Chapter 1 Limits and Continuity

1.1 Examples of Velocity, Growth Rate, and Area

1) An object moves along the x-axis so that at time t seconds (where $t \geq 0$) it is $x = \sqrt{t}$ m to the right of the origin. By calculating the average velocity of the object over time intervals [4, 4 + h] for
h = 0.1, h = 0.01, h = 0.001, and h = 0.0001, determine the velocity of the particle at time t = 4s.
A) 0.2499 m/s
B) 0.2498 m/s
C) 0.2485 m/s
D) 0.2000 m/s
E) 0.2501 m/s
Answer: A
Diff: 1

2) An object moves along the x-axis so that its velocity at time t seconds is $v(t) = \frac{2}{3t}$ m/s. What is the average acceleration (i.e., average rate of change of the velocity) of the object over the time interval
[2, 2 + h]? What is the acceleration of the object at time t = 2s?
A) $-\frac{1}{6 + 3h}$ m/s^2, $-\frac{1}{6}$ m/s^2
B) $\frac{1}{6 - 3h}$ m/s^2, $\frac{1}{6}$ m/s^2
C) $\frac{1}{2 + 3h}$ m/s^2, $\frac{1}{2}$ m/s^2
D) $-\frac{1}{6 - 3h}$ m/s^2, $-\frac{1}{6}$ m/s^2
E) $\frac{1}{6 + 3h}$ m/s^2, $\frac{1}{6}$ m/s^2
Answer: A
Diff: 1

3) The temperature of an object at time t minutes is $\frac{10}{1+t^2}$ degrees. What is the rate of change of the temperature over the time interval [t, t + h]? How fast is the temperature changing at t = 1?

A) $10\frac{2t+h}{\left(1+(t+h)^2\right)\left(1+t^2\right)}$ degrees/minute, increasing at 5 degrees/minute

B) $-10\frac{2t+h}{\left(1+(t+h)^2\right)\left(1+t^2\right)}$ degrees/minute, decreasing at 5 degrees/minute

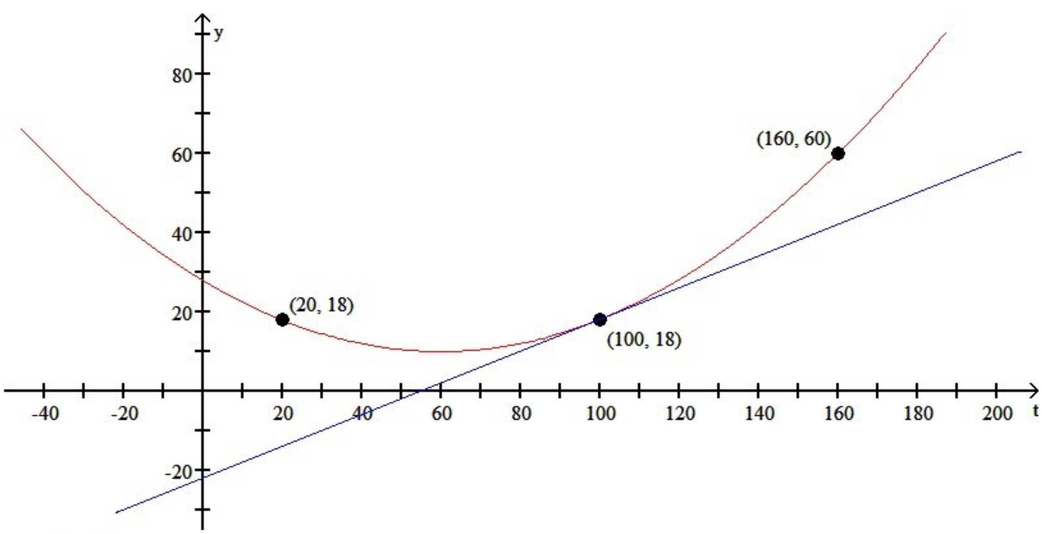
C) $\frac{5}{t}$ degrees/minute, increasing at 5 degrees/minute

D) $-10\frac{2t+h^2}{\left(1+(t+h)^2\right)\left(1+t^2\right)}$ degrees/minute, decreasing at 5 degrees/minute

E) $10\frac{2t+h^2}{\left(1+(t+h)^2\right)\left(1+t^2\right)}$ degrees/minute, decreasing at 5 degrees/minute

Answer: B
Diff: 1

4) In the following figure the curve is the graph of a quantity y that is a function of time t. Also shown is its tangent line at t = 100. What is the average rate of change of y from t = 20 to t = 160? What is the approximate rate of change of y with respect to t at t = 100?



