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Chapter 02: Derivatives

1. Suppose that $h(t)$ represents the height, in feet, of a person t years old. In real world terms, what does $h(10)$ represent? What is its unit? What does $h'(t)$ represents and what is its unit?

ANSWER: $h(10)$ represents the height of a 10-year old person. Its unit is feet. $h'(t)$ represents the rate of change of the height when a person is t years old. Its unit is feet per year.

2. The function $f(x)=9-2x+x^2$ is both continuous and differentiable at $x=0$ Write these facts as limit statements.

ANSWER: $\lim_{x\to 0}(9-2x+x^2)=f(0)=9$

Since f is continuous at $x = 0$,

$\lim_{h\to 0}\frac{-2h+2xh+h^2}{h}$

Since f is differentiable at $x = 0$, exists.

Note that there are alternate ways of writing the answer

3. Suppose $f(1)=2,\ \lim_{x\to 1^-}f(x)=2,$ and $\lim_{x\to 1^+}f(x)=2,\ \lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1}=-2,$ and $\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}=1.$ Is f continuous and/or differentiable at $x=1?$

a. f is not continuous but differentiable at $x = 1$
b. f is neither continuous nor differentiable at $x = 1$
c. f is continuous at but not differentiable at $x = 1$
d. f is both continuous and differentiable at $x = 1$

ANSWER: c

4. Suppose $f(1)=2,\ \lim_{x\to 1^-}f(x)=2,$ and $\lim_{x\to 1^+}f(x)=2,\ \lim_{h\to 0^-}\frac{f(1+h)-f(1)}{h}=-2,$ and $\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}=-2.$ Is f continuous and/or differentiable at $x=1?$

a. f is not continuous but differentiable at $x = 1$
b. f is neither continuous nor differentiable at $x = 1$
c. f is continuous but not differentiable at $x = 1$
d. f is both continuous and differentiable at $x = 1$

ANSWER: d

5. Use the definition of derivative: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(-1),\ \text{if } f(x)=x^2.$

ANSWER:

6. Use the definition of derivative: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(-2),\ \text{if } f(x)=\frac{2}{x}.$

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ANSWER: $-\frac{1}{2}$

7. Use the definition of derivative: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(1)$, if $f(x)=\frac{x+1}{x-2}$.
ANSWER: -3

8. Use the definition of derivative: $\lim_{z\to c}\frac{f(z)-f(c)}{z-c}$ to find $f'(-1)$, if $f(x)=x^2$.
ANSWER: -2

9. Use the definition of derivative: $\lim_{z\to c}\frac{f(z)-f(c)}{z-c}$ to find $f'(-2)$, if $f(x)=\frac{2}{x}$.
ANSWER: $-\frac{1}{2}$

10. Use the definition of derivative: $\lim_{z\to c}\frac{f(z)-f(c)}{z-c}$ to find $f'(1)$, if $f(x)=\frac{x+1}{x-2}$.
ANSWER: -3

11. Use the definition of derivative: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(x)$, if $f(x)=\frac{3}{x+1}$.
ANSWER: $f'(x)=\frac{-3}{(x+1)^2}$

12. Use the definition of derivative: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(x)$, if $f(x)=2\sqrt{x}$.
ANSWER: $f'(x)=\frac{1}{\sqrt{x}}$

13. Use the definition of derivative: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(x)$, if $f(x)=\frac{1}{x^2}$.
ANSWER: $f'(x)=\frac{-2}{x^3}$

14. Use the definition of derivative: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(x)$, if $f(x)=\sqrt{x+1}$.
ANSWER: $f'(x)=\frac{1}{2\sqrt{x+1}}$

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15. Given $f(x)=\begin{cases} -2x \text{ if } x<1 \\ x-3 \text{ if } x\geq 1 \end{cases}$, is f continuous and/or differentiable at $x=1$? Explain.

a. f is not continuous but differentiable at $x = 1$
b. f is neither continuous nor differentiable at $x = 1$
c. f is continuous but not differentiable at $x = 1$
d. f is both continuous and differentiable at $x = 1$

ANSWER: c

16. Given $f(x)=\begin{cases} x^2-2 \text{ if } x<2 \\ 2x+1 \text{ if } x\geq 2 \end{cases}$, is f continuous and/or differentiable at $x=2$? Explain.

a. f is continuous but not differentiable at $x = 2$
b. f is differentiable but not continuous at $x = 2$
c. f is neither continuous nor differentiable at $x = 2$
d. f is both continuous and differentiable at $x = 2$

