Chapter 2

1. Find the equation of the tangent line to $y = x^2 - 6x$ at $x = 3$.

   A) $y = -9$ B) $y = 3$ C) $y = -9x$ D) $y = 3x$
   Ans: A Difficulty: Moderate Section: 2.1

2. Find an equation of the tangent line to $y = f(x)$ at $x = 3$.
   $f(x) = x^3 + x^2 + x$

   A) $y = -12x - 36$ B) $y = 34x + 63$ C) $y = 12x - 36$ D) $y = 34x - 63$
   Ans: D Difficulty: Moderate Section: 2.1

3. Find an equation of the tangent line to $y = f(x)$ at $x = 2$.
   $f(x) = 2x^3 + 5$

   A) $y = 9x - 16$ B) $y = -24x - 27$ C) $y = 24x - 27$ D) $y = 24x + 27$
   Ans: C Difficulty: Moderate Section: 2.1

4. Find the equation of the tangent line to $y = \dfrac{2}{x+2}$ at $x = 3$.

   A) $y = \dfrac{2}{25}x + \dfrac{16}{25}$
   B) $y = -\dfrac{2}{25}x - \dfrac{16}{25}$
   C) $y = -\dfrac{2}{25}x + \dfrac{16}{25}$
   D) $y = \dfrac{2}{25}x - \dfrac{16}{25}$
   Ans: C Difficulty: Moderate Section: 2.1

5. Find the equation of the tangent line to $y = 6\sqrt{x-4}$ at $x = 5$.

   A) $y = 6x - 9$ B) $y = 3x - 9$ C) $y = 6x - 18$ D) $y = 3x - 18$
   Ans: B Difficulty: Moderate Section: 2.1

6. Compute the slope of the secant line between the points $x = -3.1$ and $x = -3$. Round your answer to the thousandths place.
   $f(x) = \sin(2x)$

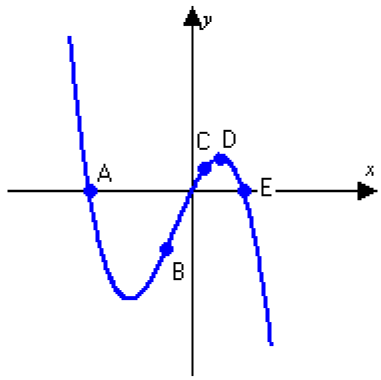
   A) –0.995 B) 1.963 C) 5.963 D) –1.991
   Ans: B Difficulty: Easy Section: 2.1

7. Compute the slope of the secant line between the points $x = 1$ and $x = 1.1$. Round your answer to the thousandths place.

$$f(x) = e^{0.5x}$$

A) 0.845 B) 5.529 C) 0.780 D) 1.691
Ans: A Difficulty: Easy Section: 2.1

8. List the points A, B, C, D, and E in order of increasing slope of the tangent line.



A) B, C, E, D, A B) A, E, D, C, B C) E, A, D, B, C D) A, B, C, D, E
Ans: B Difficulty: Easy Section: 2.1

9. Use the position function $s(t) = -4.9t^2 + 1$ meters to find the velocity at time $t = 3$ seconds.

A) –43.1 m/sec B) –29.4 m/sec C) –28.4 m/sec D) –44.1 m/sec
Ans: B Difficulty: Moderate Section: 2.1

10. Use the position function $s(t) = \sqrt{t+5}$ meters to find the velocity at time $t = -1$ seconds.

A) 2 m/sec B) 4 m/sec C) $\frac{1}{2}$ m/sec D) $\frac{1}{4}$ m/sec
Ans: D Difficulty: Moderate Section: 2.1

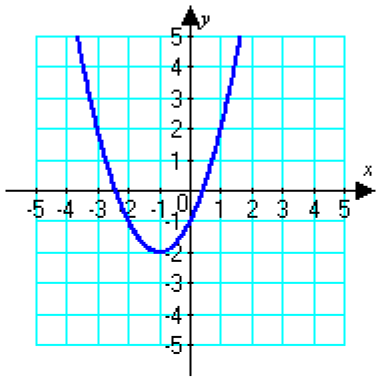
11. Find the average velocity for an object between $t = 3$ sec and $t = 3.1$ sec if $f(t) = -16t^2 + 100t + 10$ represents its position in feet.

