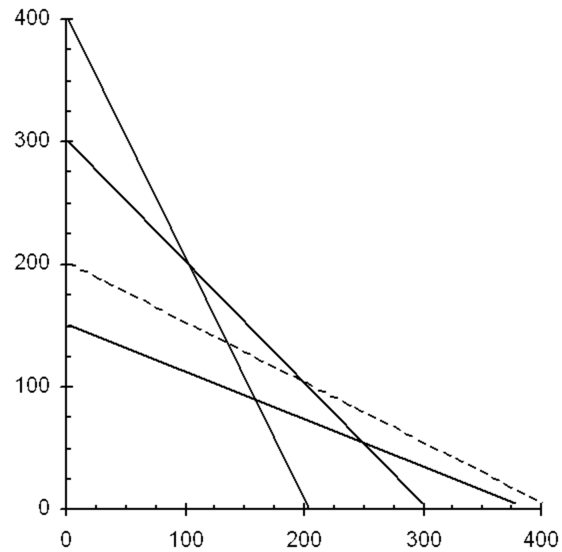
	Type A	Type B
Shade Tolerance	1	1
Traffic Resistance	2	1
Drought Resistance	2	5

ANS:
Let A = the pounds of Type A seed in the blend
Let B = the pounds of Type B seed in the blend

Min $1A + 2B$

s.t.

$$\begin{aligned} 1A + 1B &\geq 300 \\ 2A + 1B &\geq 400 \\ 2A + 5B &\geq 750 \\ A, B &\geq 0 \end{aligned}$$



The optimal solution is at A = 250, B = 50. Constraint 2 has a surplus value of 150. The cost is 350.

PTS: 1 TOP: Modeling and graphical solution

13. Muir Manufacturing produces two popular grades of commercial carpeting among its many other products. In the coming production period, Muir needs to decide how many rolls of each grade should be produced in order to maximize profit. Each roll of Grade X carpet uses 50 units of synthetic fiber, requires 25 hours of production time, and needs 20 units of foam backing. Each roll of Grade Y carpet uses 40 units of synthetic fiber, requires 28 hours of production time, and needs 15 units of foam backing.

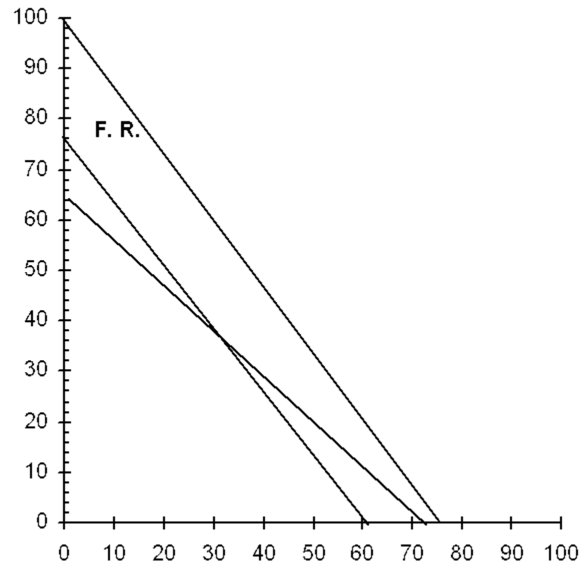
The profit per roll of Grade X carpet is \$200 and the profit per roll of Grade Y carpet is \$160. In the coming production period, Muir has 3000 units of synthetic fiber available for use. Workers have been scheduled to provide at least 1800 hours of production time (overtime is a possibility). The company has 1500 units of foam backing available for use.

Develop and solve a linear programming model for this problem.

