

CALCULUS AND ITS APPLICATIONS

Name: _____

Chapter 1, Form A

For Questions 1 – 3, consider

$$\lim_{x \rightarrow -5} f(x), \text{ where } f(x) = \frac{x^2 - 25}{x + 5}.$$

1. Numerical limits.

(a) Find the limit by completing the following input-output tables.

$x \rightarrow -5^-$	$f(x)$	$x \rightarrow -5^+$	$f(x)$
-5.1		-4.9	
-5.01		-4.99	
-5.001		-4.999	

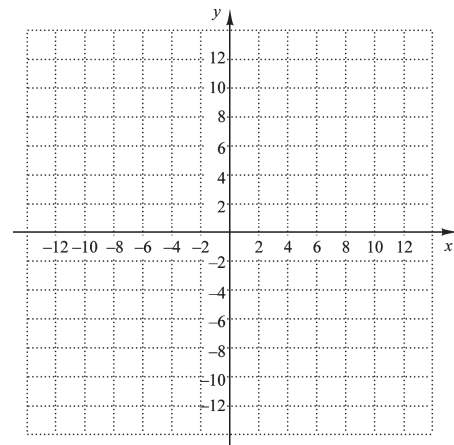
1.(a) _____

(b) Find $\lim_{x \rightarrow -5^-} f(x)$, $\lim_{x \rightarrow -5^+} f(x)$, and $\lim_{x \rightarrow -5} f(x)$, if each exists.

(b) _____

2. Graphical limits. Find the limit by graphing the function.

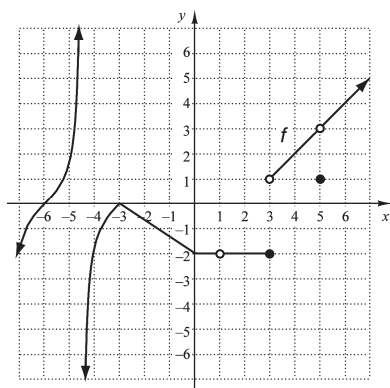
2. _____



3. Algebraic limits. Find the limit algebraically. Show all work.

3. _____

Graphical limits. For Questions 4-12, consider f graphed below.



Find the limit, if it exists.

4. $\lim_{x \rightarrow -3} f(x)$

4. _____

5. $\lim_{x \rightarrow 4.5} f(x)$

5. _____

6. $\lim_{x \rightarrow 0} f(x)$

6. _____

7. $\lim_{x \rightarrow 3} f(x)$

7. _____

8. $\lim_{x \rightarrow -5} f(x)$

8. _____

9. $\lim_{x \rightarrow 2} f(x)$

9. _____

10. Find $f'(2)$.

10. _____

11. State the value(s) of x at which f is discontinuous.

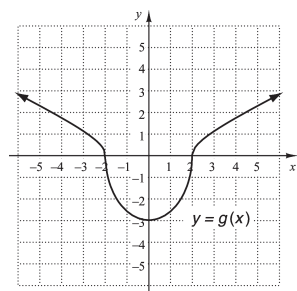
11. _____

12. State the value(s) of x for which $f'(x)$ is not defined.

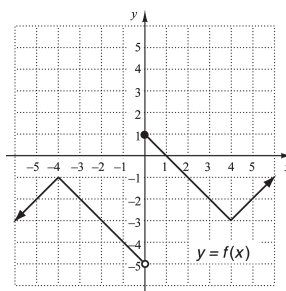
12. _____

Determine whether the function is continuous. If a function is discontinuous, state why.

13.



14.



13. _____

14. _____

Consider the function shown in Question 14, answer the following:

 15. (a) Find $\lim_{x \rightarrow 4} f(x)$.

 (b) Find $f(4)$.

 (c) Is f continuous at 4?

15.(a) _____

(b) _____

(c) _____

 16. Find $\lim_{x \rightarrow 0} f(x)$.

16. _____

Find the limit if it exists. If a limit does not exist, state why.