A) 2.4 ft/s B) 4 ft/s C) 0.8 ft/s D) 166 ft/s
Ans: A Difficulty: Moderate Section: 2.1

12. Find the average velocity for an object between $t = 1$ sec and $t = 1.1$ sec if $f(t) = 5\sin(t) + 5$ represents its position in feet. (Round to the nearest thousandth.)

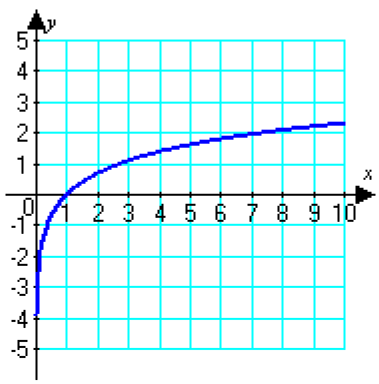
A) 2.702 B) 2.268 C) 2.487 D) –2.487
Ans: C Difficulty: Moderate Section: 2.1

13. Estimate the slope of the tangent line to the curve at $x = -2$.



A) –1 B) –2 C) 2 D) 0
Ans: B Difficulty: Easy Section: 2.1

14. Estimate the slope of the tangent line to the curve at $x = 3$.



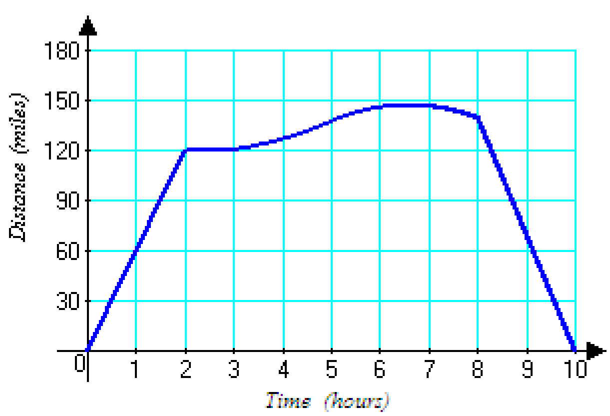
A) 3 B) –3 C) $\frac{1}{6}$ D) $\frac{1}{3}$
Ans: D Difficulty: Easy Section: 2.1

15. The table shows the temperature in degrees Celsius at various distances, d in feet, from a specified point. Estimate the slope of the tangent line at $d = 2$ and interpret the result.

d	0	1	3	5	7
$°C$	13	20	14	7	1

A) $m \approx 4.67$; The temperature is increasing 4.67 $°C$ per foot at the point 2 feet from the specified point.
B) $m \approx -0.33$; The temperature is decreasing 0.33 $°C$ per foot at the point 2 feet from the specified point.
C) $m \approx -3$; The temperature is decreasing 3 $°C$ per foot at the point 2 feet from the specified point.
D) $m \approx 20$; The temperature is increasing 20 $°C$ per foot at the point 2 feet from the specified point.
Ans: C Difficulty: Moderate Section: 2.1

16. The graph below gives distance in miles from a starting point as a function of time in hours for a car on a trip. Find the fastest speed (magnitude of velocity) during the trip. Describe how the speed during the first 2 hours compares to the speed during the last 2 hours. Describe what is happening between 2 and 3 hours.



