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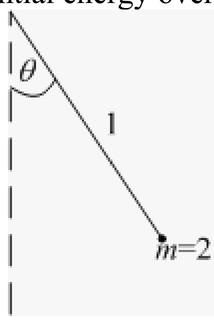
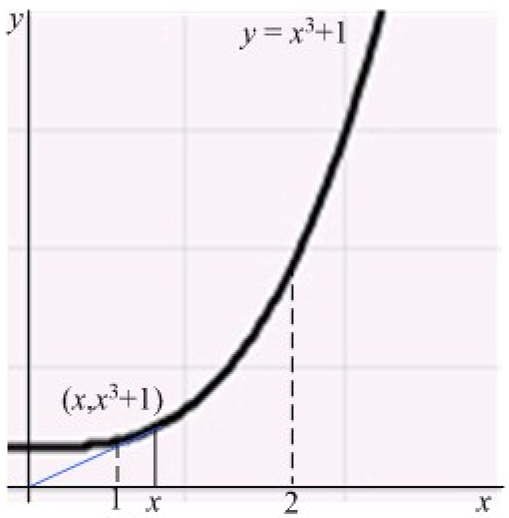
1. The potential energy V of a pendulum of length 1 and mass 2, relative to its rest position is $V = 2g(1-\cos\theta)$. Compute the average rate of change of the potential energy over the angle interval $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$.



ANSWER: $\frac{12g\left(\sqrt{2}-1\right)}{\pi} \approx 1.58g$

2. Let $S(x)$ denote the slope of the line segment connecting the origin to the point (x, y) on the graph of the equation $y = x^3 + 1$. Calculate the average rate of change of $S(x)$ for $1 \le x \le 2$.



ANSWER: $2\frac{1}{2}$

3. The flight time of a shell shot at an angle θ and initial velocity V is $T_f = \frac{2V}{g}\sin\theta$. Compute the average rate of change of the flight time for θ in the interval $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$.

ANSWER: $\frac{12\left(\sqrt{2}-1\right)V}{\pi g}$

4. Let $S(x)$ denote the slope of the line segment connecting the origin to the point (x, y) on the graph of the

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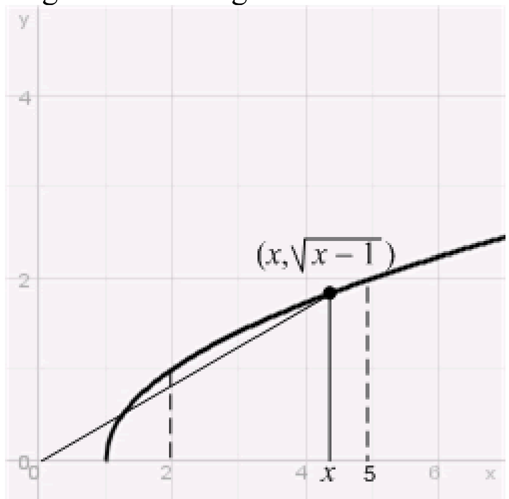
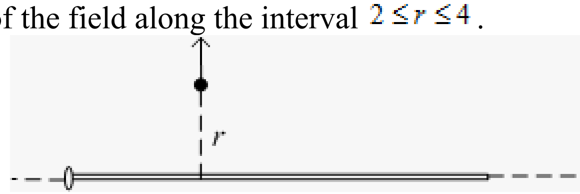
equation $y=\sqrt{x-1}$. Calculate the average rate of change of $S(x)$ for $2\le x\le 5$.



ANSWER: $-\dfrac{1}{30}$

5. The volume of a cone of radius R and height H is $V=\dfrac{\pi R^2 H}{3}$.
What is the average rate of change of V if the radius increases from 1 to 3 and the height remains unchanged?
ANSWER: $\dfrac{4\pi H}{3}$

6. The electrical field due to an infinite rod at a point at distance r from the rod is perpendicular to the rod and has a magnitude of $E(r)=\dfrac{1}{2\pi\varepsilon_0}\cdot\dfrac{\lambda}{r}$ (ε_0 is a constant and λ is the longitudinal charge density).
Find the average rate of change of the field along the interval $2\le r\le 4$.



ANSWER: $-\dfrac{\lambda}{16\pi\varepsilon_0}$

7. Let $S(x)$ denote the slope of the line segment connecting the origin to the point (x, y) on the graph of the semi-ellipse $y=2\sqrt{1-\dfrac{x^2}{9}}$. Calculate the average rate of change of $S(x)$ for $2\le x\le 2.5$

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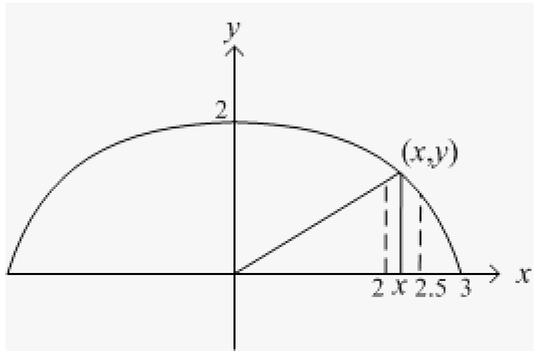


ANSWER: -0.206 per unit length

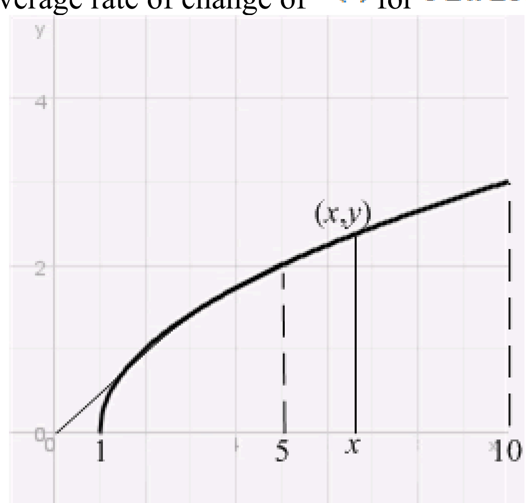
8. The electrical field caused by an electrical charge q at a point at distance r is $E=\frac{kq}{r^2}$ (k is a constant). Find the average rate of change of the field along the interval $1\le r\le 3$.

ANSWER: $-\frac{4kq}{9}$

9. The volume of a sphere of radius R is $V=\frac{4\pi R^3}{3}$. What is the average rate of change of the volume when the radius increases from $R=1$ to $R=3$?

ANSWER: $\frac{52\pi}{3}$

10. Let $S(x)$ denote the slope of the line segment connecting the origin to the point (x,y) on the graph of the equation $y=\sqrt{x-1}$. Calculate the average rate of change of $S(x)$ for $5\le x\le 10$.



ANSWER: $-\frac{1}{50}$

11. The position of a particle is given by $g(t)=2t^2+5$. Compute the average velocity over the time interval $[4, 6]$

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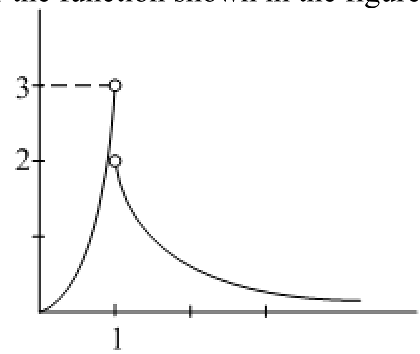
. Estimate the instantaneous velocity at $t=4$.