 17. $\lim_{x \rightarrow 4} (x^4 - 5x^2 + 2)$

17. _____

 18. $\lim_{x \rightarrow 5^+} \frac{x - 5}{4(x^2 - 25)}$

18. _____

 19. $\lim_{x \rightarrow 0} -\frac{3}{x}$

19. _____

 20. Find the simplified difference quotient for:
 $f(x) = 5x^2 - 8x$.

20. _____

 21. Find an equation of the tangent line to the graph of $y = 4x + \left(-\frac{6}{x}\right)$ at the point $(3, 10)$.

21. _____

 22. Find the points on the graph of $y = x^3 - 2x^2$ at which the tangent line is horizontal.

22. _____

Find dy/dx .

23. $y = x^{17}$

23. _____

24. $y = 5\sqrt[3]{x} + 6\sqrt{x}$

24. _____

25. $y = -\frac{8}{x^3}$

25. _____

26. $y = x^{3/5}$

26. _____

27. $y = 0.32x^4 - 7x^2 + 3$

27. _____

Differentiate.

28. $y = \frac{3}{4}x^4 - 5x^2 + 4x + 1$

28. _____

29. $y = (6\sqrt{x} + 2)(x^3 - x)$

29. _____

30. $y = \frac{x+4}{4-x}$

30. _____

31. $f(x) = (x+2)^4(3-x)^2$

31. _____

32. $y = (6x^2 - 10x + 1)^{-4}$

32. _____

33. $f(x) = x\sqrt{x^6 - 2}$

33. _____

34. For $y = 4x^6 - 9x^3$, find $\frac{d^3y}{dx^3}$.

34. _____

- 35.** *Volume of a scoop of ice cream.* The volume of a spherical scoop of ice cream with radius r is given by $V = \frac{4}{3}\pi r^3$, where r is measured in inches.

(a) Find the rate of change of the volume of the scoop of ice cream with respect to the radius.

35. (a) _____

(b) What is the volume when the radius is 0.5 in.?

(b) _____

(c) Find the rate of change of the volume of the scoop of ice cream when $r = 0.5$.

(c) _____

- 36.** *Business: average revenue, cost and profit.* Given revenue and cost functions $R(x) = 30x$ and $C(x) = x^{2/3} + 400$, where x is the number of items produced and $R(x)$ and $C(x)$ are in dollars, find:

(a) The average revenue, the average cost and the average profit when x items are produced.

36. (a) _____

(b) The rate at which average cost is changing when 12 items are produced.

(b) _____

For Questions 37 and 38, let $f(x) = 2x^2 - x$ and $g(x) = x + 5$.

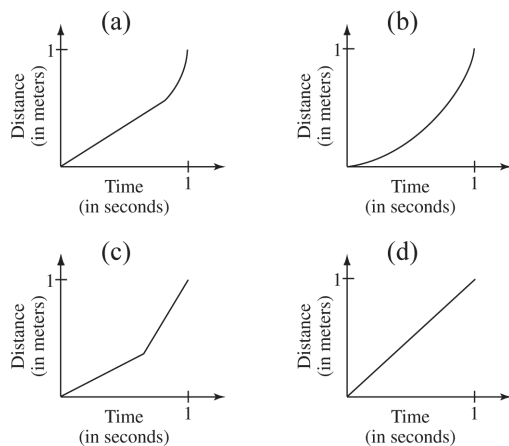
37. Find $\frac{d}{dx}(f \circ g)(x)$.

37. _____

38. Find $\frac{d}{dx}(g \circ f)(x)$.

38. _____

39. A marble rolls along a level table at a constant rate and then down an incline plane. Let $y = s(t)$ represent the marble's distance after starting to roll. Which graph best represents s



39. _____

40. Differentiate $y = \sqrt{(2-3x)^{2/3}(5+x)^{1/2}}$.

40. _____

41. Find $\lim_{x \rightarrow -1} \frac{1+x^3}{1+x}$.

41. _____

42. Graph f and f' over the interval $[-3, 3]$. Then estimate points at which the line tangent to f is horizontal.

$$f(x) = 3x^5 - 15x^2 + 15x; [-3, 3]$$

42. _____



43. Find the following limit by creating a table of values:

$$\lim_{x \rightarrow 6} \frac{\sqrt{2x+4} - 4}{x-6}.$$

Start with $\Delta Tbl = 0.1$ and then go to 0.01 and 0.001. When you think you know the limit, graph

$$y = \frac{\sqrt{2x+4} - 4}{x-6}$$

and use TRACE to verify your assertion.