A) $\frac{3}{10}, \frac{1}{5}$

B) $\frac{3}{10}, \frac{2}{5}$

C) $\frac{3}{8}, \frac{3}{10}$

D) $\frac{1}{3}, \frac{4}{9}$

E) none of the above
Answer: B
Diff: 3

1.2 Limits of Functions

1) Evaluate the following limits:

(i) $\lim\limits_{t \to -3} \frac{t+2}{t-7}$

(ii) $\lim\limits_{x \to -3} \frac{x^2-9}{x+3}$

A) (i) $\frac{1}{10}$ (ii) -6

B) (i) $\frac{-1}{10}$ (ii) 6

C) (i) 1 (ii) 1

D) (i) $\frac{-2}{7}$ (ii) does not exist

E) (i) $\frac{1}{10}$ (ii) does not exist

Answer: A
Diff: 1

2) Find the limit of f(x) as x approaches 2 if f is defined by

$f(x) = \begin{cases} 1 & \text{if } x \neq 2 \\ 0 & \text{if } x = 2 \end{cases}$

A) 1
B) 2
C) 3
D) 0
E) does not exist
Answer: A
Diff: 1

3) Evaluate $\lim\limits_{x \to 2} \frac{2x-13+\frac{36}{x+2}}{2-x}$.

A) $\frac{1}{4}$

B) 0

C) $\frac{7}{6}$

D) - $\frac{1}{4}$

E) - ∞
Answer: A
Diff: 2

4) Evaluate $\lim\limits_{x \to -2} \sqrt{4x^2-3}$.

A) $\sqrt{13}$
B) $2\sqrt{3}$
C) $\sqrt{5}$
D) 4
E) does not exist
Answer: A
Diff: 1

5) Evaluate $\lim\limits_{x \to 0} \dfrac{x^2 + 1}{x - 1}$.

A) 1
B) 0
C) -1
D) -2
E) does not exist
Answer: C
Diff: 1

6) Evaluate $\lim\limits_{t \to -3} \dfrac{t^2 + 2t - 3}{2t^2 + 5t - 3}$.

A) 4/7
B) 2/5
C) 0
D) 1/2
E) does not exist
Answer: A
Diff: 2

7) Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 3} - \sqrt{3}}{x}$.

A) $\dfrac{1}{2\sqrt{3}}$

B) $\dfrac{1}{3\sqrt{3}}$

C) $\dfrac{3}{2\sqrt{3}}$

D) 0
E) does not exist
Answer: A
Diff: 2

8) Evaluate $\lim\limits_{x \to 3} \frac{x - 3}{x^2 - 2x - 3}$.

A) $\frac{1}{4}$

B) 0

C) $\frac{3}{4}$

D) - $\frac{1}{4}$

E) does not exist
Answer: A
Diff: 2

9) If $\lim\limits_{x \to 2} \frac{f(x)+7x}{x-2}$ = - 5, find $\lim\limits_{x \to 2}$ f(x).

A) 14
B) - 14
C) 0
D) ∞
E) - 19
Answer: B
Diff: 2

10) Evaluate $\lim\limits_{x \to 3^-} \frac{|x - 3|}{x - 3}$.

A) -1
B) -2
C) 1
D) 0
E) does not exist
Answer: A
Diff: 2

11) Let $f(x) = \sqrt{2-x}$ and $g(x) = x^2 - 2$. Which of the following is true?