Ans: The fastest speed occurred during the last 2 hours of the trip when the car traveled at about 70 mph. The speed during the first 2 hours is 60 mph while the speed from 8 to 10 hours is about 70 mph. Between 2 and 3 hours the car was stopped.
Difficulty: Moderate Section: 2.1

17. Compute $f'(3)$ for the function $f(x) = 5x^3 - 5x$.

A) 150 B) 130 C) 120 D) –130
Ans: B Difficulty: Moderate Section: 2.2

18. Compute $f'(4)$ for the function $f(x)=\dfrac{2}{x^2+4}$.

A) $\dfrac{1}{4}$ B) $\dfrac{1}{25}$ C) $-\dfrac{2}{25}$ D) $-\dfrac{1}{25}$

Ans: D Difficulty: Moderate Section: 2.2

19. Compute the derivative function $f'(x)$ of $f(x)=\dfrac{7}{3x-1}$.

A) $f'(x)=\dfrac{-21}{(3x-1)^2}$

B) $f'(x)=\dfrac{-3}{(3x-1)^2}$

C) $f'(x)=\dfrac{-7}{(3x-1)^2}$

D) $f'(x)=\dfrac{21}{(3x-1)^2}$

Ans: A Difficulty: Moderate Section: 2.2

20. Compute the derivative function $f'(x)$ of $f(x)=\sqrt{4x^2+9}$.

A) $f'(x)=\dfrac{-8x}{\sqrt{4x^2+9}}$

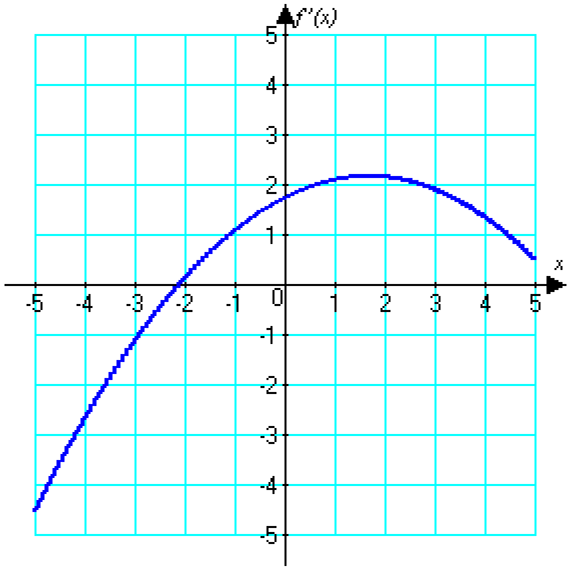
B) $f'(x)=\dfrac{4x}{\sqrt{4x^2+9}}$

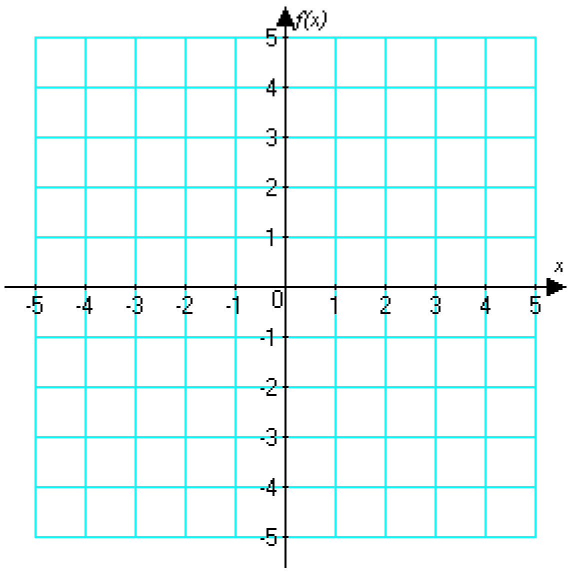
C) $f'(x)=\dfrac{-4x}{\sqrt{4x^2+9}}$

D) $f'(x)=\dfrac{-4x}{\sqrt{8x+9}}$

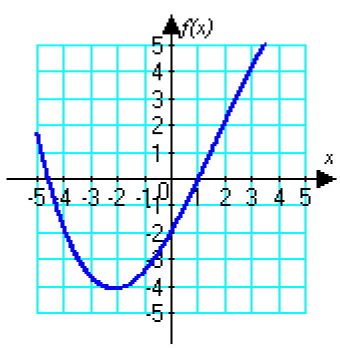
Ans: B Difficulty: Moderate Section: 2.2