43. _____

Chapter 1, Form B

For Questions 1 – 3, consider

$$\lim_{x \rightarrow 2} f(x), \text{ where } f(x) = \frac{x^2 - 4}{x - 2}.$$

1. *Numerical limits.*

(a) Find the limit by completing the following input-output-tables.

$x \rightarrow 2^-$	$f(x)$	$x \rightarrow 2^+$	$f(x)$
1.9		2.1	
1.99		2.01	
1.999		2.001	

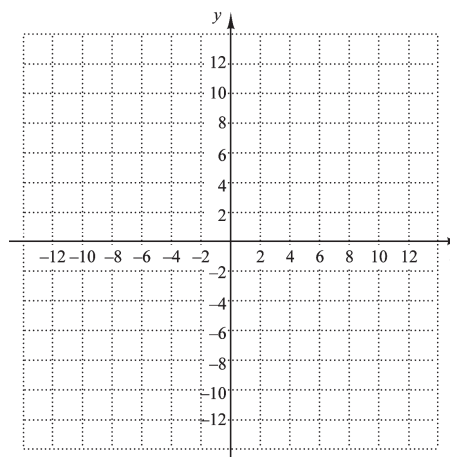
(b) Find $\lim_{x \rightarrow 2^-} f(x)$, $\lim_{x \rightarrow 2^+} f(x)$, and $\lim_{x \rightarrow 2} f(x)$, if each exists.

1. (a) _____

(b) _____

2. *Graphical limits.* Find the limit by graphing the function.

2. _____

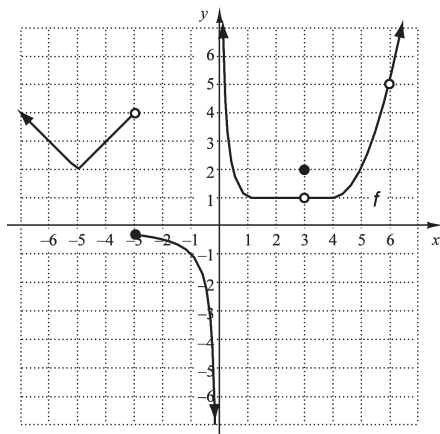


3. *Algebraic limits.* Find the limit algebraically. Show all work.

3. _____

42 CALCULUS AND ITS APPLICATIONS Chapter 1, Form B

Graphical limits. For Questions 4 – 12, consider f graphed below.



Find the limit, if it exists.

4. $\lim_{x \rightarrow -5} f(x)$ 4. _____

5. $\lim_{x \rightarrow -3} f(x)$ 5. _____

6. $\lim_{x \rightarrow 0} f(x)$ 6. _____

7. $\lim_{x \rightarrow 3} f(x)$ 7. _____

8. $\lim_{x \rightarrow 5} f(x)$ 8. _____

9. $\lim_{x \rightarrow 6} f(x)$ 9. _____

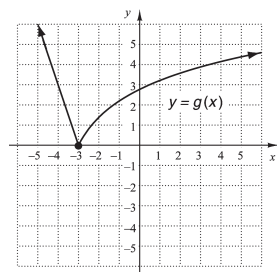
10. Find $f'(2)$. 10. _____

11. State the value(s) of x at which f is not continuous. 11. _____

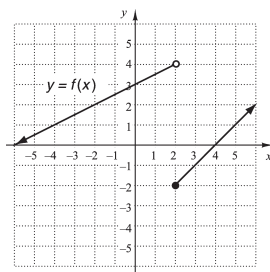
12. State the value(s) of x for which $f'(x)$ is not defined. 12. _____

Determine whether the function is continuous. If a function is discontinuous, state why.

13.



14.



13. _____

14. _____

Consider the function shown in Question 14, answer the following:

15. (a) Find $\lim_{x \rightarrow 0} f(x)$.

(b) Find $f(0)$.

(c) Is f continuous at 0?

15. (a) _____

(b) _____

(c) _____

16. Find $\lim_{x \rightarrow 2} f(x)$.

16. _____

Find the limit if it exists. If a limit does not exist, state why.