A) $\lim_{x \to -2} f\big(g(x)\big) = 0$

B) $\lim_{x \to 2} g\big(f(x)\big) = -2$

C) $\lim_{x \to 2^+} f\big(g(x)\big) = 0$

D) $\lim_{x \to 2^+} g\big(f(x)\big) = -2$

E) $\lim_{x \to 2^-} g\big(f(x)\big) = -2$

Answer: E
Diff: 2

12) Evaluate $\lim_{x \to 2^-} \dfrac{\left|x^2 + 4x - 12\right|}{x^2 + 10x - 24}$.

A) $-\dfrac{4}{7}$

B) -2
C) 1
D) $\dfrac{2}{5}$

E) $\dfrac{4}{7}$

Answer: A
Diff: 2

13) Evaluate $\lim_{x \to 2} \dfrac{x^3 - 8}{x^2 - 4}$.

A) 3
B) 1
C) 4
D) 0
E) does not exist
Answer: A
Diff: 2

14) Evaluate $\lim\limits_{x \to 3^-} \left(\frac{2x^2}{3x^2 + 1} + \sqrt{9 - x^2} \right)$.

A) $\frac{9}{14}$

B) $\frac{1}{3}$

C) $\frac{9}{5}$

D) ∞
E) does not exist
Answer: A
Diff: 1

15) Find the following limit:
$\lim\limits_{x \to -3} \left(\frac{5}{x + 3} + \frac{30}{x^2 - 9} \right)$.

A) $-\frac{5}{6}$

B) $\frac{5}{6}$

C) $\frac{1}{6}$

D) $-\frac{1}{6}$

E) does not exist
Answer: A
Diff: 3

16) Evaluate $\lim\limits_{t \to 2} \frac{|3t - 7| - |3 - 2t|}{t - 2}$.

A) - 5
B) 2
C) 5
D) $-\infty$
E) - 3
Answer: A
Diff: 2

17) Evaluate $\lim\limits_{x \to -4} \frac{x+4}{|x+4|}$.

A) -1
B) 1
C) 4
D) ±1
E) does not exist
Answer: E
Diff: 2

18) Evaluate $\lim\limits_{x \to 2} \frac{\sqrt{x^2+5}-3}{x^2-2x}$.

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) 1

D) $\frac{2}{3}$

E) does not exist
Answer: A
Diff: 3

19) Let $f(x) = \sqrt{x+11}$ and $g(x) = 14 - x^2$. Compute $\lim\limits_{x \to 3} \frac{4 - f \circ g(x)}{g \circ f(x)}$.

A) $-\frac{3}{4}$

B) $\frac{1}{11}$

C) $-\frac{4}{3}$

D) $\frac{3}{4}$

E) $\frac{1}{2}$

Answer: A
Diff: 3

20) If $\lim\limits_{x \to k} \dfrac{x^4 - k^4}{x - k} = -32$, find k.

Answer: First observe that $x^4 - k^4 = (x^2 - k^2)(x^2 + k^2) = (x - k)(x + k)(x^2 + k^2)$

Hence, $\lim\limits_{x \to k} \dfrac{x^4 - k^4}{x - k} = \lim\limits_{x \to k} \dfrac{(x - k)(x + k)(x^2 + k^2)}{x - k} = \lim\limits_{x \to k} (x + k)(x^2 + k^2) = (2k)(2k^2) = 4k^3$

Now $4k^3 = -32 \Rightarrow k^3 = -8$, therefore $k = \sqrt[3]{-8} = -2$.
Diff: 3

21) Evaluate $\lim\limits_{x \to 2} \dfrac{2x^2 - 9x + 10}{16 - x^4}$.

A) $-\infty$
B) $-\dfrac{1}{32}$
C) $\dfrac{7}{48}$
D) $\dfrac{1}{32}$
E) 0
Answer: D
Diff: 2

22) Evaluate $\lim\limits_{x \to 2} \dfrac{x^2 - 7x + 10}{x^2 - 5x + 6}$.

A) 3
B) -3
C) $\dfrac{5}{7}$
D) 1
E) does not exist
Answer: A
Diff: 2

23) Evaluate the following limit:
$\lim\limits_{x \to 1^+} x + \sqrt{1 - x}$.