ANSWER: c

17. Use the Intermediate Value Theorem to show that $f(x)=x^2-2$ has at least one zero on [0, 2].

ANSWER: f is continuous on [0, 2], and $f(0)=-2<0, \text{ and } f(2)=2>0.$

18. Use the Intermediate Value Theorem to show that $f(x)=x^3+2$ has at least one zero on [- 2, 1].

ANSWER: f is continuous on [- 2, 1], and $f(-2)=-6<0, \text{ and } f(1)=3>0.$

19. Suppose f is a piecewise-defined function, equal to $g(x) \text{ if } x<3, \text{ and } h(x) \text{ if } x\geq 3,$ where $g \text{ and } h$ are continuous and differentiable everywhere. If $g'(3)=h'(3),$ is the function f differentiable at $x=3$? Explain why or why not.

ANSWER: Yes.

20. Suppose $f, g \text{ and } h$ are functions with values $f(1)=-3,\ g(1)=2,\ f'(1)=0,\ g'(1)=3.$ Find $(fg)'(1).$

a. 7
b. 9
c. – 9
d. – 7

ANSWER: c

21. Suppose $f, g \text{ and } h$ are functions with values $f(1)=-3,\ g(1)=2,\ f'(1)=0,\ g'(1)=3.$ Find $(2f-5g)'(1)$

a. 15

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b. – 12
c. 12
d. – 15

ANSWER: d

22. Suppose $f, g \text{ and } h$ are functions with values $f(1)=-3,\ g(1)=2,\ f'(1)=0,\ g'(1)=3.$ Find $\left[\frac{g}{f}\right]'(1).$

a. 3
b. $-\frac{9}{4}$
c. – 1
d. 1

ANSWER: c

23. Find constants $a \text{ and } b$ so that $f(x)=\begin{cases} ax+b & \text{if } x<1 \\ bx^2+1 & \text{if } x\geq 1 \end{cases}$, is continuous and differentiable everywhere?

a. $a=2,\ b=1$
b. $a=1,\ b=1$
c. $a=1,\ b=\frac{-1}{2}$
d. $a=1,\ b=\frac{1}{2}$

ANSWER: d

x	g(x)	h(x)	g'(x)	h'(x)
-3	0	3	1	0
-2	1	2	2	-3
-1	3	0	-1	-2
0	2	3	-2	3
1	0	-1	-2	-2
2	-2	-2	-1	0
3	-3	0	0	1

24. Use the table above to find $f'(3)$ if $f(x)=5g(x)-4h(x)$.

a. 4
b. – 1

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c. 1
d. – 4

ANSWER: d

25. Use the table above to find $f'(2)$ if $f(x)=\frac{2g(x)}{h(x)}$.
a. 4
b. – 1
c. 1
d. – 4

ANSWER: c

26. Use the table above to find $f'(-1)$ if $f(x)=\frac{g(x)h(x)+1}{g(x)}$.
a. 19/9
b. – 19/9
c. 17/9
d. – 17/9

ANSWER: d

27. Use the table above to find $f'(-2)$ if $f(x)=g(h(x))$.
a. – 3
b. 3
c. – 1
d. 1

ANSWER: b

28. Use the table above to find $f'(-2)$ if $f(x)=h(g(x))$.
a. 4
b. – 1
c. 1
d. – 4

ANSWER: d

29. Use the table above to find $f'(1)$ if $f(x)=(h(x))^3$.
a. 3
b. – 3
c. 6

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d. – 6

ANSWER: d

30. Use the table above to find $f'(-1)$ if $f(x)=\sqrt{g(x)}$.

a. $\frac{\sqrt{3}}{2}$

b. $\frac{2}{\sqrt{3}}$

c. $\frac{1}{2\sqrt{3}}$

d. $\frac{-1}{2\sqrt{3}}$

ANSWER: d

31. Use the table above to find $f'(-1)$ if $f(x)=h(x^2g(x))$.

a. 1

b. – 1

c. 7

d. – 7

ANSWER: d

32. Use the table above to find $f'(1)$ if $f(x)=h(3-2x^2)$.

a. 2

b. – 2

c. 8

d. – 8

ANSWER: c

33. Find the derivative of $f(x)=5-2e+3x-2x^5+\frac{1}{\sqrt[3]{x}}$.

a. $-1-10x^4-\frac{1}{3}x^{-\frac{1}{3}}$

b. $3-10x^4-\frac{1}{3}x^{-\frac{1}{3}}$

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c. $3-10x^4-\frac{1}{3}x^{-\frac{4}{3}}$