17. $\lim_{x \rightarrow 3} (-2x^3 + 6x^2 - 4)$

17. _____

18. $\lim_{x \rightarrow 5^+} \frac{x-5}{x^2-x-20}$

18. _____

19. $\lim_{x \rightarrow -3} \frac{4}{x+3}$

19. _____

20. Find the simplified difference quotient for:
 $f(x) = -5x^2 - 3$.

20. _____

22. Find the points on the graph of $y = 6x^3 + 9x^2$ at which the tangent line is horizontal.

22. _____

44 CALCULUS AND ITS APPLICATIONS Chapter 1, Form BFind dy/dx .

23. $y = x^{85}$

23. _____

24. $y = 5\sqrt[3]{x} + 4\sqrt{x}$

24. _____

25. $y = \frac{120}{x^5}$

25. _____

26. $y = x^{2/5}$

26. _____

27. $y = 4.1x^4 - 5x^2 + 7$

27. _____

Differentiate.

28. $y = \frac{2}{3}x^3 - 4x^2 + 10x + 6$

28. _____

29. $y = \frac{3x}{3-x}$

29. _____

30. $(2\sqrt{x} + 3)(x - x^2)$

30. _____

31. $f(x) = (x + 2)^3(2 - x)^2$

31. _____

32. $y = (4x^3 - 2x^2 + 5)^{-4}$

32. _____

33. $f(x) = x^2\sqrt{x-5}$

33. _____

34. For $y = 280x - 3x^5$, find $\frac{d^3y}{dx^3}$.

34. _____

35. *Medicine: temperature during an illness.* The temperature T , in degrees Fahrenheit, of a patient taking fever-reducing medicine is given by $T = 0.17t^2 - 1.5t + 102.5$, where t is time in hours.

- (a) Find the rate of change of the patient's temperature with respect to time.
- (b) What is the patient's temperature 3 hours after taking the medicine?
- (c) Find the rate of change of the patient's temperature 3 hours after taking the fever-reducing medicine.

35. (a) _____

(b) _____

(c) _____

36. *Business: average revenue, cost and profit.* Given revenue and cost functions $R(x) = 35x$ and $C(x) = x^{2/5} + 500$, where x is the number of items produced and $R(x)$ and $C(x)$ are in dollars, find:

- (a) The average revenue, the average cost and the average profit when x items are produced.
- (b) The rate at which average cost is changing when 15 items are produced.

36. (a) _____

(b) _____

For Questions 37 and 38, let $f(x) = 4x + x^2$ and $g(x) = 9x^5$.

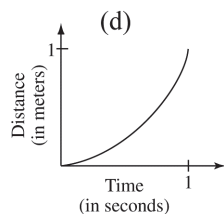
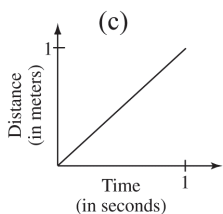
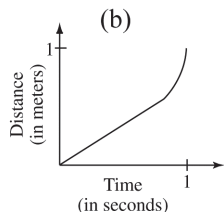
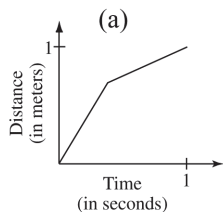
37. Find $\frac{d}{dx}(f \circ g)(x)$.

37. _____

38. Find $\frac{d}{dx}(g \circ f)(x)$.

38. _____

39. A marble rolls unimpeded along a level table. Let $y = s(t)$ represent the marble's distance after starting to roll. Which graph best represents s .



39. _____

40. Differentiate $y = \sqrt{(6 - 3x)^{1/3}(10 + x)^{4/3}}$.

40. _____

41. Find $\lim_{x \rightarrow 3} \frac{27 - x^3}{3 - x}$.

41. _____

42. Graph f and f' over the interval $[0, 5]$. Then estimate points at which the line tangent to f is horizontal.

$$f(x) = 4x^3 - 25x^2 + 32x + 4\sqrt{x}; \quad [0, 5]$$

42. _____



43. Find the following limit by creating a table of values:

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{x}.$$

Start with $\Delta Tbl = 0.1$ and then go to 0.01 and 0.001. When you think you know the limit, graph

$$y = \frac{\sqrt{x^2 + 1} - 1}{x}$$

and use TRACE to verify your assertion.

43. _____

Chapter 1, Form C

For Questions 1 – 3, consider

$$\lim_{x \rightarrow -3} f(x), \text{ where } f(x) = \frac{x^2 - 9}{x + 3}.$$

1. *Numerical limits.*

(a) Find the limit by completing the following input-output tables.