A) 0
B) 1
C) 2
D) -1
E) does not exist
Answer: E
Diff: 2

24) Evaluate the following limit:
$\lim_{x \to 0^-} x\sqrt{3x + \frac{4}{x^2}}$.

A) -2
B) 0
C) 2
D) -1
E) does not exist
Answer: A
Diff: 3

25) Compute the following limit: $\lim_{x \to a} \frac{x^3 - a^3}{x^2 - a^2}$.

A) $\frac{3}{2}a$

B) $\frac{3}{4}a$

C) $-\frac{3}{2}a$

D) $\frac{3}{2}$

E) does not exist
Answer: A
Diff: 2

26) True or False: If $\lim_{x \to 0^+} f(x) = A$ and $\lim_{x \to 0^-} f(x) = B$, where $A \neq B$,

then $\lim_{x \to 0^-} f\left(x^3 - x\right) = B$.

Answer: FALSE
Diff: 2

27) True or False: If $\lim_{x \to a} |f(x)| = |L|$, then $\lim_{x \to a} f(x) = L$.

Answer: FALSE
Diff: 2

28) True or False: If $\lim_{x \to a} f(x) = L$, then $\lim_{x \to a} \left(f(x)\right)^3 = L^3$.

Answer: TRUE
Diff: 2

29) True or False: If $\lim_{x \to a} f(x) = L$, then $\lim_{x \to a} \left(f(x)\right)^{-3} = L^{-3}$.

Answer: FALSE
Diff: 2

1.3 Limits at Infinity and Infinite Limits

1) Evaluate $\lim_{t \to \infty} \frac{t^3 - 5t^2 + t + 8}{4t^3 - 3t^2 + 7t + 3}$.

A) $\frac{1}{4}$

B) $-\frac{1}{3}$

C) $-\frac{1}{4}$

D) $\frac{8}{3}$

E) does not exist
Answer: A
Diff: 1

2) Evaluate $\lim_{x \to +\infty} (\sqrt{x^2 + x} - x)$.

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $-\frac{1}{3}$

D) $-\frac{1}{2}$

E) does not exist
Answer: A
Diff: 2

3) Evaluate $\lim_{x \to 2} \frac{x^2 - 10x - 24}{|x-2|}$.

A) $+\infty$
B) 14
C) $-\infty$
D) -14
E) 0
Answer: C
Diff: 1

4) Let $g(x) = \begin{cases} -\dfrac{5}{x} & \text{if } -\infty < x < 0 \\ -3 & \text{if } x = 0 \\ \dfrac{4}{x^2} & \text{if } 0 < x < \infty \end{cases}$.

Evaluate $\lim\limits_{x \to 0} g(x)$.

A) ∞
B) -3
C) $-\infty$
D) 1
E) none of the above
Answer: A
Diff: 1

5) Evaluate $\lim\limits_{x \to \infty} \dfrac{3x^2 + 27}{x^3 - 27}$.

A) 0
B) 1
C) ∞
D) -1
E) 3
Answer: A
Diff: 2

6) Evaluate $\lim\limits_{x \to -\infty} (3x^4 - x^2 + x - 7)$.

A) $-\infty$
B) 0
C) 1
D) ∞
E) none of the above
Answer: D
Diff: 2

7) True or False: If f and g are functions such that $\lim\limits_{x \to c} f(x) = -\infty$ and $\lim\limits_{x \to c} g(x) = -\infty$, then $\lim\limits_{x \to c} [f(x) - g(x)] = 0$.

Answer: FALSE
Diff: 1

8) Evaluate $\lim\limits_{x \to 2} \frac{3}{x - 2}$.

A) ∞
B) $-\infty$
C) 1
D) 0
E) does not exist
Answer: E
Diff: 2

9) Find all values of the real number k so that $\lim\limits_{x \to -\infty} \frac{\sqrt{k^2x^2 - 3x - 5}}{k - 12x} = \frac{1}{4}$.