d. $3-10x^4-\frac{1}{3}x^{-\frac{2}{3}}$

ANSWER: c

34. Find the derivative of $f(x)=-2x^3+3x+4\sqrt{x}+5$.

a. $6x^2+3+2x^{\frac{1}{2}}$

b. $-6x^2+3+2x^{\frac{1}{2}}$

c. $6x^2+3+2x^{-\frac{1}{2}}$

d. $-6x^2+3+2x^{-\frac{1}{2}}$

ANSWER: d

35. Find the derivative of $f(x)=\frac{x^2}{1-2x^3}$.

a. $\frac{-2x^4+2x}{(1-2x^3)^2}$

b. $\frac{-2x^4-2x}{(1-2x^3)^2}$

c. $\frac{2x^4-2x}{(1-2x^3)^2}$

d. $\frac{2x^4+2x}{(1-2x^3)^2}$

ANSWER: d

36. Find the derivative of $f(x)=\frac{3+2x}{4-5x}$.

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a. $\frac{-23}{(4-5x)^2}$

b. $\frac{23}{(4-5x)^2}$

c. $\frac{-7}{(4-5x)^2}$

d. $\frac{7}{(4-5x)^2}$

ANSWER: b

37. Find the derivative of $f(x)=\left(\sqrt[3]{x}+2\sqrt{x}\right)^3$.

ANSWER: $\left(\sqrt[3]{x}+2\sqrt{x}\right)^2\left(\frac{1}{\sqrt[3]{x^2}}+\frac{3}{\sqrt{x}}\right)$

38. Find the derivative of $f(x)=\frac{\sqrt[3]{x^5}-3x^5}{x^3}$.

a. $\frac{4}{3}x^{-\frac{3}{5}}-6x$

b. $\frac{4}{3}x^{-\frac{3}{7}}-6x$

c. $-\frac{4}{3}x^{-\frac{7}{3}}-6x$

d. $\frac{4}{3}x^{-\frac{7}{3}}-6x$

ANSWER: c

39. Find the derivative of $f(x)=\frac{(x+2)^2}{(x^2-4)(x+2)}$.

a. $\frac{1}{x+2}$

b. $\frac{1}{(x+2)^2}$

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c. $\dfrac{1}{(x-2)^2}$

d. $-\dfrac{1}{(x-2)^2}$

ANSWER: d

40. Find the derivative of $f(x)=|2x+1|$.

ANSWER:
$$f'(x)=\begin{cases} -2 & \text{if } x<-\frac{1}{2} \\ \text{DNE} & \text{if } x=-\frac{1}{2} \\ 2 & \text{if } x>-\frac{1}{2} \end{cases}$$

41. Find the derivative of $f(x)=|1-3x|$.

ANSWER:
$$f'(x)=\begin{cases} -3 & \text{if } x<\frac{1}{3} \\ \text{DNE} & \text{if } x=\frac{1}{3} \\ 3 & \text{if } x>\frac{1}{3} \end{cases}$$

42. Find the derivative of $f(x)=\begin{cases} 4x-1 & \text{if } x<2 \\ 2x^2+3 & \text{if } x\geq 2 \end{cases}$.

ANSWER:
$$f'(x)=\begin{cases} 4 & \text{if } x<2 \\ \text{DNE} & \text{if } x=2 \\ 4x & \text{if } x>2 \end{cases}$$

43. Find the derivative of $f(x)=\dfrac{2}{4-5x^4}$.

a. $\dfrac{-20x^3}{(4-5x^4)^2}$

b. $\dfrac{40x^3}{(4-5x^4)^2}$

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c. $\dfrac{20x^3}{(4-5x^4)^2}$

d. $\dfrac{-40x^3}{(4-5x^4)^2}$

ANSWER: b

44. Differentiate $f(x)=\left(\dfrac{x^3-1}{\sqrt{x}}\right)^3$ in three ways: (a) with the chain rule, (b) with the quotient rule but not chain rule, (c) without the chain or quotient rules.