ANSWER: Average velocity over $[4,6]$: 20
Instantaneous velocity at $t=4$: 16

12. A balloon is blown up and takes the shape of a sphere. What is the average rate of change of the surface area of the balloon as the radius increases from 3 to 4 cm?

ANSWER: 28π

13. Determine $\lim_{x\to 1+} f(x)$ and $\lim_{x\to 1-} f(x)$ for the function shown in the figure.



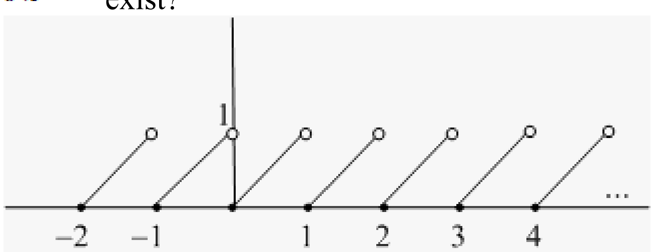
ANSWER: $\lim_{x\to 1+} f(x)=2$ $\lim_{x\to 1-} f(x)=3$

14. The greatest integer function is defined by $[x]=n$, where n is the unique integer such that $n\le x<n+1$. The graph of $f(x)=x-[x]$ is shown in the figure.

A) For which values of c does $\lim_{x\to c-} f(x)$ exist?

B) For which values of c does $\lim_{x\to c+} f(x)$ exist?

C) For which values of c does $\lim_{x\to c} f(x)$ exist?



ANSWER: A) All c
B) All c
C) Every real number c that is not an integer

15. The graph of a function $y=f(x)$ is shown in the figure.
Determine the following limits or state that the limit does not exist (if the limit is infinite, write ∞ or $-\infty$):

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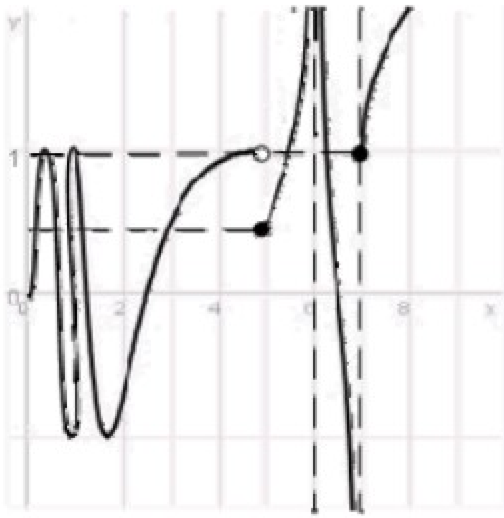
A) $\lim_{x\to 0+} f(x)$ B) $\lim_{x\to 5-} f(x)$ C) $\lim_{x\to 5+} f(x)$ D) $\lim_{x\to 5} f(x)$ E) $\lim_{x\to 6-} f(x)$

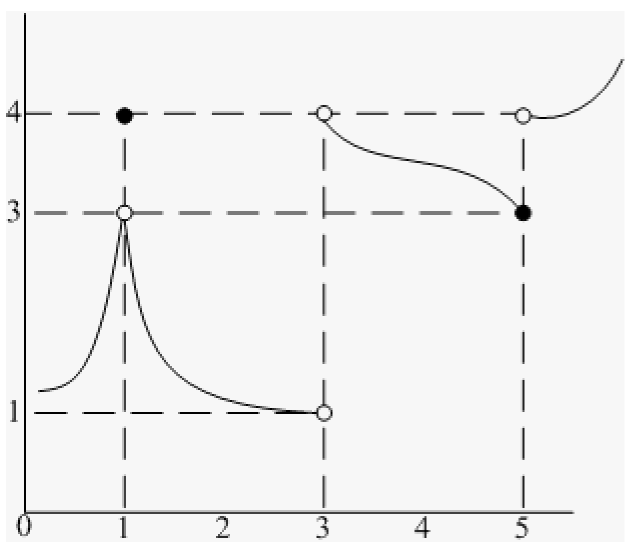
F) $\lim_{x\to 6+} f(x)$ G) $\lim_{x\to 6} f(x)$ H) $\lim_{x\to 7-} f(x)$ I) $\lim_{x\to 7+} f(x)$ J) $\lim_{x\to 7} f(x)$



ANSWER: A) 0 B) 1 C) $\frac{1}{2}$ D) Does not exist E) ∞

F) ∞ G) ∞ H) $-\infty$ I) 1 J) Does not exist

16. Determine the one-sided limits at $c = 1, 3, 5$ of the function $f(x)$ shown in the figure and state whether the limit exists at these points.



ANSWER: $\lim_{x\to 1-} f(x) = \lim_{x\to 1+} f(x) = 3$; limit exists

$\lim_{x\to 3-} f(x) = 1,\ \lim_{x\to 3+} f(x) = 4$; limit does not exist

$\lim_{x\to 5-} f(x) = 3,\ \lim_{x\to 5+} f(x) = 4$; limit does not exist

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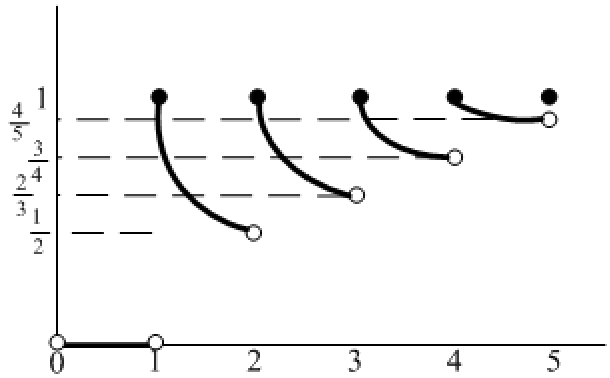
17. Consider the function $f(x)=\dfrac{[x]}{x}$ for $x>0$. (Here, [x] denotes the greatest integer function.)

A) Write $f(x)$ in piecewise form.

What is $f(n)$ for positive integers n?

B) Determine $\lim_{x\to 3-} f(x)$ and $\lim_{x\to 3+} f(x)$.

C) For which values of c does $\lim_{x\to c} f(x)$ exist?



ANSWER:

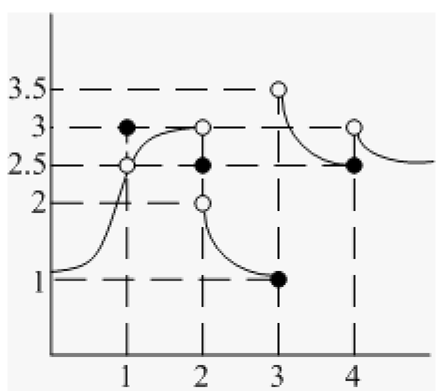
$$f(x)=\begin{cases}0 & 0<x<1\\ \dfrac{n}{x} & n\le x<n+1 \quad n=1,2,\ldots\end{cases}$$

A) $f(n)=1 \quad n=1,2,\ldots$

B) $\dfrac{2}{3},1$

C) The limit exists for all positive real numbers that are not integers.

18. Determine the one-sided limits at $c=1,2,3,4$ of the function shown in the figure and state whether the limit exists at these points.