1.(a) _____

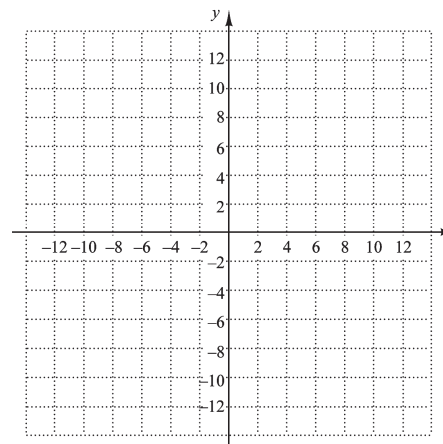
$x \rightarrow -3^-$	$f(x)$	$x \rightarrow -3^+$	$f(x)$
-3.1		-2.9	
-3.01		-2.99	
-3.001		-2.999	

(b) Find $\lim_{x \rightarrow -3^-} f(x)$, $\lim_{x \rightarrow -3^+} f(x)$, and $\lim_{x \rightarrow -3} f(x)$, if each exists.

(b) _____

2. *Graphical limits.* Find the limit by graphing the function.

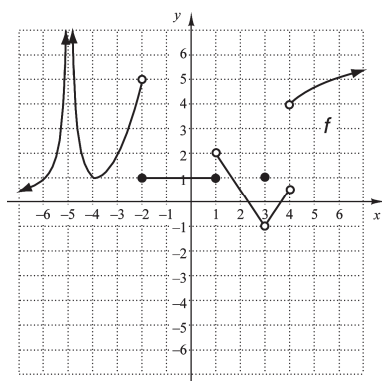
2. _____



3. *Algebraic limits.* Find the limit algebraically. Show all work.

3. _____

Graphical limits. For Questions 4 – 12, consider f graphed below.



Find the limit, if it exists.

4. $\lim_{x \rightarrow -5} f(x)$

4. _____

5. $\lim_{x \rightarrow -2} f(x)$

5. _____

6. $\lim_{x \rightarrow 0} f(x)$

6. _____

7. $\lim_{x \rightarrow 3} f(x)$

7. _____

8. $\lim_{x \rightarrow 4} f(x)$

8. _____

9. $\lim_{x \rightarrow -4} f(x)$

9. _____

10. Find $f'(-1)$.

10. _____

11. State the value(s) of x at which f is not continuous.

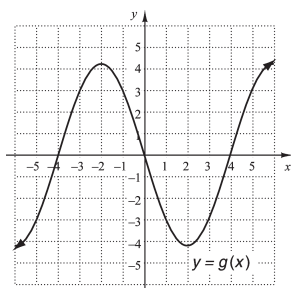
11. _____

12. State the value(s) of x at which $f'(x)$ is not defined.

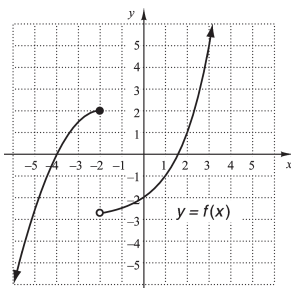
12. _____

Determine whether the function is continuous. If a function is discontinuous, state why.

13.



14.



13. _____

14. _____

Consider the function shown in Question 14, answer the following:

15. (a) Find $\lim_{x \rightarrow -2} f(x)$.

(b) Find $f(-2)$.

(c) Is f continuous at -2 ?

15. (a) _____

(b) _____

(c) _____

16. Find $\lim_{x \rightarrow 3} f(x)$.

16. _____

Find the limit if it exists. If the limit does not exist, state why.

17. $\lim_{x \rightarrow -2} (4x^3 - 6x^2 - 3x + 1)$

17. _____

18. $\lim_{x \rightarrow -3^+} \frac{x+3}{x(x^2-9)}$

18. _____

19. $\lim_{x \rightarrow 0} \frac{15}{x}$

19. _____

20. Find the simplified difference quotient for:
 $f(x) = 4x^2 - 6x$

20. _____

21. Find an equation of the tangent line to the graph of $y = -x + \left(\frac{4}{x}\right)$ at the point $(2, 0)$.

21. _____

22. Find the points on the graph of $y = x^3 - 3x^2$ at which the tangent line is horizontal.

22. _____

Find dy/dx .