ANSWER: $\left(x^{\frac{5}{2}}-x^{-\frac{1}{2}}\right)^2\left(\dfrac{15}{2}x^{\frac{3}{2}}+\dfrac{3}{2}x^{-\frac{3}{2}}\right)$

45. Differentiate $f(x)=\left(\dfrac{2x^3+1}{\sqrt[3]{x}}\right)^3$ in three ways: (a) with the chain rule, (b) with the quotient rule but not chain rule, (c) without the chain or quotient rules.

ANSWER: $\left(2x^{\frac{8}{3}}+x^{-\frac{1}{3}}\right)^2\left(\dfrac{48}{3}x^{\frac{5}{3}}-x^{-\frac{4}{3}}\right)$

46. Find the derivative of $f(x)=\left(\sqrt[3]{x}+4\right)^6$.

a. $\dfrac{2\left(x^{\frac{1}{3}}+4\right)^5}{x^{\frac{1}{3}}}$

b. $\dfrac{2\left(x^{\frac{1}{3}}+4\right)^5}{x^{\frac{2}{3}}}$

c. $2\left(x^{\frac{1}{3}}+4\right)^5$

d. $\dfrac{2\left(x^{\frac{1}{3}}+4\right)^4}{x^{\frac{2}{3}}}$

ANSWER: b

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47. Find the derivative of $f(x)=2x(4x^2+1)^7$.

a. $(112x^2+2)(4x^2+1)^6$

b. $(120x^2+2)(4x^2+1)^7$

c. $(120x^2+2)(4x^2+1)^6$

d. $112x^2(4x^2+1)^6$

ANSWER: c

48. Find the derivative of $f(x)=\dfrac{2x-1}{\sqrt{x^2+2}}$.

a. $\dfrac{-x+4}{(x^2+2)^{\frac{3}{2}}}$

b. $\dfrac{x+4}{(x^2+2)^{\frac{3}{2}}}$

c. $\dfrac{x^2+4}{(x^2+2)^{\frac{3}{2}}}$

d. $\dfrac{-x-4}{(x^2+2)^{\frac{3}{2}}}$

ANSWER: b

49. Find the derivative of $f(x)=\left(3x\sqrt{x^2+1}\right)^{-3}$.

ANSWER: $\dfrac{-(2x^2+1)}{9x^4(x^2+1)^{\frac{5}{2}}}$

50. Find the derivative of $f(x)=\dfrac{(1-\sqrt{x})^2}{2x^3-5x+1}$.

ANSWER: $\dfrac{\left(1-x^{\frac{1}{2}}\right)\left(4x^{\frac{5}{2}}-6x^2-x^{-\frac{1}{2}}+5\right)}{(2x^3-5x+1)^2}$

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51. Find the derivative of $f(x)=\sqrt{5-\sqrt{2x+1}}$.

ANSWER: $\dfrac{-1}{2\sqrt{10x+5-(2x+1)\sqrt{2x+1}}}$

52. Find the derivative of $f(x)=\left(\sqrt[3]{x}-3x\right)^{-2}$.

a. $\dfrac{18x^{\frac{1}{3}}-2}{3x^{\frac{2}{3}}\left(x^{\frac{1}{3}}-3x\right)^3}$

b. $\dfrac{18x^{\frac{1}{3}}-2}{3x^{\frac{1}{3}}\left(x^{\frac{1}{3}}-3x\right)^3}$

c. $\dfrac{18x^{\frac{2}{3}}-2}{3x^{\frac{2}{3}}\left(x^{\frac{1}{3}}-3x\right)^3}$

d. $\dfrac{18x^{\frac{2}{3}}-2}{2x^{\frac{2}{3}}\left(x^{\frac{1}{3}}-3x\right)^3}$

ANSWER: c

53. Find the derivative of $f(x)=\left(1-2x^3\right)^3\left(4x^2+1\right)^6$.

a. $3(1-2x^3)^2(4x^2+1)^4(4x^2+16x+1)$

b. $3(1-2x^3)^2(4x^2+1)^4(32x^4+4x^2+16x+1)$

c. $3(1-2x^3)^2(4x^2+1)^5(4x^2+16x+1)$

d. $6x(1-2x^3)^2(4x^2+1)^5(-28x^3-3x+8)$

ANSWER: d

54. Find the derivative of $f(x)=2\left(\left(x^2+1\right)^5-4x\right)^{-5/2}$.

a. $\dfrac{50x(x^2+1)^4}{((x^2+1)^5-4x)^{\frac{5}{2}}}$

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b. $\dfrac{20-50x(x^2+1)^4}{((x^2+1)^5-4x)^{\frac{7}{2}}}$