ANSWER:

c:Left-sided:Right-sided:Limit
1:2.5:2.5:Exists (2.5)
2:3:2:Does not exist
3:1:3.5:Does not exist
4:2.5:3:Does not exist

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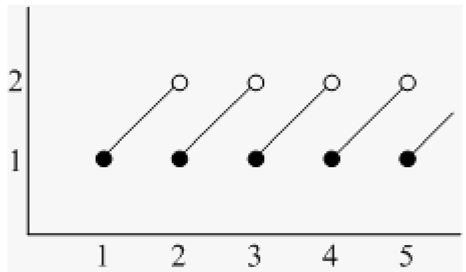
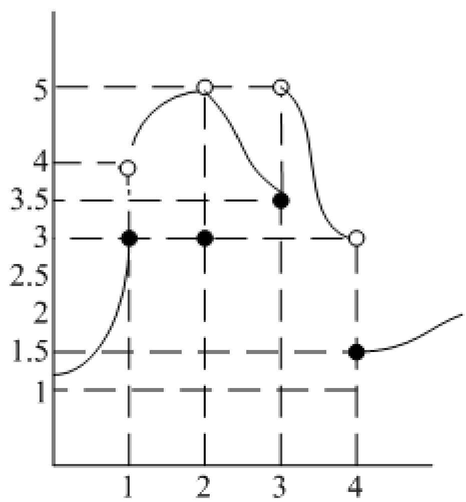
19. Consider the function $f(x)=x+1-[x]$ for $x\geq 1$. (Here, [*x*] denotes the greatest integer function.)
A) Write f in piecewise form. What is $f(n)$ for positive integers $n\geq 1$?
B) Find $\lim_{x\to 2-} f(x)$ and $\lim_{x\to 2+} f(x)$.
C) For which values of c does the limit $\lim_{x\to c} f(x)$ fail to exist?



ANSWER: A) $f(x)=x+1-n$ for $n\leq x<n+1$, $f(n)=1$
B) $2, 1$
C) The limit fails to exist for all positive integers.

20. Determine the one-sided limits at $c=1, 2, 3, 4$ of the function shown in the figure and state whether the limit exists at these points.



ANSWER: *c*:Left-sided:Right-sided:Limit
1:3:4:Does not exist
2:5:5:Exists (5)
3:3.5:5:Does not exist
4:3:1.5:Does not exist

21. Let $f(x)$ be the following function defined for $-0.5\leq x\leq 4.5$:

$$f(x)=\begin{cases} 1, & \text{if } \sin\left(\frac{\pi x}{2}\right)>0 \\ -1, & \text{if } \sin\left(\frac{\pi x}{2}\right)<0 \\ 0, & \text{if } \sin\left(\frac{\pi x}{2}\right)=0 \end{cases}$$

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Write $f(x)$ as a piecewise-defined function where the intervals are in terms of x instead of $\sin\left(\frac{\pi x}{2}\right)$, sketch its graph, and determine the points where the limit of $f(x)$ does not exist. Find the one-sided limits at these points.

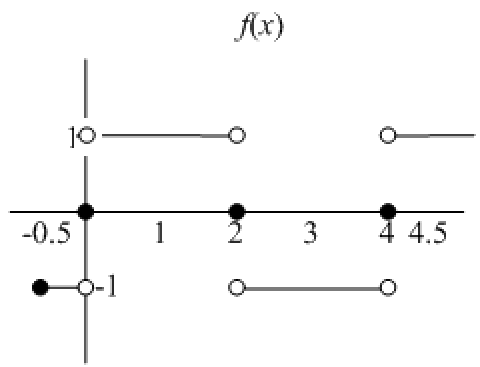
ANSWER:

$$f(x)=\begin{cases}-1 & -0.5\le x<0\\ 0 & x=0\\ 1 & 0<x<2\\ 0 & x=2\\ -1 & 2<x<4\\ 0 & x=4\\ 1 & 4<x\le 4.5\end{cases}$$

$x=-0.5,0,2,4,4.5$

$\lim_{x\to -0.5+} f(x)=-1$ $\lim_{x\to 0-} f(x)=-1$ $\lim_{x\to 0+} f(x)=1$ $\lim_{x\to 2-} f(x)=1$

$\lim_{x\to 2+} f(x)=-1$ $\lim_{x\to 4-} f(x)=-1$ $\lim_{x\to 4+} f(x)=1$ $\lim_{x\to 4.5-} f(x)=1$



22. Find a real number c such that $\lim_{x\to 1} f(x)$ exists and compute the limit.

$$f(x)=\begin{cases}x-\dfrac{3}{x-2} & x<1\\ 10 & x=1\\ \dfrac{c}{(x+1)^2} & x>1\end{cases}$$

ANSWER: $c=16;\ \lim_{x\to 1} f(x)=4$

23. Let $\lim_{x\to a} f(x)=L$. Determine whether each of the following statements is always true, never true, or sometimes true.

A) $\lim_{x\to a-} f(x)=L$

B) $4f(a)=3L$

C) $\lim_{x\to a-} f(x)-\lim_{x\to a+} f(x)<0$

D) $\dfrac{\lim_{x\to a-} f(x)}{\lim_{x\to a+} f(x)}=1$

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ANSWER: A) Always
B) Sometimes
C) Never
D) Sometimes (note the case when $\lim_{x \to a} f(x) = 0$)

24. Compute the following one-sided limits:

A) $\lim_{x \to 2-} \frac{\sqrt{2-x}}{x^2+5x}$

B) $\lim_{\theta \to 0+} \frac{\theta^3 \cos^2\theta}{\sin\theta}$

C) $\lim_{\theta \to \frac{\pi}{2}+} \frac{\theta^3}{\tan\theta}$

ANSWER: A) 0
B) 0
C) 0

25. Evaluate the limits using the Limit Laws:

A) $\lim_{x \to -1} \left(x^3 - 2x^2 + 1\right)$

B) $\lim_{t \to 1} \frac{t^2 - t}{t+1}$

C) $\lim_{x \to 0} \frac{1+\cos x}{x^3+2}$

D) $\lim_{t \to 0} \frac{3\sin t}{2t}$

ANSWER: A) -2
B) 0
C) 1
D) $\frac{3}{2}$

26. Which of the following functions are examples of the existence of the limit $\lim_{x \to 0} \frac{f(x)}{g(x)}$, although the limits of $f(x)$ and $g(x)$ as $x \to 0$ do not exist?

a. $f(x) = x \quad g(x) = \frac{1}{x}$

b. $f(x) = \frac{\sin x}{x} \quad g(x) = \frac{1}{x}$

c. $f(x) = \frac{1}{x} \quad g(x) = \frac{1}{x^3}$

d. $f(x) = x^2 \quad g(x) = \cos x$

e. $f(x)=\dfrac{x}{\sin x} \quad g(x)=\dfrac{1}{x}$

ANSWER: c

27. Let $f(x)$, $g(x)$ be functions and let $F(x)=f(x)-g(x)$. Consider the following statement: If $\lim_{x\to x_0} F(x)$ and $\lim_{x\to x_0} g(x)$ exist, then $\lim_{x\to x_0} f(x)$ also exists. To prove this statement, we should use which of the following?

a. The statement is not true.

b. The Product Rule applied to $\dfrac{F+g}{g}$ and g.

c. The Quotient Rule applied to $(F+g)g$ and g.

d. The Sum Rule applied to F and g.