A) 0
B) $-\frac{1}{3}$
C) $\pm\frac{1}{3}$
D) ± 3
E) -3
Answer: D
Diff: 2

10) Evaluate $\lim\limits_{x \to -\infty} \frac{5x - \sqrt{81x^2 + 16}}{35x}$.

A) $\frac{2}{5}$
B) 0
C) $+\infty$
D) $-\frac{2}{5}$
E) $-\infty$
Answer: A
Diff: 3

11) Find $\lim\limits_{x \to \infty} \frac{3x + 9}{2x^2 - 8x - 1}$.

A) 0
B) ∞
C) $\frac{3}{2}$
D) 2
E) does not exist
Answer: A
Diff: 2

12) Evaluate $\lim\limits_{x \to 1^-} \frac{x}{1 - x^2}$.

A) 0
B) 1
C) -1
D) ∞
E) -∞
Answer: D
Diff: 2

13) Let $f(x) = \frac{x^4 - 1}{(x - 1)^4}$. Find $\lim_{x \to 1^-} f(x)$.

A) +∞
B) 0
C) 1
D) -∞
E) -1
Answer: D
Diff: 2

14) Find the limit $\lim_{x \to \infty} \frac{5x^3 - 4x + 2}{11x^3 + 5}$.

A) $-\frac{5}{11}$

B) $\frac{1}{11}$

C) $\frac{2}{5}$

D) $\frac{5}{11}$

E) does not exist
Answer: D
Diff: 2

15) Evaluate $\lim_{x \to 1^-} \frac{x^2 - 3x}{x - 1}$.

A) 1
B) -∞
C) ∞
D) 0
E) none of the above
Answer: C
Diff: 3

16) Evaluate $\lim_{t \to \infty} \frac{t + 1}{t^2 + 1}$.

A) 1
B) 0

C) -1
D) 2
E) does not exist
Answer: B
Diff: 3

17) Evaluate $\lim_{\theta \to 0} \frac{\tan^2 \theta}{1 - \sec \theta}$.

A) -2
B) -1
C) 0
D) 2
E) does not exist
Answer: A
Diff: 3

18) Evaluate $\lim_{x \to -\infty} \frac{4x - 1}{\sqrt{x^2 + 2}}$.

A) -4
B) $2\sqrt{2}$
C) 4
D) $-2\sqrt{2}$
E) does not exist
Answer: A
Diff: 3

19) Evaluate $\lim_{x \to 0^+} \frac{x + 2}{x^2}$.

A) ∞
B) $-\infty$
C) 1
D) -1
E) 2
Answer: A
Diff: 3

20) Evaluate $\lim_{x \to \infty} \frac{x^3}{\sqrt{1 + x^6}}$.

A) ∞
B) 0
C) 1
D) -1
E) $\frac{1}{\sqrt{2}}$

Answer: C
Diff: 3

21) Evaluate $\lim_{x \to \infty} x^2 \sin x$.

A) ∞
B) $-\infty$
C) 0
D) 1
E) does not exist
Answer: E
Diff: 3

22) Evaluate $\lim_{x \to 0^-} \frac{x + 2}{x^2}$.

A) ∞
B) 1
C) $-\infty$
D) -1
E) 2
Answer: A
Diff: 2

23) Evaluate $\lim_{x \to 0} \frac{x + 2}{x^2}$.

A) ∞
B) 1
C) 0
D) $-\infty$
E) 2
Answer: A
Diff: 2

24) Evaluate $\lim\limits_{x \to -\infty} \dfrac{-4x^3 + 7x}{2x^2 - 3x - 10}$.

A) ∞
B) $-\infty$
C) 0
D) -1
E) -2
Answer: A
Diff: 3

25) Evaluate $\lim\limits_{x \to 2^+} \dfrac{2x - 5}{x - 2}$.

A) ∞
B) $-\infty$
C) 2
D) $-\dfrac{5}{2}$
E) none of the above
Answer: B
Diff: 3

26) Evaluate $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2 + 4} - \sqrt[3]{8x^3 + x - 15}}{4x-5}$.