c. $\dfrac{50x(x^2+1)^4-20}{((x^2+1)^5-4x)^{\frac{7}{2}}}$

d. $\dfrac{20-50x(x^2+1)^4}{((x^2+1)^5-4x)^{\frac{5}{2}}}$

ANSWER: b

55. If $f(x)=\left(x\sqrt{x+2}\right)^{-2}$, find $f''(x)$.

ANSWER: $\dfrac{12x^4+32x^3+24x^2}{x^6(x+2)^3}$

56. Use implicit differentiation to find $\dfrac{dy}{dx}$ if $xy^2+2x^3+y^2=10$.

a. $\dfrac{6x^2+y^2}{2xy+2y}$

b. $\dfrac{-6x^2+y^2}{2xy+2y}$

c. $\dfrac{6x^2-y^2}{2xy+2y}$

d. $\dfrac{-6x^2-y^2}{2xy+2y}$

ANSWER: d

57. Use implicit differentiation to find $\dfrac{dy}{dx}$ if $5xy+x^2y-y^2x=10$.

a. $\dfrac{10}{5x-x^2+2xy}$

b. $\dfrac{y^2+5y+2xy}{5x-x^2+2xy}$

c. $\dfrac{y^2-5y}{5x+x^2}$

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d. $\dfrac{y^2-2xy-5y}{5x+x^2-2xy}$

ANSWER: d

58. Use implicit differentiation to find $\dfrac{dy}{dx}$ if $\sqrt{2y+1}=4xy$.

a. $\dfrac{1-8y\sqrt{2y+1}}{8x\sqrt{2y+1}}$

b. $\dfrac{1-4y\sqrt{2y+1}}{2x\sqrt{2y+1}}$

c. $\dfrac{4y\sqrt{2y+1}}{1-4x\sqrt{2y+1}}$

d. $\dfrac{4y\sqrt{2y+1}}{1+2x\sqrt{2y+1}}$

ANSWER: c

59. Find the equation of the tangent lines to the circle $x^2+y^2=4$ at the points with x-coordinate $x=1.$

ANSWER: $y-\sqrt{3}=-\dfrac{\sqrt{3}}{3}(x-1)$ and $y+\sqrt{3}=\dfrac{\sqrt{3}}{3}(x-1)$

60. Find the equation of the tangent lines to the graph of $-x^2+2y^2+3x=3$ at the points with x-coordinate $x=2.$

ANSWER: $y-\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{2}}{4}(x-2)$ and $y+\dfrac{\sqrt{2}}{2}=-\dfrac{\sqrt{2}}{4}(x-2)$

61. Find the derivative of $f(x)=\dfrac{(x^3-4)^5(5x-2)}{(x+1)^{-3}(2+x^2)^4}$.

ANSWER: $\dfrac{(x^3-4)^5(5x-2)}{(x+1)^{-3}(2+x^2)^4}\left(\dfrac{15x^2}{x^3-4}+\dfrac{5}{5x-2}+\dfrac{3}{x+1}-\dfrac{8x}{2+x^2}\right)$

62. Find the derivative of $f(x)=\dfrac{\sqrt{2x+1}(x^2-1)^4}{(x-3)^3(2+x)}$.

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ANSWER: $\dfrac{\sqrt{2x+1}\left(x^2-1\right)^4}{(x-3)^3(2+x)}\left(\dfrac{1}{2x+1}+\dfrac{8x}{x^2-1}-\dfrac{3}{x-3}-\dfrac{1}{2+x}\right)$

63. Find the derivative of $f(x)=\dfrac{1}{3+e^{2x}}$.

a. $\dfrac{e^{2x}}{\left(3+e^{2x}\right)^2}$

b. $\dfrac{2e^{2x}}{\left(3+e^{2x}\right)^2}$

c. $\dfrac{-e^{2x}}{\left(3+e^{2x}\right)^2}$

d. $\dfrac{-2e^{2x}}{\left(3+e^{2x}\right)^2}$

ANSWER: d

64. Find the derivative of $f(x)=e^{5x}\ln\left(2x^2+1\right)$.

a. $e^{5x}\ln\left(2x^2+1\right)+\dfrac{e^{5x}}{\ln\left(2x^2+1\right)}$

b. $e^{5x}\ln\left(2x^2+1\right)+\dfrac{e^{5x}}{2x^2+1}$

c. $5e^{5x}\ln\left(2x^2+1\right)+\dfrac{e^{5x}}{2x^2+1}$

d. $5e^{5x}\ln\left(2x^2+1\right)+\dfrac{4xe^{5x}}{2x^2+1}$

ANSWER: d

65. Find the derivative of $f(x)=\dfrac{3+x^2}{e^{2x}}$.

a. $\dfrac{2x^2-2x+6}{e^{2x}}$

b. $\dfrac{x^2-2x+3}{e^{2x}}$

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c. $\dfrac{-2x^2+2x-6}{e^{2x}}$