ANSWER: d

28. Evaluate the limits using the Limit Laws:

A) $\lim_{t\to(-2)} (2t+1)(t^2+2)$

B) $\lim_{x\to(-1)} \dfrac{x^2+3x}{x-1}$

C) $\lim_{x\to\frac{\pi}{4}} \dfrac{\sin x+\cos x}{2\tan x}$

D) $\lim_{x\to 4} \dfrac{2x^{-1}+x^{-\frac{1}{2}}}{x+3}$

ANSWER:
A) -18
B) 1
C) $\dfrac{\sqrt{2}}{2}$
D) $\dfrac{1}{7}$

29. Determine whether the following statement is correct: If $\lim_{x\to 0} x\,g(x)=0$, then $\lim_{x\to 0} g(x)$ exists. If yes, prove it; otherwise, give a counterexample

ANSWER: False; $g(x)=\begin{cases}1 & x\ge 0\\ -1 & x<0\end{cases}$

30. Evaluate the limits using the Limit Laws:

A) $\lim_{t\to 3}\left(t^2+t-1\right)\sin\dfrac{\pi t}{2}$

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B) $\lim_{x \to -1} \frac{x^3+5}{x^2+2x-1}$

C) $\lim_{y \to 4} \frac{y^{-\frac{1}{2}} \tan\left(\frac{\pi y}{16}\right)}{\sqrt{y^2+9}}$

ANSWER:

A) -11
B) -2
C) $\frac{1}{10}$

31. A) Can the Product Rule be used to compute the limit $\lim_{x \to 0} [x]x$? (Here, [x] denotes the greatest integer function.) Explain.

B) Show that $\lim_{x \to 0} [x]x$ exists and find it. *Hint:* Compute the one-sided limits.

ANSWER:

A) No. The limit $\lim_{x \to 0} [x]$ does not exist.
B) 0

32. Let $f(x)$, $g(x)$, and $F(x) = f(x) + g(x)$. To prove that if $\lim_{x \to x_0} f(x)$ and $\lim_{x \to x_0} F(x)$ exist then also $\lim_{x \to x_0} g(x)$ exists, we should use which of the following?

a. The Product Rule applied to $\frac{F-f}{f}$ and f.
b. The Quotient Rule applied to $(F-f)f$ and f.
c. The Sum Rule applied to F and $-f$.
d. The statement is not true.
e. Both A and C

ANSWER: c

33. Evaluate the limits using the Limit Laws:

A) $\lim_{x \to -1} \frac{x+2}{x^3-x-1}$

B) $\lim_{x \to 1} \left(x^2 - x^{-3} + x\right)(x+1)$

C) $\lim_{x \to \frac{\pi}{4}} \frac{\sin x + \cos x}{x}$

ANSWER:

A) -1
B) 2
C) $\frac{4\sqrt{2}}{\pi}$

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34. Consider this statement: If $\lim_{x\to x_0} f(x) = c \neq 0$ and $\lim_{x\to x_0} g(x) = 0$, then $\frac{f(x)}{g(x)}$ does not converge to a finite limit as $x \to x_0$.

To prove this statement, we assume that $\lim_{x\to x_0} \frac{f(x)}{g(x)} = M$ exists and is finite. Then, by the Quotient Rule, $\lim_{x\to x_0} \frac{g(x)}{f(x)} = 0$ and by the Product Rule, $\lim_{x\to x_0} \left(\frac{f(x)}{g(x)} \cdot \frac{g(x)}{f(x)}\right) = 0$.
Which of the following statements completes the proof?

a. From $\lim_{x\to x_0} \left(\frac{f(x)}{g(x)} \cdot \frac{g(x)}{f(x)}\right) = 0$, it follows that 1 = 0, which is a contradiction.

b. From $\lim_{x\to x_0} \frac{g(x)}{f(x)} = 0$, we can conclude that $\lim_{x\to x_0} \frac{f(x)}{g(x)} = 0$, which contradicts our assumption.

c. From $\lim_{x\to x_0} \frac{g(x)}{f(x)} = 0$, we can conclude that $\lim_{x\to x_0} \frac{f(x)}{g(x)} = \infty$, which contradicts our assumption.

d. From $\lim_{x\to x_0} \left(\frac{f(x)}{g(x)} \cdot \frac{g(x)}{f(x)}\right) = 0$, we can conclude that $\lim_{x\to x_0} \frac{f(x)}{g(x)} = 1$, which contradicts our assumption.

ANSWER: a

35. Which of the following functions are examples of the existence of the limit $\lim_{x\to 0} \frac{f(x)}{g(x)}$, although the limits $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist?

a. $f(x) = \frac{1}{x}$, $g(x) = x^2$

b. $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{\sin x}$

c. $f(x) = [x]$, $g(x) = x$ (Here, [x] denotes the greatest integer function.)

d. $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$

e. $f(x) = \frac{1}{1-x}$, $g(x) = \frac{1}{1-\sin x}$

ANSWER: b

36. Assume a and L are nonzero real numbers. If $\lim_{x\to a} 2f(x) = L$ and $\lim_{x\to a} \frac{g(x)}{4} = 0$, calculate the following limits, if possible. If not, state that it is not possible.

A) $\lim_{x\to a} f(x) \cdot g(x)$

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B) $\lim_{x\to a} \frac{f(x)}{g(x)}$

C) $\lim_{x\to a} \frac{f(x)+x^2}{g(x)+a}$

ANSWER: A) 0
B) Not possible
C) $\frac{\frac{L}{2}+a^2}{a}$

37. Determine the points at which the following functions are not continuous and state the type of discontinuity: removable, jump, infinite, or none of these.

A) $f(x) = \frac{x^2-1}{|x-3|}$

B) $g(x) = \frac{\sin x}{x}$

C) $h(x) = x-[x]$ (Here, [x] denotes the greatest integer function.)

D) $j(x) = \left|\sin\frac{1}{x}\right|$

E) $k(x) = \frac{3x^2-27}{3+x}$

ANSWER: A) $x=3$; infinite
B) $x=0$; removable
C) Integers; jump
D) $x=0$; none of these
E) $x = -3$; removable

38. At each point of discontinuity, state whether the function is left or right continuous:

A) $f(x) = \begin{cases} \sin\left(\frac{\pi x}{2}\right)+4 & |x|\le 2 \\ |x-2| & |x|>2 \end{cases}$

B) $f(x) = \begin{cases} 1 & x\le 0 \\ \frac{\sin x}{x} & 0<x\le\frac{\pi}{2} \\ \frac{2x}{\pi-x} & \frac{\pi}{2}<x<\pi \\ x-\pi & \pi\le x \end{cases}$

ANSWER: A) $x=2$; left continuous
B) $x=\frac{\pi}{2}$; left continuous

$x=\pi$; right continuous

39. Determine real numbers *a, b,* and c that make the function continuous:

$$f(t)=\begin{cases} a & t<0 \\ \frac{1}{4}t(t+8) & 0\le t<b \\ t+3 & b\le t<4 \\ c & 4\le t \end{cases}$$

ANSWER: $a=0, b=2, c=7$

40. Find the points of discontinuity for each of these functions and state the type of discontinuity: removable, jump, infinite, or none of these.

A) $f(x) = \frac{|4+x|}{4+x}$

B) $g(x) = \frac{[x]}{x}, \ x>0$ (Here, [*x*] denotes the greatest integer function.)

C) $h(x) = \frac{1-x}{x^2+4x-5}$

ANSWER: A) $x = -4$; jump
B) x = positive integer; jump
C) $x=1$ removable; $x=-5$ infinite

41. Determine whether the function is left or right continuous at each of its points of discontinuity:

A) $f(x)=\begin{cases} \cos \pi x & |x|\le \frac{1}{2} \\ x-\frac{1}{2} & |x|>\frac{1}{2} \end{cases}$

B) $f(x)=x^2[x], x\ge 0$ (Here, [*x*] denotes the greatest integer function.)