A) $-\dfrac{3}{4}$
B) $+\infty$
C) $-\dfrac{1}{4}$
D) 0
E) $-\dfrac{2}{3}$
Answer: A
Diff: 3

27) Evaluate $\lim\limits_{x \to 1^-} \dfrac{x^2 + 1}{\cot \pi x}$.

A) ∞
B) -2
C) 0
D) $-\infty$
E) π
Answer: D
Diff: 2

28) Evaluate $\lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{4x^2 + 1}}$.

A) -2
B) $\frac{1}{2}$
C) 0
D) - $\frac{1}{2}$
E) does not exist
Answer: D
Diff: 2

29) Evaluate $\lim\limits_{x \to -\infty} \frac{3x^2 + 1}{x^2 + x}$.

A) -3
B) 3
C) 1
D) -1
E) does not exist
Answer: B
Diff: 2

30) Evaluate $\lim\limits_{x \to 0^+}$ (2 tan x + 3 cot x).

A) 3
B) 2
C) ∞
D) $-\infty$
E) 5
Answer: C
Diff: 3

31) Given that $\lim\limits_{t \to 0} \frac{\sin t}{t} = 1$, evaluate $\lim\limits_{x \to \infty} (5x^2 + 7x - 3) \sin \frac{1}{x^2}$.

A) 5
B) 7
C) 0
D) -3
E) does not exist
Answer: A
Diff: 3

32) Given that $\lim_{t \to 0} \frac{\sin t}{t} = 1$, evaluate $\lim_{x \to 0} \frac{x \sin(2x) \csc(4x)}{\sin(3x)}$.

A) $\frac{1}{6}$

B) 0

C) $\frac{3}{8}$

D) $\frac{8}{3}$

E) does not exist
Answer: A
Diff: 3

33) True or False: If $\lim_{x \to a} f(x) = \infty$, then $\lim_{x \to a} f(x)$ does not exist.

Answer: TRUE
Diff: 3

34) True or False: If $\lim_{x \to a} f(x) = 0$, then $\lim_{x \to a} \frac{1}{f(x)} = \infty$

Answer: FALSE
Diff: 3

35) True or False: If $\lim_{x \to a+} f(x) = 0$, then $\lim_{x \to a} \frac{1}{|f(x)|} = \infty$

Answer: FALSE
Diff: 3

1.4 Continuity

1) Courier rates for delivery of mail up to 200 g are given in the following chart:

Up to and including	30 g	50 g	100 g	200 g
Mailing cost	\$0.38	\$0.59	\$0.76	\$1.14

Draw the graph of the cost (in dollars) of mailing a letter as a function of its mass (in grams). Where are the discontinuities of this function? Is the cost function right or left continuous at these points?
A) discontinuous at 30, 50, 100; left but not right continuous there
B) discontinuous at 30, 50, 100; right but not left continuous there
C) discontinuous at 30, 50, 100; neither left nor right continuous there
D) There are no discontinuities.
E) discontinuous at 30, 50, 100; right continuous at 30 and 50, left continuous at 100
Answer: A
Diff: 1

2) Let $g(x) = \begin{cases} \dfrac{k^2 - 2x}{3-x} & \text{if } -\infty < x < 2 \\ 12 & \text{if } x = 2 \\ 4x + k & \text{if } 2 < x < \infty \end{cases}$

where k is a constant real number. If g has a removable discontinuity at x = 2, then k is equal to:
A) -3
B) 4
C) ±4
D) 3
E) -4 or 3
Answer: A
Diff: 3

3) Let f(x) be defined by

$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$ Where, if anywhere, is f discontinuous?
A) at x = 0
B) at x = 3
C) nowhere
D) at x = -3
E) at x = 6
Answer: C
Diff: 2

4) Is the function $f(x) = \dfrac{x^2 - 9}{x - 3}$ discontinuous at any point of its domain? Would your answer change if we also define f(3) = 6?
A) no, yes
B) yes, yes
C) no, no
D) yes, no
E) none of the above
Answer: B
Diff: 2

5) True or False: If $h(x) = \dfrac{x^2 + 2x - 3}{x - 1}$ if $x \neq 1$ and h(1) = 4, then h is continuous at every x.
Answer: TRUE
Diff: 2