d. $\dfrac{-x^2+2x-3}{e^{2x}}$

ANSWER: c

66. Find the derivative of $f(x)=\sqrt[3]{\ln(2x^2+1)}$.

a. $\dfrac{1}{(6x^2+3)\sqrt[3]{\ln(2x^2+1)}}$

b. $\dfrac{4x}{(6x^2+3)\sqrt[3]{[\ln(2x^2+1)]^2}}$

c. $\dfrac{4x}{(6x^2+3)\sqrt[3]{\ln(2x^2+1)}}$

d. $\dfrac{1}{(6x^2+3)\sqrt[3]{[\ln(2x^2+1)]^2}}$

ANSWER: b

67. Find the derivative of $f(x)=e^{5x}\ln(x^2+1)$.

ANSWER: $\dfrac{e^{5x}[5(x^2+1)\ln(x^2+1)+2x]}{x^2+1}$

68. Find the derivative of $f(x)=x^3\ln(2x^2)$.

a. $3x^2\ln(2x^2)+\dfrac{1}{2}x$

b. $3x^2\ln(2x^2)+2x$

c. $3x^2\ln(2x^2)+2x^2$

d. $3x\ln(2x^2)+2x^2$

ANSWER: c

69. Find the derivative of $f(x)=x^2e^{3x}$.

a. $2xe^{3x}+x^2e^{3x}$

b. $2xe^{3x}+3x^2e^{3x}$

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c. $x^2e^{3x}+3xe^{3x}$

d. $2e^{3x}+3x^2e^{3x}$

ANSWER: b

70. Find the derivative of $f(x)=\ln\left(\frac{x^5}{x^3+2x+1}\right)$.

a. $\frac{5}{x}+\frac{3x+2}{x^3+2x+1}$

b. $\frac{5}{x}+\frac{3x^2+2}{x^3+2x+1}$

c. $\frac{5}{x}-\frac{3x^2+2}{x^3+2x+1}$

d. $\frac{5}{x}-\frac{3x^2-2}{x^3+2x+1}$

ANSWER: c

71. Find the derivative of $f(x)=5^x+\ln\sqrt{x}$.

a. $5^x+\frac{1}{x}$

b. $5^x\ln 5+\frac{1}{x}$

c. $5^x+\frac{1}{2x}$

d. $5^x\ln 5+\frac{1}{2x}$

ANSWER: d

72. Find the derivative of $f(x)=xe^{3x^2}+5x\ln\sqrt[3]{x}$.

a. $6x^2e^{3x^2}+\frac{5}{3}$

b. $6x^2e^{3x^2}+5\ln\sqrt[3]{x}+\frac{5}{3}$

c. $6x^2e^{3x^2}+e^{3x^2}+\frac{5}{3}$

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d. $6x^2e^{3x^2}+e^{3x^2}+5\ln\sqrt[3]{x}+\frac{5}{3}$

ANSWER: d

$$f(x)=\begin{cases}\ln(x+3) & \text{if } x<1\\ 4x^2-1 & \text{if } x\geq 1\end{cases}$$

73. Find the derivative of .

a. $f'(x)=\begin{cases}\frac{1}{x+3} & \text{if } x<1\\ 4x^2+1 & \text{if } x\geq 1\end{cases}$

b. $f'(x)=\begin{cases}\frac{1}{x+3} & \text{if } x<1\\ \text{DNE} & \text{if } x=1\\ 4x & \text{if } x>1\end{cases}$

c. $f'(x)=\begin{cases}\frac{1}{x+3} & \text{if } x<1\\ 8x & \text{if } x\geq 1\end{cases}$

d. $f'(x)=\begin{cases}\frac{1}{x+3} & \text{if } x<1\\ \text{DNE} & \text{if } x=1\\ 8x & \text{if } x>1\end{cases}$