ANSWER:
A) $x=-\frac{1}{2}$ right continuous
B) Right continuous at the positive integers

42. Determine real numbers a, b, and c that make the following function continuous:

$$f(t)=\begin{cases} t+a & t<0 \\ t^2+t+b+\frac{a}{2} & 0\le t<1 \\ t-b & 1\le t<2 \\ c & 2\le t \end{cases}$$

ANSWER: $a=-\frac{2}{3},\ b=-\frac{1}{3},\ c=\frac{7}{3}$

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43. Determine the points where the function is not continuous and state the type of the discontinuity: removable, jump, infinite, or none of these.

A) $f(x) = \dfrac{x^2+2x-8}{|x-2|}$

B) $g(x) = \dfrac{x}{[x]}, x \geq 1$ (Here, [x] denotes the greatest integer function.)

C) $h(x) = \dfrac{(x^3-3x+2)\sin 2x}{x}$

D) $j(x) = \dfrac{4}{|x|-3}$

ANSWER: A) $x=2$, jump
B) $x=2,3,4,\ldots$; jump
C) $x=0$; removable
D) $x = 3$, $x = -3$; infinite

44. At each point of discontinuity, state whether the function is left or right continuous.

A) $f(x) = \begin{cases} \sin\frac{1}{x} & x<0 \\ 1+x^2 & 0 \leq x < 2 \\ (x+1)^2-4 & 2 \leq x < 3 \\ 10 & 3 \leq x \end{cases}$

B) $f(x) = \begin{cases} |x-1| & x \leq 2 \\ x^2-3 & 2 < x \leq 4 \\ \frac{1}{x-4} & 4 < x < 5 \\ 6 & 5 \leq x \end{cases}$

ANSWER: A) $x=0$; right continuous
$x=3$; right continuous
B) $x=4$; left continuous
$x=5$; right continuous

45. Determine real numbers a, b, and c that make the following function continuous:

$$f(x) = \begin{cases} a & x \leq -1 \\ \dfrac{x}{[x]} & -1 < x < 0 \\ \dfrac{\sin\left(\frac{\pi x}{2}\right)+b+c}{x^2+1} & 0 \leq x < 1 \\ b & 1 \leq x \end{cases}$$

(Here, [x] denotes the greatest integer function.)

ANSWER: $a=1;\ b=\frac{1}{2};\ c=-\frac{1}{2}$

46. Determine the points where the function is not continuous and state the type of discontinuity: removable, jump, infinite, or none of these:

A) $f(x) = \frac{x^2+x-6}{x-2}$

B) $g(x) = \frac{1}{x-2}+\sin\frac{1}{x}$

C) $h(x) = [x]x$ (Here, [x] denotes the greatest integer function.)

D) $j(x) = \frac{x^2+x-6}{x-3}$

ANSWER: A) $x=2$; removable
B) $x=2$; infinite
$x=0$; none of these
C) Nonzero integers; jump
D) $x=3$; infinite

47. At each point of discontinuity state whether the function is left continuous, right continuous, or neither

A) $f(x)=\begin{cases}\frac{1}{x-2} & x<1\\ \cos\pi x & 1\le x\le 2\\ \frac{1+x}{(x-3)^2} & 2<x\end{cases}$

B) $f(x)=\begin{cases}0 & x<0\\ \cos\pi x & 0\le x\le 1\\ 2\cos\pi x & 1<x\le 2\\ 2 & 2<x\end{cases}$

ANSWER: A) $x=2$; left continuous
$x=3$; none of these
B) $x=0$; right continuous
$x=1$; left continuous

48. Determine real numbers a, b, and c that make the function continuous:

$$f(x)=\begin{cases}a & t<0\\ x^2+1 & 0\le t<b\\ 5x-c & b\le t<7\\ 42 & 7\le t\end{cases}$$

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ANSWER: $a=1;\ b=6;\ c=-7$

49. Consider the function

$$f(x)=\begin{cases}-1 & x\le 0\\ 1 & x>0\end{cases}$$

The function $f(x)+g(x)$ is continuous for which of the following functions *g*?

a. $g(x)=2$ if $x\ne 0,\ g(0)=0$

b. $g(x)=0$ if $x\ne 0,\ g(0)=2$

c. $g(x)=2$ if $x\le 0,\ g(x)=0$ if $x>0$

d. $g(x)=2$ if $x<0,\ g(x)=0$ if $x\ge 0$

e. A and C both correct

ANSWER: c

50. Let $f(x)$ be a discontinuous function. Is it possible to find a continuous function $g(x)$ such that $f(x)+g(x)$ is continuous? Explain.

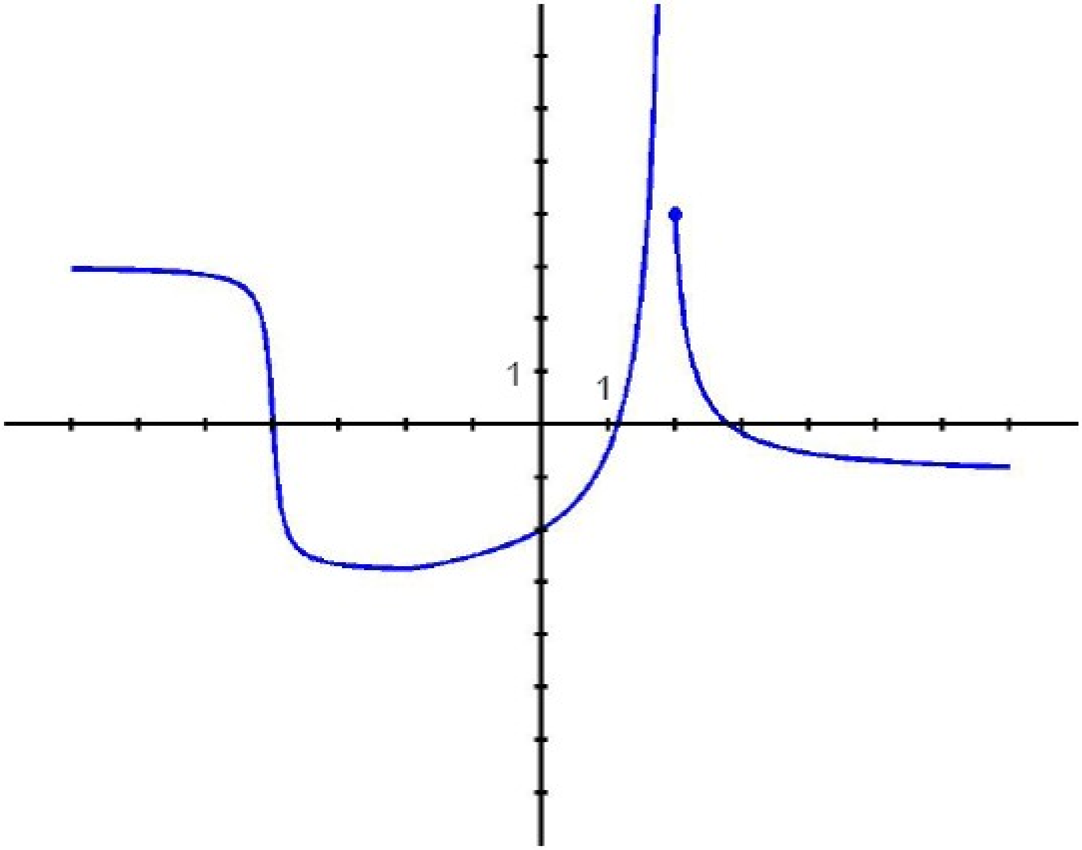
ANSWER: No. If $F(x) = f(x) + g(x)$ is continuous, then $f(x) = F(x) - g(x)$ is continuous by the continuity laws.