6) Let f(x) have values

$$\begin{cases} -2 & \text{if } x \le -5 \\ \frac{2}{5}x & \text{if } -5 < x \le 5 \\ 2 & \text{if } x > 5 \end{cases}$$

Where is f discontinuous?
A) nowhere
B) at x = -5 only
C) at x = 5 only
D) at both x = -5 and x = 5
E) none of the above
Answer: A
Diff: 2

7) True or False: If $\lim\limits_{x \to c} f(x)$ exists and is finite, then either f is continuous at x = c or f has a continuous extension to x = c.
Answer: TRUE
Diff: 1

8) True or False: The polynomial function $P(x) = x^4 + x^3 - 7x^2 - 2x + 10$ has a zero in the closed interval [1 , 2].
Answer: TRUE
Diff: 2

9) In which of the intervals (-2, -1), (-1, 0), (0, 1), and (1, 2) does the intermediate value theorem imply that $f(x) = 2x^3 - 4x^2 + 5x - 4$ must have a zero?
A) (1, 2)
B) (0, 1)
C) (-1, 0)
D) (-2, -1)
E) none of the above
Answer: A
Diff: 2

10) For what value of the constant c is the function $f(x) = \begin{cases} x + c & \text{if } x < 2 \\ cx^2 + 1 & \text{if } x \geq 2 \end{cases}$

continuous everywhere?

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) 1
D) 0
E) none of the above
Answer: A
Diff: 3

11) Given $f(x) = \begin{cases} 12 - kx & \text{if } x < k \\ 3 & \text{if } x = k \\ x^2 - 2k & \text{if } x > k \end{cases}$

find the value of the constant real number k such that f is continuous at x = k.
A) - 1
B) -3
C) 3
D) -2
E) 1
Answer: C
Diff: 2

12) Let $f(x) = \begin{cases} x^2 & \text{if } x \neq 1 \\ 4 & \text{if } x = 1 \end{cases}$

Which of the following statements, I, II, or III, are true?
I. $\lim\limits_{x \to 1} f(x)$ exists

II. f(1) exists
III. f is continuous at x = 1.
A) only I
B) only II
C) only I and II
D) only II and III
E) I, II, and III
Answer: C
Diff: 3

13) If f(x) is a function defined on the closed interval [a , b] such that f(a) < 0 and f(b) > 0 , then the function f must have at least one zero in the interval [a , b].
Answer: FALSE
Diff: 1

14) Let f and g be continuous functions on the interval $(-\infty, \infty)$ such that f has a zero on the closed interval I, and g has a zero on the closed interval.
Which of the following functions must have at least one zero on the closed interval I ?
A) $f \circ g$, $g \circ f$
B) $f + g$, $f - g$
C) $f g$, $g f$
D) $\frac{f}{g}$, $\frac{g}{f}$
E) none of the above.
Answer: C
Diff: 3

15) Let $f(x) = \begin{cases} \frac{\sqrt{1-2x}-1}{x} & \text{if } x < 0 \\ -1 & \text{if } x = 0 \\ x^2 + 1 & \text{if } x > 0 \end{cases}$
Which of the following statements is true?
A) The function f is continuous at $x = 0$.
B) The function f is continuous from the left at $x = 0$.
C) The function f is continuous from the right at $x = 0$.
D) The function f has continuous extension to $x = 0$.
E) The function f has a removable discontinuity at $x = 0$.
Answer: B
Diff: 2

16) A function is continuous on the interval [3, 5] and takes the values $f(3) = 0$ and $f(5) = \pi - 2$.
Which of the numbers 1, 2, 3, 4, or 5 must belong to the range of f?
A) only 1
B) only 4
C) only 1 and 2
D) only 3, 4, and 5
E) only 3 and 5
Answer: A
Diff: 2

17) For what values of the constants c and k is the function

$$f(x) = \begin{cases} x^3 + k & \text{if } -1 \le x \le 3 \\ c\,x & \text{if } x < -1 \text{ or } x > 3 \end{cases}$$

continuous at all x?
A) $c = 7$, $k = -6$
B) $c = -6$, $k = 7$
C) $c = 6$, $k = 7$
D) $c = -6$, $k = -7$
E) $c = -7$, $k = 6$
Answer: A
Diff: 2

1.5 The Formal Definition of Limit

1) To the nearest 1/1,000, how close need x be chosen to 1 to ensure that $|x^3 - 1| < \frac{1}{10}$?