23. $y = x^{46}$ 23. _____

24. $y = 6\sqrt[4]{x} - 2\sqrt{x}$ 24. _____

25. $y = -\frac{7}{x^7}$ 25. _____

26. $y = x^{4/5}$ 26. _____

27. $y = 6.3x^3 - 4x^2 - 5$ 27. _____

Differentiate.

28. $y = \frac{3}{4}x^4 + 8x^2 - 161x + 25$ 28. _____

29. $y = (5\sqrt{x} - 8)(x + x^2)$ 29. _____

30. $y = \frac{2x - 1}{x^4}$ 30. _____

31. $f(x) = (x + 1)^3(3 - x)^4$ 31. _____

32. $y = (3x^3 - 5x^2 + 8)^{-3}$ 32. _____

33. $f(x) = x^2\sqrt{x^3 - 5}$ 33. _____

34. For $y = 3x^6 - 4x^3$, find $\frac{d^3y}{dx^3}$. 34. _____

- 35. Ozone level.** The ozone level (in parts per billion) in a metropolitan area is modeled by $P = 60 + 15t - t^2$, where t is time in hours and $t = 0$ corresponds to 8:00am.

(a) Find the rate of change of the ozone level with respect to time.

(b) What is the ozone level at $t = 6$?

(c) Find the rate of change of the ozone level at $t = 6$.

35. (a) _____

(b) _____

(c) _____

- 36. Business: average revenue, cost and profit.** Given revenue and cost functions $R(x) = 25x$ and $C(x) = x^{1/3} + 1000$, where x is the number of items produced and $R(x)$ and $C(x)$ are in dollars, find:

(a) The average revenue, the average cost and the average profit when x items are produced.

(b) The rate at which average cost is changing when 25 items are produced.

36. (a) _____

(b) _____

For Questions 37 and 38, let $f(x) = 5x^3$ and $g(x) = x^2 + 8x$.

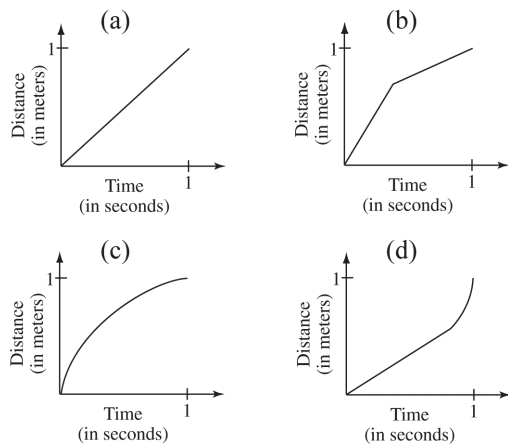
37. Find $\frac{d}{dx}(f \circ g)(x)$.

37. _____

38. Find $\frac{d}{dx}(g \circ f)(x)$.

38. _____

39. A marble rolls on a smooth level surface, then along a carpeted surface. Let $y = s(t)$ represent the marble's distance after starting to roll. Which graph best represents s ?



39. _____

40. Differentiate $y = \sqrt{(4-3x)^{6/5}(1+x)^{2/5}}$.

40. _____

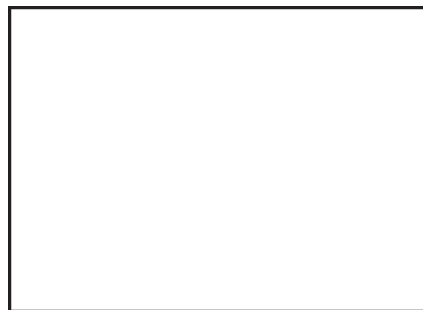
41. Find $\lim_{x \rightarrow -4} \frac{x^3 + 64}{x + 4}$

41. _____

42. Graph f and f' over the interval $[-3, 3]$. Then estimate points at which the line tangent to f is horizontal.

$$f(x) = 2x^5 - 5x^2 - x + 2; [-3, 3]$$

42. _____



43. Find the following limit by creating a table of values:

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{x}.$$

Start with $\Delta Tbl = 0.1$ and then go to 0.01 and 0.001. When you think you know the limit, graph

$$y = \frac{\sqrt{x^2 + 1} - 1}{x}$$

and use TRACE to verify your assertion.