21. Below is a graph of $f'(x)$. Sketch a plausible graph of a continuous function $f(x)$.



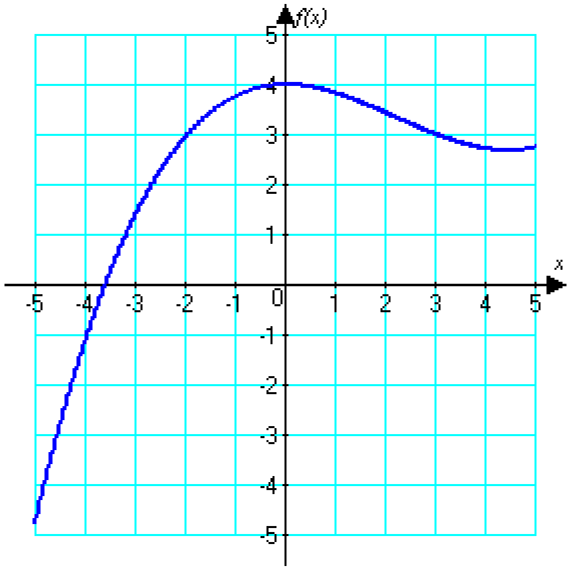


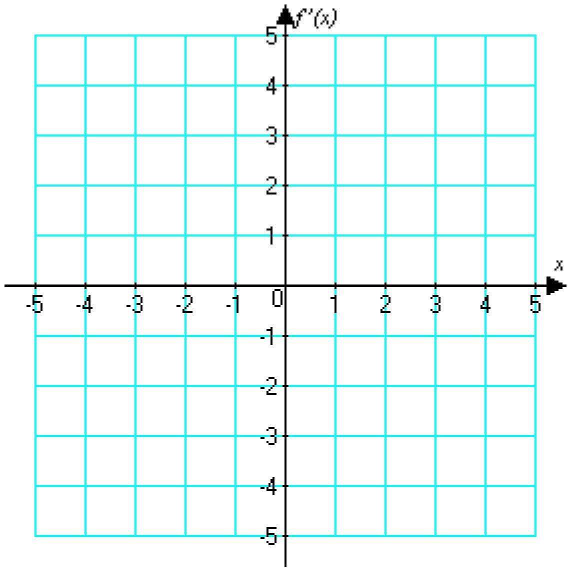
Ans: Answers may vary. Below is one possible answer.



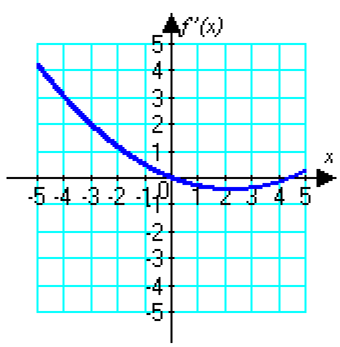
Difficulty: Moderate Section: 2.2

22. Below is a graph of $f(x)$. Sketch a graph of $f'(x)$.





Ans:



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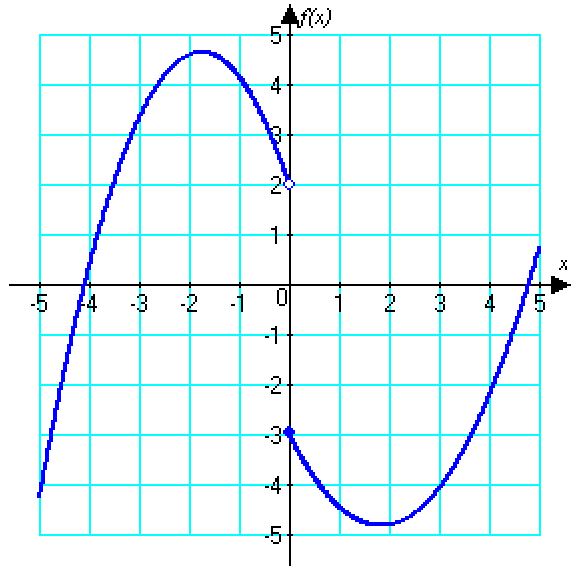
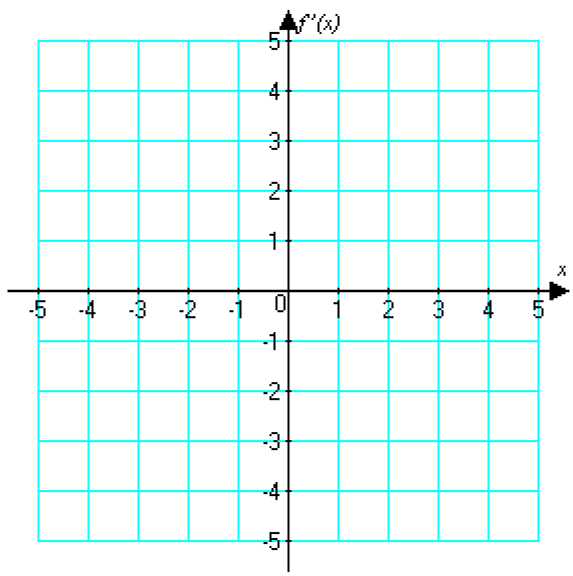
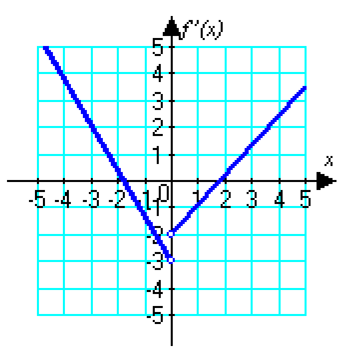
Difficulty: Moderate Section: 2.2

23. Below is a graph of $f(x)$. Sketch a graph of $f'(x)$.



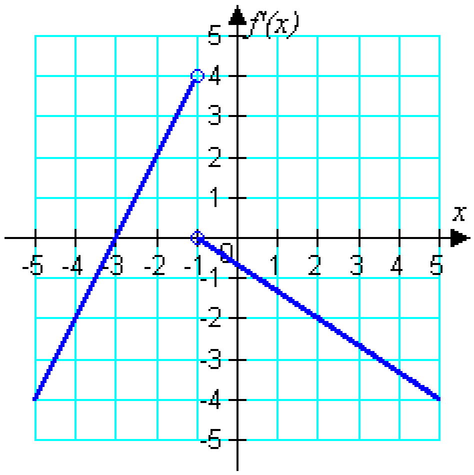


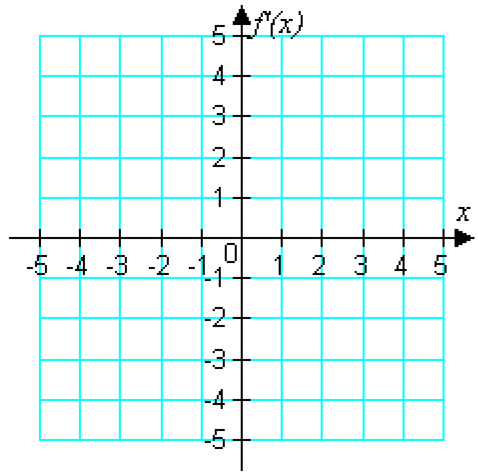
Ans:



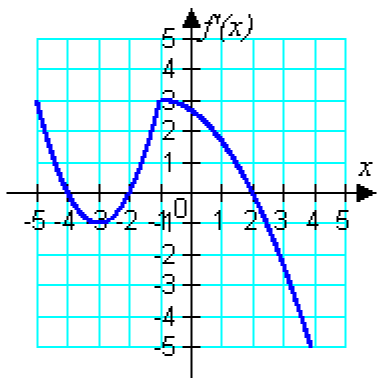
Difficulty: Difficult Section: 2.2

24. Below is a graph of $f'(x)$. Sketch a plausible graph of a continuous function $f(x)$.





Ans: Answers may vary. Below is one possible answer.