ANS:
Let X = the number of rolls of Grade X carpet to make
Let Y = the number of rolls of Grade Y carpet to make

Max $200X + 160Y$

s.t.
$$
\begin{aligned}
50X + 40Y &\leq 3000 \\
25X + 28Y &\geq 1800 \\
20X + 15Y &\leq 1500 \\
X, Y &\geq 0
\end{aligned}
$$



The complete optimal solution is X = 30, Y = 37.5, Z = 12000, S_1 = 0, S_2 = 0, S_3 = 337.5

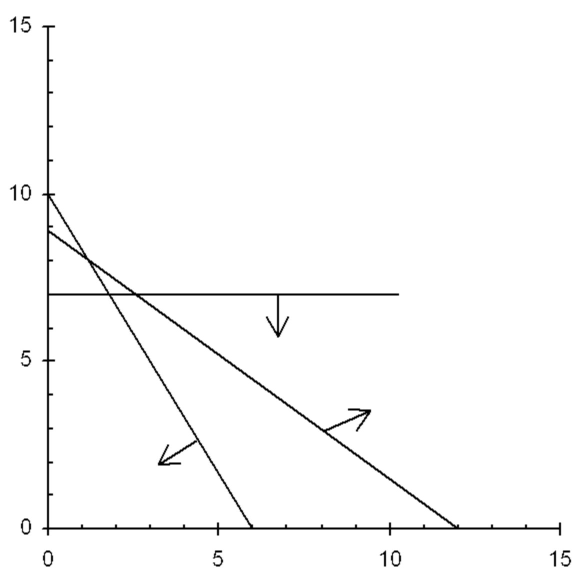
PTS: 1 TOP: Modeling and graphical solution

14. Does the following linear programming problem exhibit infeasibility, unboundedness, or alternate optimal solutions? Explain.

Min 1X + 1Y

s.t.
$5X + 3Y \leq 30$
$3X + 4Y \geq 36$
$Y \leq 7$
$X, Y \geq 0$

ANS:
The problem is infeasible.



PTS: 1 TOP: Special cases

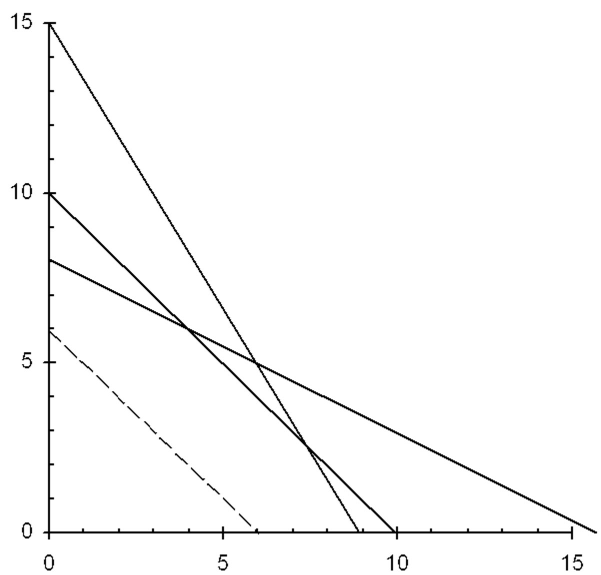
15. Does the following linear programming problem exhibit infeasibility, unboundedness, or alternate optimal solutions? Explain.

Min $3X + 3Y$

s.t.

$$
\begin{aligned}
1X + 2Y &\le 16 \\
1X + 1Y &\le 10 \\
5X + 3Y &\le 45 \\
X, Y &\ge 0
\end{aligned}
$$

ANS:
The problem has alternate optimal solutions.



PTS: 1 TOP: Special cases

16. A businessman is considering opening a small specialized trucking firm. To make the firm profitable, it is estimated that it must have a daily trucking capacity of at least 84,000 cu. ft. Two types of trucks are appropriate for the specialized operation. Their characteristics and costs are summarized in the table below. Note that truck 2 requires 3 drivers for long haul trips. There are 41 potential drivers available and there are facilities for at most 40 trucks. The businessman's objective is to minimize the total cost outlay for trucks.

Truck	Cost	Capacity (Cu. ft.)	Drivers Needed
Small	$18,000	2,400	1
Large	$45,000	6,000	3

Solve the problem graphically and note there are alternate optimal solutions. Which optimal solution:

a. uses only one type of truck?
b. utilizes the minimum total number of trucks?

c. uses the same number of small and large trucks?

ANS:

a. 35 small, 0 large
b. 5 small, 12 large
c. 10 small, 10 large

PTS: 1 TOP: Alternative optimal solutions