ANSWER: d

$$f(x)=\frac{e^{3x}(x^3+2)^5}{x^2(5e^{2x}+1)}$$

74. Find the derivative of .

a. $f'(x)=\frac{e^{3x}(x^3+2)^5}{x^2(5e^{2x}+1)}\left(3x+\frac{5}{x^3+2}-\frac{2}{x}-\frac{10e^{2x}}{5e^{2x}+1}\right)$

b. $f'(x)=\frac{e^{3x}(x^3+2)^5}{x^2(5e^{2x}+1)}\left(3e^{3x}+\frac{5x^2}{x^3+2}-\frac{2}{x}-\frac{10e^{2x}}{5e^{2x}+1}\right)$

c. $f'(x)=\frac{e^{3x}(x^3+2)^5}{x^2(5e^{2x}+1)}\left(3+\frac{5x^2}{x^3+2}-\frac{2}{x}-\frac{10e^{2x}}{5e^{2x}+1}\right)$

d. $f'(x)=\frac{e^{3x}(x^3+2)^5}{x^2(5e^{2x}+1)}\left(3+\frac{15x^2}{x^3+2}-\frac{2}{x}-\frac{10e^{2x}}{5e^{2x}+1}\right)$

ANSWER: d

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$$f(x)=\left(\frac{x^2}{x-1}\right)^x$$

75. Find the derivative of .

a. $f'(x)=\left(\frac{x^2}{x-1}\right)^x(2\ln x+1-\ln(x-1))$

b. $f'(x)=\left(\frac{x^2}{x-1}\right)^x\left[2\ln x+2-\ln(x-1)-\frac{x}{x-1}\right]$

c. $f'(x)=\left(\frac{x^2}{x-1}\right)^x\left[2\ln x-\ln(x-1)-\frac{x}{x-1}\right]$

d. $f'(x)=\left(\frac{x^2}{x-1}\right)^x\left[2-\ln(x-1)-\frac{x}{x-1}\right]$

ANSWER: b

76. Find the derivative of $f(x)=\sin^2 x+3\cos^2 x.$

a. $2\sin x+6\cos x$

b. $2\sin 2x$

c. $4\sin 2x$

d. $-2\sin 2x$

ANSWER: d

77. Find the derivative of $f(x)=2^x\tan x+2x$.

a. $x2^{x-1}\tan x+2^x\sec^2 x+2$

b. $x2^{x-1}\tan x+2^x\sec x\tan x+2$

c. $2^x\ln 2\tan x+2^x\sec^2 x+2$

d. $2^x\ln 2\tan x+2^x\sec x\tan x+2$

ANSWER: c

$$f(x)=\frac{x^2\ln(3x^2)}{\sin 3x}$$

78. Find the derivative of .

ANSWER: $\dfrac{x\ln(3x^2)(2\sin 3x-3x\cos 3x)+2x\sin 3x}{\sin^2 3x}$

$$f(x)=3x^2\tan^{-1}x^3.$$

79. Find the derivative of

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a. $6x \tan^{-1} x^3 + \frac{3x^2}{1+x^9}$

b. $6x \tan^{-1} x^3 + \frac{3x^2}{1+x^6}$

c. $6x \tan^{-1} x^3 + \frac{9x^2}{1+x^6}$

d. $6x \tan^{-1} x^3 + \frac{9x^4}{1+x^6}$

ANSWER: d

80. Find the derivative of $f(x) = 2x\sqrt{\sin 2x \cos 2x}$.

ANSWER: $\frac{2 \sin 4x + 4x \cos 4x}{\sqrt{2 \sin 4x}}$

81. Find the derivative of $f(x) = \cos^2 (\tan^{-1} x)$.

a. $\frac{\cos 2(\tan^{-1} x)}{1+x^2}$

b. $\frac{2 \cos (\tan^{-1} x)}{1+x^2}$

c. $\frac{-\sin 2(\tan^{-1} x)}{1+x^2}$

d. $\frac{2 \sin (\tan^{-1} x)}{1+x^2}$

ANSWER: c

82. Find the derivative of $f(x) = 2(\sin^{-1} x)^3$.

a. $\frac{6(\sin^{-1} x)^2}{1+x^2}$

b. $\frac{6(\sin^{-1} x)^2}{\sqrt{1-x^2}}$

c. $\frac{6(\sin^{-1} x)}{\sqrt{1-x^2}}$

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d. $\dfrac{6(\sin^{-1} x)}{1+x^2}$