51. Sketch the graph of a function $f(x)$ that satisfies all of the following conditions:

$\lim_{x\to 2^-} f(x)=\infty,\quad \lim_{x\to 2^+} f(x)=4,\quad \lim_{x\to 0} f(x)=-2,$

$\lim_{x\to \infty} f(x)=-1,\quad \lim_{x\to -\infty} f(x)=3$

ANSWER:



52. Evaluate each limit or state that it does not exist:

A) $\lim_{x\to 3}\dfrac{x^4-3x^3+x^2-9}{x-3}$

B) $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x^2+3x-1}-\sqrt{2x+1}}$

C) $\lim_{x\to 1}\dfrac{\sqrt{x^2-2x+1}}{x-1}$

ANSWER:

A) 33

B) $\dfrac{2\sqrt{3}}{3}$

C) Does not exist

53. Evaluate each limit or state that it does not exist:

A) $\lim_{x\to \frac{\pi}{2}}\dfrac{\cos^2 x}{(1+\cos 2x)(2+\cos 2x)}$

B) $\lim_{x\to 0}\left(\dfrac{1}{x^2}-\dfrac{\sqrt{1+x^2}}{x^2}\right)$

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C) $\lim_{\theta \to 0} \frac{1+\sin\theta-\cos\theta}{\sin\theta}$

ANSWER:

A) $\frac{1}{2}$

B) $-\frac{1}{2}$

C) 1

54. Evaluate the limits in terms of the constants involved:

A) $\lim_{x \to 0} \frac{\sqrt{(x+h)^2+1}-\sqrt{h^2+1}}{x}$

B) $\lim_{h \to a} \frac{\frac{1}{\sqrt{a}}-\frac{1}{\sqrt{h}}}{h-a}, a>0$

ANSWER:

A) $\frac{h}{\sqrt{1+h^2}}$

B) $\frac{1}{2a\sqrt{a}}$

55. Evaluate each limit or state that it does not exist:

A) $\lim_{x \to 2} \frac{x^4-x^2-12}{x-2}$

B) $\lim_{x \to 1} \frac{\sqrt{x^2+1}-\sqrt{x+1}}{x^2+x-2}$

C) $\lim_{x \to 2} \frac{\sqrt{x^2-4x+4}}{x-2}$

ANSWER:

A) 28

B) $\frac{\sqrt{2}}{12}$

C) Does not exist

56. Evaluate the limit:

$\lim_{x \to 1} \frac{\sqrt{x+\sqrt{x}}-\sqrt{x+1}}{\sqrt{x}-1}$

ANSWER:

$\frac{\sqrt{2}}{4}$

57. Evaluate each limit or state that it does not exist:

A) $\lim_{x \to 2} \frac{x^3-2x^2+3x-6}{x-2}$

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B) $\lim_{x\to -1}\frac{\sqrt{3x+4}+x}{x^2-x-2}$

C) $\lim_{x\to 1}\frac{|x^2+x-2|}{x-1}$

ANSWER:

A) 7

B) $-\frac{5}{6}$

C) Does not exist

58. Determine a real number c for which the limit exists and then compute the limit:

$\lim_{x\to 0+}\left(\frac{1}{\sqrt{x+c^2}}-\frac{1}{\sqrt{x^2+x}}\right)$

ANSWER: $c=0$, the limit is 0

59. Evaluate each limit or state that it does not exist:

A) $\lim_{x\to 2}\left(\frac{x+1}{x-2}-\frac{x-5}{x^2-5x+6}\right)$

B) $\lim_{x\to 5}\frac{\sqrt{x^2-6}-\sqrt{4x-1}}{x-5}$

C) $\lim_{x\to 8}\frac{\sqrt{x+1}-3}{x-8}$

ANSWER:

A) (-1)

B) $\frac{3\sqrt{19}}{19}$

C) $\frac{1}{6}$

60. Determine a real number a for which the limit exists and then compute the limit:

$\lim_{x\to 2}\frac{x^2+ax+6}{\sqrt{x^2+2x-4}-\sqrt{x+2}}$

ANSWER:

$a=-5$; limit is $-\frac{4}{5}$

61. Let $f(x) = 2x + 3$. Compute $\lim_{h\to 0}\frac{f(-3+h)-f(-3)}{h}$.

ANSWER: 2

62. Compute $\lim_{\theta\to 0}\left(\cot^2\theta-\csc^2\theta\right)$.

ANSWER: –1

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63. Compute $\lim_{\theta \to \frac{7\pi}{6}} \frac{2\sin^2\theta - 5\sin\theta - 3}{2\sin\theta + 1}$.

ANSWER: $-\frac{7}{2}$

64. Evaluate the limits:

A) $\lim_{x \to 0} \frac{\sin^2 x}{\sin x^2}$

B) $\lim_{x \to 0} x\cos\frac{1}{x^3}$

C) $\lim_{x \to 0} |\sin x|\left(1 - \cos\frac{1}{x}\right)$

ANSWER: A) 1
B) 0
C) 0

65. Show that $0 \le x - [x] < 1$ for all x. (Here, [*x*] denotes the greatest integer function.) Then use the above inequality and the Squeeze Theorem to evaluate $\lim_{x \to 0} x(x - [x])$.

ANSWER: 0

66. Evaluate the limits in terms of the constants involved:

A) $\lim_{x \to h} \frac{\sin(x-h)}{x^2 + (1-h)x - h}$

B) $\lim_{x \to a} \frac{\frac{1}{x^2} - \frac{1}{a^2}}{x - a}$

ANSWER:
A) $\frac{1}{h+1}$
B) $\left(-\frac{2}{a^3}\right)$

67. Evaluate the limits using the Squeeze Theorem, trigonometric identities, and trigonometric limits, as necessary:

A) $\lim_{x \to 0} \frac{\sin\frac{1}{x}\sin^2\frac{x}{2}}{x}$

B) $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x}$

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C) $\lim_{x\to 0}\frac{\sin^3 x}{\sin\left(x^3\right)}$

ANSWER:

A) 0
B) 1
C) 1

68. Show that $0 \le x-[x]<1$ for all x. (Here, [*x*] denotes the greatest integer function.) Then use this inequality with the Squeeze Theorem to evaluate $\lim_{x\to\pi}(x-[x])\tan x$.

ANSWER: 0

69. Determine a real number c such that the following limit exists, and then evaluate the limit for this value:

$\lim_{x\to 0}\frac{3\sin\frac{x}{2}+(c-1)^2}{\sin x-\cos x+1}$

ANSWER:

$c=1$; limit is $\frac{3}{2}$

70. Evaluate each limit or state that it does not exist:

A) $\lim_{t\to 0}\frac{1-\cos t}{\sqrt{t}}$

B) $\lim_{x\to 0}\frac{1-\cos x}{\sin 2x\sin x}$

C) $\lim_{x\to 0}\frac{\sin 2x-\sin x}{x}$

ANSWER:

A) 0
B) $\frac{1}{4}$
C) 1

71. Evaluate the limits:

A) $\lim_{x\to 0}\frac{1-\cos^4 x}{x^2}$

B) $\lim_{x\to 0}\frac{\cos\left(x+\frac{\pi}{4}\right)-\sin\left(x+\frac{\pi}{4}\right)}{x}$

ANSWER:

A) 2
B) $-\sqrt{2}$

72. Evaluate the limits:

A) $\lim_{x\to 0}\frac{\sin 5x}{x^2-x}$

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Hint: Factor the denominator.