A) within distance 0.032
B) within distance 0.033
C) within distance 0.034
D) within distance 0.040
E) within distance 0.031
Answer: A
Diff: 1

2) Given f(x) = 4x + 7 and a number ε > 0, find the largest value of δ for which $0 < |x - 2| < \delta$ will imply that $|f(x) - f(2)| < \varepsilon$.

A) $\delta = \frac{\varepsilon}{4}$

B) $\delta = \frac{\varepsilon}{5}$

C) $\delta = 4\varepsilon$

D) $\delta = \frac{\varepsilon}{2}$

E) $\delta = \varepsilon$
Answer: A
Diff: 1

3) In order to prove that $\lim\limits_{x \to 2} x^2 = 4$, we need to find, for any given ε > 0, a corresponding number $\delta > 0$ such that if $0 < |x - 2| < \delta$, then $|x^2 - 4| < \varepsilon$. Which of the following values of δ will do that for a given ε?

A) $\delta = \frac{\varepsilon}{4}$

B) $\delta = \frac{\varepsilon}{4 + \varepsilon}$

C) $\delta = \frac{\varepsilon}{2}$

D) $\delta = \min\{1, \varepsilon\}$
E) $\delta = \sqrt{\varepsilon + 4} - 2$
Answer: E
Diff: 2

4) Which of the following conditions will guarantee that $\left|\frac{1}{x} - \frac{1}{2}\right| < \frac{1}{100}$?

A) $-\frac{2}{51} < x - 2 < \frac{2}{49}$

B) $|x - 2| < \frac{2}{50}$

C) $|x - 2| < \frac{2}{49}$

D) all of the above
E) none of the above
Answer: A
Diff: 2

5) Use the formal definition of the limit to verify the addition property of limits:

If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = K$, then $\lim_{x \to a} [f(x) + g(x)] = L + K$.

Answer: Let $\varepsilon > 0$. Since $\lim_{x \to a} f(x) = L$, there exists δ_1 such that if $0 < |x - a| < \delta_1$, then $|f(x) - L| < \frac{\varepsilon}{2}$. Since $\lim_{x \to a} g(x) = K$, there exists δ_2 such that if $0 < |x - a| < \delta_2$, then $|g(x) - K| < \frac{\varepsilon}{2}$. If $0 < |x - a| < \delta = \min\{\delta_1, \delta_2\}$, then

$|(f(x) + g(x)) - (L + K)| \le |f(x) - L| + |g(x) - K| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$. Thus $\lim_{x \to a} (f(x) + g(x)) = L + K$.

Diff: 3

6) Complete the following definition: We say $\lim_{x \to a-} f(x) = L$ if for every $\varepsilon > 0$ there exists $\delta > 0$ depending on ε such that
A) if $a - \delta < x < a$, then $|f(x) - L| < \varepsilon$
B) if $|x - a| < \delta$, then $|f(x) - L| < \varepsilon$
C) if $|x - a| < \delta$, then $f(x) - L < \varepsilon$
D) if $a < x < a + \delta$, then $|f(x) - L| < \varepsilon$
E) none of the above
Answer: A
Diff: 3

7) Use the formal definition of limit to verify that $\lim_{x \to a} (x-a) = 0$.

Answer: Let $\varepsilon > 0$ be given. We must find $\delta > 0$ so that:
$0 < |x - a| < \delta$ implies $|(x - a) - 0| < \varepsilon$
That is to say: we must find $\delta > 0$ so that
$0 < |x - a| < \delta$ implies $|x - a| < \varepsilon$
We can simply take $\delta = \varepsilon$, and the statement above will be true.
Diff: 1