Difficulty: Difficult Section: 2.2

25. Compute the right-hand derivative $D_+f(0) = \lim_{h\to 0^+} \dfrac{f(h)-f(0)}{h}$ and the left-hand derivative $D_-f(0) = \lim_{h\to 0^-} \dfrac{f(h)-f(0)}{h}$.

$$f(x) = \begin{cases} 4x+8 & \text{if } x<0 \\ -8x+8 & \text{if } x \ge 0 \end{cases}$$

A) $D_+f(0) = -8$, $D_-f(0) = 4$

B) $D_+f(0) = 4$, $D_-f(0) = -8$

C) $D_+f(0) = 8$, $D_-f(0) = 8$

D) $D_+f(0) = -2$, $D_-f(0) = -2$

Ans: A Difficulty: Moderate Section: 2.2

26. Numerically estimate the derivative $f'(0)$ for $f(x)=5xe^{3x}$.

A) 0 B) 1 C) 3 D) 5
Ans: D Difficulty: Moderate Section: 2.2

27. The table below gives the position $s(t)$ for a car beginning at a point and returning 5 hours later. Estimate the velocity $v(t)$ at two points around the third hour.

t (hours)	0	1	2	3	4	5
$s(t)$ (miles)	0	15	50	80	70	0

Ans: The velocity is the change in distance traveled divided by the elapsed time. From hour 3 to 4 the average velocity is (70 − 80)/(4 − 3) = −10 mph. Likewise, the velocity between hour 2 and hour 3 is about 30 mph.
Difficulty: Easy Section: 2.2

28. Use the distances $f(t)$ to estimate the velocity at $t = 2.2$. (Round to 2 decimal places.)

t	1.6	1.8	2	2.2	2.4	2.6	2
$f(t)$8	49	54	59.5	64	68.5	73.5	79

A) –2250.00 B) 29.09 C) 22.50 D) 25.00
Ans: C Difficulty: Easy Section: 2.2

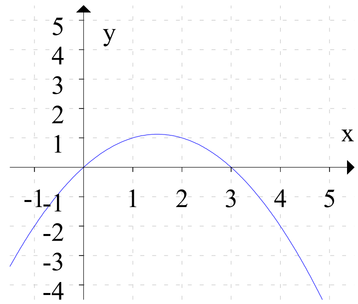
29. For $f(x)=\begin{cases}5x^2-6x & \text{if } x<0\\ ax+b & \text{if } x\geq 0\end{cases}$ find all real numbers a and b such that $f'(0)$ exists.

A) $a=10$, b any real number
B) $a=4$, $b=0$
C) $a=-6$, b any real number
D) $a=-6$, $b=0$

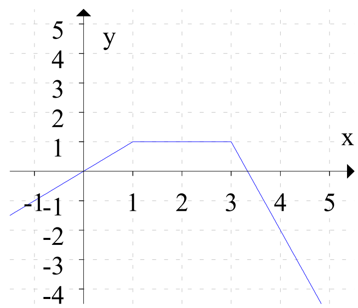
Ans: D Difficulty: Moderate Section: 2.2

30. Sketch the graph of a function with the following properties: $f(0)=0$, $f(2)=1$, $f(4)=-2$, $f'(0)=1$, $f'(2)=0$, and $f'(4)=-3$.

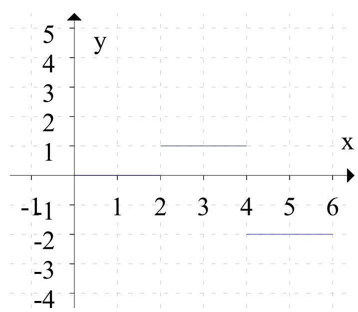
A)



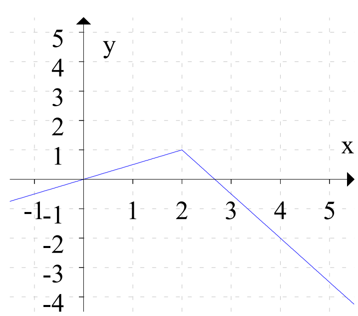
B)



C)



D)



Ans: B Difficulty: Moderate Section: 2.2

31. Suppose a sprinter reaches the following distances in the given times. Estimate the velocity of the sprinter at the 6 second mark. Round to the nearest integer.

t sec	5	5.5	6	6.5	7
$f(t)$ ft	120.7	142.1	158.3	174.5	193.5

A) 32 ft/sec B) 36 ft/sec C) 26 ft/sec D) 28 ft/sec
Ans: A Difficulty: Moderate Section: 2.2

32. $\lim_{h\to 0}\frac{(1+h)^3+(1+h)-2}{h}$ equals $f'(a)$ for some function $f(x)$ and some constant a. Determine which of the following could be the function $f(x)$ and the constant a.