B) $\lim_{x\to 0}\frac{1-\cos^2 x}{2x^2-x}$

Hint: Factor the two expressions.

C) $\lim_{x\to 0}\frac{\sin 3x\sin 8x}{x^3+x^2}$

ANSWER:

A) -5
B) 0
C) 24

73. Use the Squeeze Theorem to evaluate the limit

$\lim_{x\to\pi}(1+\cos x)\sin\frac{1}{x}$

ANSWER: 0

74. If $3x^2-4\le f(x)\le x$ on the interval [0,4], then $\lim_{x\to 1} f(x)$ must exist.

a. True
b. False

ANSWER: b

75. Calculate the limits:

A) $\lim_{x\to\infty}\frac{2x^5-x^4+1}{8x^5+x^3+x-2}$

B) $\lim_{x\to-\infty}\frac{3x^2+x-1}{4x-7}$

C) $\lim_{x\to\infty}\left(\frac{6x^3}{2x^2+1}-3x\right)$

ANSWER:

A) $\frac{1}{4}$
B) $-\infty$
C) 0

76. Calculate the limits:

A) $\lim_{x\to-\infty}\frac{\sqrt[3]{2x^3-x+1}}{\sqrt{x^2+x-2}}$

B) $\lim_{x\to\infty}\frac{(4x+1)^{15}(3x-1)^{10}}{(9x+7)^5(4x+11)^{20}}$

C) $\lim_{x\to-\infty}\frac{\sqrt{|x^2-5|}}{x}$

ANSWER:

A) $-\sqrt[3]{2}$
B) $\frac{1}{1024}$
C) -1

77. Calculate the following limits:

A) $\lim_{x\to-\infty}\frac{x^2+1}{\sqrt{x^4-2}}$
B) $\lim_{x\to-\infty}\frac{2x-1}{\sqrt[3]{x^3+1}}$
C) $\lim_{x\to\infty}\left(\sqrt{x^2-x}-\sqrt{x^2+5x}\right)$

ANSWER:

A) 1
B) 2
C) -3

78. Compute the following limits:

A) $\lim_{x\to\infty}\frac{2x^2-6x+1}{x^2-3}$
B) $\lim_{x\to-\infty}\left(-x^5+2x^4-x^2+1\right)$
C) $\lim_{x\to-\infty}\frac{x^7-6x^3+1}{2x^7+x^2-2}$

ANSWER:

A) 2
B) ∞
C) $\frac{1}{2}$

79. Compute the following limits:

A) $\lim_{x\to\infty}\frac{3(x+7)^3-(x-7)^3}{2(x+2)^3-(x-2)^3}$
B) $\lim_{x\to-\infty}\frac{x-1}{2x^2+1}$
C) $\lim_{x\to-\infty}\frac{2x^2-6x+1}{x-3}$

ANSWER:

A) 2
B) 0
C) $-\infty$

80. Compute the following limits:

A) $\lim_{x\to\infty}x\left(\sqrt{4x^2-1}-2x\right)$

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Hint: Multiply and divide by the conjugate expression.

B) $\lim_{x\to-\infty}\dfrac{2x+7}{\sqrt{x^2-1}}$

Hint: For $x<0, \sqrt{x^2}=-x$.

C) $\lim_{x\to\infty}\dfrac{x^{\frac{5}{3}}-3x^{\frac{2}{3}}}{x^{\frac{18}{5}}+x}$

ANSWER:

A) $-\frac{1}{4}$
B) -2
C) 0

81. Compute the following limits:

A) $\lim_{x\to\infty}\left(\dfrac{1}{x-1}-\dfrac{2}{x^2-1}\right)$

B) $\lim_{x\to-\infty}\dfrac{x^3+x^2-2x+1}{1-2x^3}$

C) $\lim_{x\to-\infty}\dfrac{x^4+2x-1}{x^3+x}$

ANSWER:

A) 0
B) $-\frac{1}{2}$
C) $-\infty$

82. Compute the following limits:

A) $\lim_{x\to-\infty}\dfrac{1-\sqrt{3+x^2}}{1+\sqrt{4x^2+1}}$

B) $\lim_{x\to\infty}\dfrac{x+3-\sqrt{x^2+2}}{\sqrt{x^2+1}-5}$

C) $\lim_{x\to\infty}\left(\sqrt{x^2-3x+7}-x\right)$

ANSWER:

A) $-\frac{1}{2}$
B) 0
C) $-\frac{3}{2}$

83. The Intermediate Value Theorem guarantees that the equation $x\cos x-\sin x=0$ has a solution in which of the following intervals?

a. $(2\pi, 3\pi)$

b. $\left(\frac{\pi}{2}, \pi\right)$

c. $\left(\frac{3\pi}{2}, 2\pi\right)$

d. $\left(\frac{\pi}{4}, \pi\right)$

e. $(\pi, 3\pi)$

ANSWER: a

84. The polynomial $P(x) = x^3 - x - 5$ must have a root in which of the following intervals?

a. $(3, 4)$

b. $(1, 2)$

c. $(0, 1)$

d. $\left(\frac{1}{2}, 1\right)$

e. $(-1, 1)$

ANSWER: b

85. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem, which states: If $f(x)$ assumes all the values between $f(a)$ and $f(b)$ in the interval $[a, b]$, then f is continuous on $[a, b]$.

a. $f(x) = x - 1$ on $[0, 2]$

b. $f(x) = \frac{1}{x-1}$ on $[0, 2]$

c. $f(x) = \frac{\sin x}{x}$ on $(0, 2)$, $f(0) = 1$ $f(x) = \begin{cases} \frac{\sin x}{x}, & 0 < x \le 2 \\ 1, & x = 0 \end{cases}$ on [0,2]

d. $f(x) = [x]$ on $[0, 2]$ (Here, $[x]$ denotes the greatest integer function.)

e. $f(x) = \frac{1}{(x-1)^2}$ on $[-3, 2]$

ANSWER: e

86. Which of the following functions has a zero in the interval $[-1, 3]$?

a. $f(x) = \frac{x}{x-4}$

b. $f(x) = x^2 - 3x + 3$

c. $f(x) = \frac{x^2}{x-2}$

d. $f(x) = \cos\frac{x}{\pi}$

e. Both A and C

ANSWER: e

87. The Intermediate Value Theorem guarantees that the equation $\tan x = x$ has a solution in which of the following intervals?

a. $\left[-\frac{\pi}{8}, \frac{3\pi}{8}\right]$

b. $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

c. $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

d. $\left[\frac{\pi}{4}, \pi\right]$

e. Both A and C

ANSWER: e

88. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem, which states: If $f(x)$ assumes all the values between $f(a)$ and $f(b)$ in the interval $[a,b]$, then f is continuous on $[a,b]$.

a. $f(x) = x^2$ for $x \in (1,3)$, $f(1) = 9$, $f(3) = 1$ on [1,3]

b. $f(x) = \frac{1}{x-2}$ on $[1,3]$

c. $f(x) = \frac{1-\cos x}{x}$ on $(0, \frac{\pi}{2}]$, $f(0) = 0$ on $[0, \frac{\pi}{2}]$

d. $f(x) = [x]$ on $[1,3]$ (Here, [*x*] denotes the greatest integer function.)

e. Both A and C

ANSWER: a

89. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem: If f assumes all the values between $f(a)$ and $f(b)$ in the interval $[a,b]$, then f is continuous on $[a,b]$.

a. $f(x) = \frac{1}{x-1}$ if $1 < x \le 3$, $f(1) = 2$ on [1,3]

b. $f(x) = [x]$ on $1 < x \le 3$ (Here, [*x*] denotes the greatest integer function.)

c. $f(x) = \frac{1}{x-4}$ on $1 < x \leq 3$

d. $f(x) = x^2$ for $1 < x \leq 3$ and $x \neq 2, f(2) = 1$

e. Both A and D

ANSWER: a

90. Assume $g(x)$ is continuous on $[-3,9]$, $g(-3) = 14$, and $g(9) = 72$. Determine whether each of the following statements is always true, never true, or sometimes true.