43. _____

Chapter 1, Form D

For Questions 1 – 3, consider

$$\lim_{x \rightarrow 4} f(x), \text{ where } f(x) = \frac{x^2 - 16}{x - 4}.$$

1. *Numerical limits.* Find the limit by completing the following input-output tables.

$x \rightarrow 4^-$	$f(x)$	$x \rightarrow 4^+$	$f(x)$
3.9		4.1	
3.99		4.01	
3.999		4.001	

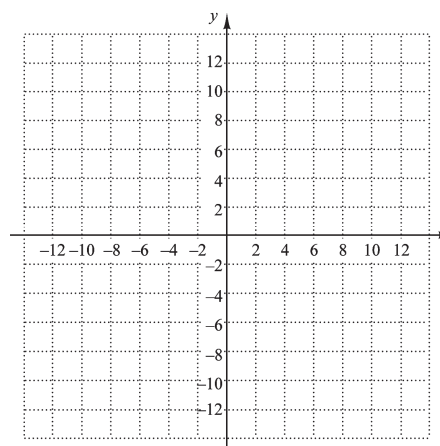
- (b) Find $\lim_{x \rightarrow 4^-} f(x)$, $\lim_{x \rightarrow 4^+} f(x)$, and $\lim_{x \rightarrow 4} f(x)$, if each exists

1. (a) _____

(b) _____

2. *Graphical limits.* Find the limit by graphing the function.

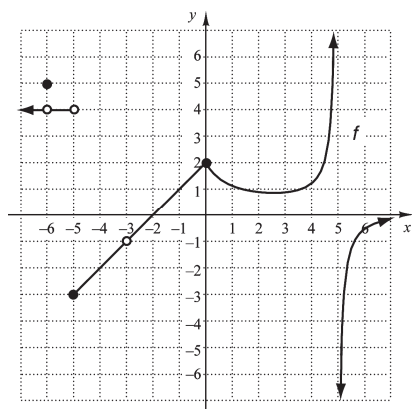
2. _____



3. *Algebraic limits.* Find the limit algebraically. Show all work.

3. _____

Graphical limits. For Questions 4 – 12, consider f graphed below.



Find the limit, if it exists.

4. $\lim_{x \rightarrow -5} f(x)$

4. _____

5. $\lim_{x \rightarrow -3} f(x)$

5. _____

6. $\lim_{x \rightarrow 0} f(x)$

6. _____

7. $\lim_{x \rightarrow -2} f(x)$

7. _____

8. $\lim_{x \rightarrow 5} f(x)$

8. _____

9. $\lim_{x \rightarrow -6} f(x)$

9. _____

10. Find $f'(3)$.

10. _____

11. State the value(s) of x at which f is not continuous.

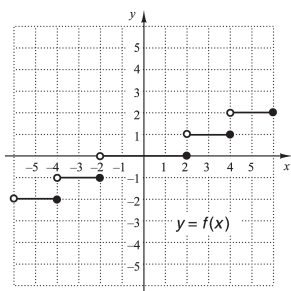
11. _____

12. State the value(s) of x for which $f'(x)$ is not defined.

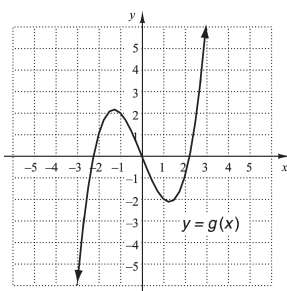
12. _____

Determine whether the function is continuous. If a function is discontinuous, state why.

13.



14.



13. _____

14. _____

Consider the function in Question 13, answer the following.

 15. (a) Find $\lim_{x \rightarrow -3} f(x)$.

 (b) Find $f(-3)$.

 (c) Is f continuous at -3 ?

15. (a) _____

(b) _____

(c) _____

 16. Find $\lim_{x \rightarrow 4} f(x)$.

16. _____

Find the limit if it exists. If a limit does not exist, state why.