ANSWER: b

83. Find the derivative of $f(x)=4\sec 5x+e^{5x}$.

a. $20\sec^2 5x+e^{5x}$

b. $4\sec 5x\tan 5x+e^{5x}$

c. $20\sec^2 5x+5e^{5x}$

d. $20\sec 5x\tan 5x+5e^{5x}$

ANSWER: d

84. Find the derivative of $f(x)=\tan^3(x^2+1)$.

a. $3\tan^2(x^2+1)\sec^2(x^2+1)$

b. $3x\tan^2(x^2+1)\sec^2(x^2+1)$

c. $6x\tan^2(x^2+1)\sec^2(x^2+1)$

d. $6x\tan^2(x^2+1)$

ANSWER: c

85. Find the derivative of $f(x)=\cos^{-1}(\ln x)$.

a. $\dfrac{1}{x\sqrt{1-\ln^2 x}}$

b. $\dfrac{1}{\sqrt{1-\ln^2 x}}$

c. $\dfrac{-1}{x\sqrt{1-\ln^2 x}}$

d. $\dfrac{-1}{\sqrt{1-\ln^2 x}}$

ANSWER: c

86. Find the derivative of $f(x)=\ln(x\sin 3x)$.

a. $\dfrac{1}{3x}+3\tan 3x$

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b. $\frac{1}{3x} + 3 \cot 3x$

c. $\frac{1}{x} + 3 \tan 3x$

d. $\frac{1}{x} + 3 \cot 3x$

ANSWER: d

87. Find the derivative of $f(x) = \csc^3 (e^{5x})$.

a. $e^{5x} \csc^2 (e^{5x}) \cot (e^{5x})$

b. $e^{5x} \csc^3 (e^{5x}) \cot (e^{5x})$

c. $15e^{5x} \csc^3 (e^{5x}) \cot (e^{5x})$

d. $-15e^{5x} \csc^3 (e^{5x}) \cot (e^{5x})$

ANSWER: d

88. Find the derivative of $f(x) = 2x \sinh^2 x.$

a. $2 \sinh^2 x + 2x \sinh x \cosh x$

b. $2 \sinh^2 x + 4x \sinh x \cosh x$

c. $2 \sinh^2 x + 4x \sinh x$

d. $4x \sinh x \cosh x$

ANSWER: b

89. Find the derivative of $f(x) = \frac{\sin^{-1} x}{\cos^{-1} x}$.

ANSWER: $\frac{\cos^{-1} x + \sin^{-1} x}{(\cos^{-1} x)^2 \sqrt{1 - x^2}}$

90. Find the derivative of $f(x) = (\sin 3x)^{2x}$.

a. $f'(x) = (\sin 3x)^{2x} (2 \ln (\sin 3x) + 3x \cot 3x)$

b. $f'(x) = (\sin 3x)^{2x} (2 \ln (\sin 3x) + 3x \tan 3x)$

c. $f'(x) = (\sin 3x)^{2x} (2 \ln (\sin 3x) + 6x \cot 3x)$

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d. $f'(x) = (\sin 3x)^{2x}\,(2 \ln (\sin 3x) + 6x)$

ANSWER: c

91. Find the derivative of $f(x) = \cosh (\ln (x^3 + 1))$.

a. $\dfrac{\sinh (\ln (x^3 + 1))}{x^3 + 1}$

b. $\dfrac{3x^2 \sinh (\ln (x^3 + 1))}{x^3 + 1}$

c. $\dfrac{-\sinh (\ln (x^3 + 1))}{x^3 + 1}$

d. $\sinh (\ln (x^3 + 1))$

ANSWER: b

92. Find the derivative of $f(x) = \dfrac{\tanh \sqrt{x}}{\sinh 3x}$.

ANSWER: $\dfrac{\sec h^2 \sqrt{x} \sinh 3x - 6\sqrt{x} \cosh 3x \tanh \sqrt{x}}{2\sqrt{x} \sinh^2 3x}$

93. Find the derivative of $f(x) = \ln (3e^{4x}) + \sin^2 x^3$.

ANSWER: $4 + 3x^2 \sin 2x^3$

94. Use implicit differentiation to find $\dfrac{dy}{dx}$ if $\sin (xy) = x^2 + y^3$.

ANSWER: $\dfrac{2x - y \cos (xy)}{x \cos (xy) - 3y^2}$