A) $g(c) = 0$: no solution with $c \in [-3,9]$

B) $g(c) = 60$: no solution with $c \in [-2,9]$

C) $g(c) = 21$: no solution with $c \in [-3,9]$

D) $g(c) = -1{,}000{,}000$: exactly one solution with $c \in [-2,9]$

E) $g(c) = 49.5$: a solution with $c \in [-3,9]$

ANSWER: A) Sometimes true
B) Sometimes true
C) Never true
D) Sometimes true
E) Always true

91. Draw the graph of a function $g(x)$ on $[-2, 2]$ such that the graph does not satisfy the conclusion of the Intermediate Value Theorem.

ANSWER: Answers may vary. A sample answer is:

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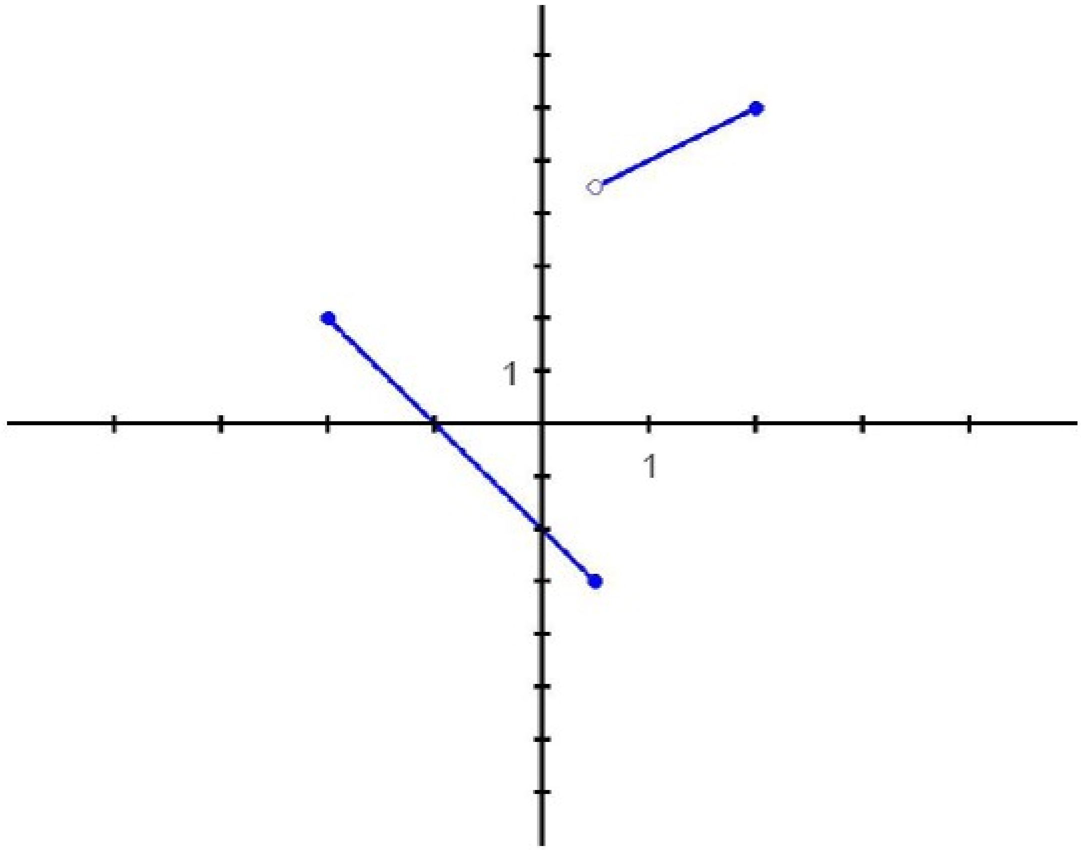

92. Which of the following properties can be used to prove that $f(x) = \cos x$ is continuous for all x?

a. $|\cos x| \le 1$ for all x

b. $|\cos x - \cos y| \le |x - y|$ for all x and y

c. $\cos x - \cos y \le x - y$ for all x and y

d. The limit $\lim_{x \to 0} \frac{1 - \cos x}{x}$ exists

e. $|\cos x - \cos y| \ge |x - y|$ for all x and y

ANSWER: b

93. Which of the following statements imply that $\frac{1}{x}$ is not continuous at $x = 0$?

a. $\frac{1}{x}$ has opposite signs on the two sides of $x = 0$.

b. $\left|\frac{1}{x}\right| < 0.01$ implies that $x > 100$.

c. For any $\varepsilon > 0$, $\left|\frac{1}{x}\right| < \varepsilon$ implies that $|x| > \frac{1}{\varepsilon}$.

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d. If $x < \varepsilon$, then $\left|\frac{1}{x}\right| < \frac{1}{\varepsilon}$.

e. A and C are correct.

ANSWER: c

94. To show that L is not the limit of $f(x)$ as $x \to x_0$, we should show that:

a. For any $\varepsilon > 0$, there exists $\delta > 0$ such that if $|x - x_0| > \delta$ then $|f(x) - L| < \varepsilon$.

b. For any $\varepsilon > 0$, there exists $\delta > 0$ such that if $|x - x_0| > \delta$ then $|f(x) - L| > \varepsilon$.

c. There exists $\varepsilon > 0$, such that for any $\delta > 0$ the inequalities $0 < |x - x_0| < \delta$ and $|f(x) - L| \geq \varepsilon$ have a solution x.

d. There exist $\varepsilon > 0$ and $\delta > 0$ such that if $0 < |x - x_0| < \delta$, then $|f(x) - L| > \varepsilon$.

e. A and C are both correct.

ANSWER: c

95. Suppose there exists a value of $\varepsilon > 0$ so that for any value of $\delta > 0$, we can find a value of x satisfying $0 < |x - x_0| < \delta$ and $|f(x) - L| > \varepsilon$. We may conclude that:

a. L is the limit of f as $x \to x_0$.

b. L is not the limit of f as $x \to x_0$.

c. The limit of f as $x \to x_0$ does not exist.

d. The limit of f as $x \to x_0$ exists but is not equal to L.

e. None of the above.

ANSWER: b

96. To show that L is not the limit of $f(x)$ as $x \to x_0$, we should show that:

a. There exists $\varepsilon > 0$ such that for any $\delta > 0$ there exists a solution to the inequalities $|x - x_0| < \delta$ and $|f(x) - L| > \varepsilon$.

b. There exists $\varepsilon > 0$ such that for any $\delta > 0$ there exists a solution to the inequalities $|x - x_0| > \delta$ and $|f(x) - L| < \varepsilon$.

c. There exists $\delta > 0$ such that for any $\varepsilon > 0$, if $|f(x) - L| < \varepsilon$, then $|x - x_0| < \delta$.

d. For any $\varepsilon > 0$ and $\delta > 0$, if $|x - x_0| < \delta$, then $|f(x) - L| > \varepsilon$.

e. A and D are both correct.

ANSWER: a