 17. $\lim_{x \rightarrow -1} (5x^4 + 3x^3 - 6x^2 - 4x)$

17. _____

 18. $\lim_{x \rightarrow -2^-} \frac{x+2}{3x(x^2-4)}$

18. _____

 19. $\lim_{x \rightarrow -6} \frac{6}{x+6}$

19. _____

 20. Find the simplified difference quotient for:
 $f(x) = 3x^2 - 7x$.

20. _____

 21. Find an equation of the tangent line to the graph of $y = 3x + \left(\frac{8}{x}\right)$ at the point $(2, 10)$.

21. _____

 22. Find the points on the graph of $y = 2x^3 - 3x^2$ at which the tangent line is horizontal.

22. _____

Find dy/dx .

23. $y = x^{113}$

23. _____

24. $y = 2\sqrt[4]{x} - 4\sqrt{x}$

24. _____

25. $y = \frac{3}{x^4}$

25. _____

26. $y = x^{2/7}$

26. _____

27. $y = 0.59x^4 - 6x^2 + 8$

27. _____

Differentiate.

28. $y = \frac{1}{10}x^5 + 3x^4 - 6x - 6$

28. _____

29. $y = (4\sqrt{x} - 1)(x^2 + x)$

29. _____

30. $y = \frac{4x^2 + 1}{x^4}$

30. _____

31. $f(x) = (x + 1)^3(6 - x)^2$

31. _____

32. $y = (6x^2 + 2x^5 + x^6)^{-4}$

32. _____

33. $f(x) = x^2\sqrt{x^4 - 1}$

33. _____

34. For $y = 4x^6 - 3x^2$, find $\frac{d^3y}{dx^3}$.

34. _____

- 35. Social Sciences: memory.** In a certain memory experiment, a person is able to memorize M words after t minutes, where $M = -0.002t^3 + 0.1t^2$.

(a) Find the rate of change of the number of words memorized with respect to time.

(b) How many words are memorized during the first 20 minutes (at $t = 20$)?

(c) Find the rate at which words are being memorized after 20 minutes.

35. (a) _____

(b) _____

(c) _____

- 36. Business: average revenue, cost and profit.** Given revenue and cost functions $R(x) = 60x$ and $C(x) = x^{3/4} + 300$, where x is the number of items produced and $R(x)$ and $C(x)$ are in dollars, find:

(a) The average revenue, the average cost and the average profit when x items are produced.

(b) The rate at which average cost is changing when 16 items are produced

36. (a) _____

(b) _____

For Questions 37 and 38, let $f(x) = \sqrt{x+4}$ and $g(x) = x^2 + x$.

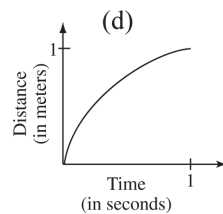
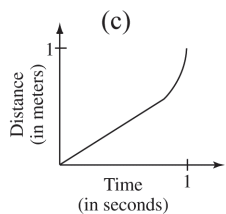
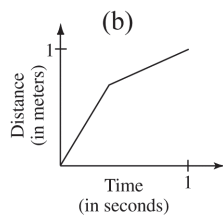
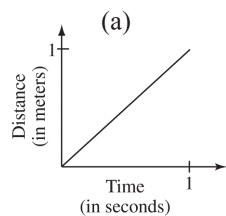
37. Find $\frac{d}{dx}(f \circ g)(x)$.

37. _____

38. Find $\frac{d}{dx}(g \circ f)(x)$.

38. _____

39. A small ball rolls along a level table at a constant rate and then down an inclined plane. Let $y = s(t)$ represent the marble's distance after starting to roll. Which graph best represents s ?



40. Differentiate $y = \sqrt{(8-2x)^{3/2}(4+x)^{1/3}}$.

40. _____

41. Find $\lim_{x \rightarrow 5} \frac{x^3 - 125}{x - 5}$.

41. _____

42. Graph f and f' over the interval $[-5, 5]$. Then estimate points at which the line tangent to f is horizontal.

$$f(x) = 2x^5 + 4x^2 - 7x; [-5, 5]$$

42. _____



43. Find the following limit by creating a table of values:

$$\lim_{x \rightarrow 5} \frac{\sqrt{6x-5}-5}{x-5}.$$

Start with $\Delta Tbl = 0.1$ and then go to 0.01 and 0.001. When you think you know the limit, graph

$$y = \frac{\sqrt{6x-5}-5}{x-5}$$

and use TRACE to verify your assertion.

43. _____