A) $f(x)=x^3-x$ and $a=-1$
B) $f(x)=x^3+x^2$ and $a=0$
C) $f(x)=x^3+x-20$ and $a=0$
D) $f(x)=x^3+x$ and $a=1$

Ans: D Difficulty: Moderate Section: 2.2

33. $\lim_{h\to 0}\frac{\frac{1}{(h+3)^2}-\frac{1}{9}}{h}$ equals $f'(a)$ for some function $f(x)$ and some constant a. Determine which of the following could be the function $f(x)$ and the constant a.

A) $f(x)=\frac{1}{x^2}$ and $a=3$
B) $f(x)=\frac{3}{x^2}$ and $a=3$
C) $f(x)=-\frac{1}{x^2}$ and $a=4$
D) $f(x)=-\frac{1}{x^2}$ and $a=-3$

Ans: A Difficulty: Moderate Section: 2.2

34. Find the derivative of $f(x)=x^2+3x+2$.

A) $x+3$ B) $2x^2+2$ C) $2x+3$ D) $-2x-3$
Ans: C Difficulty: Easy Section: 2.3

35. Differentiate the function.

$$f(t)=5t^3-2\sqrt{t}$$

A) $f'(t)=15t^2-4\sqrt{t}$
B) $f'(t)=15t^2-4$
C) $f'(t)=\frac{15t^{5/2}-1}{\sqrt{t}}$
D) $f'(t)=\frac{15t^2-1}{\sqrt{t}}$

Ans: C Difficulty: Moderate Section: 2.3

36. Find the derivative of $f(x)=\frac{4}{x}+4x-3$.

A) $f'(x)=\frac{4}{x^2}+4$
B) $f'(x)=-\frac{4}{x^2}+4$
C) $f'(x)=-\frac{4}{x}+4$
D) $f'(x)=-\frac{4}{x^2}+8x^2$

Ans: B Difficulty: Easy Section: 2.3

37. Differentiate the function.

$$f(s)=5s^{3/2}-7s^{-1/3}$$

A) $f'(s)=\frac{45s^{5/3}+2}{6s^{2/3}}$
B) $f'(s)=\frac{45s^{1/2}+2s^{1/3}}{6}$
C) $f'(s)=\frac{45s^{1/2}+2s^{2/3}}{6}$
D) $f'(s)=\frac{45s^{11/6}+14}{6s^{4/3}}$

Ans: D Difficulty: Moderate Section: 2.3

38. Find the derivative of $f(x)=\frac{x^2+5x-2}{4x}$.

A) $f'(x)=\frac{2x+5}{4}$
B) $f'(x)=-\frac{x}{2}-\frac{5}{4}$
C) $f'(x)=\frac{1}{4}+\frac{1}{2x^2}$
D) $f'(x)=\frac{x^2}{4}+\frac{5x}{4}-\frac{1}{2x}$

Ans: C Difficulty: Moderate Section: 2.3

39. Find the derivative of $f(x)=\frac{-5x^2-7x-7}{\sqrt{x}}$.

A) $f'(x)=-\frac{15\sqrt{x}}{2}-\frac{7}{2\sqrt{x}}+\frac{7}{2\sqrt{x^3}}$
B) $f'(x)=-\frac{20x+14}{x}$
C) $f'(x)=-\frac{15\sqrt{x}}{2}+\frac{7}{2\sqrt{x}}-\frac{7}{2\sqrt{x^3}}$
D) $f'(x)=-15\sqrt{x}-\frac{7}{\sqrt{x}}-\frac{7}{\sqrt{x^3}}$

Ans: A Difficulty: Moderate Section: 2.3

40. Differentiate the function.

$$f(x) = x\left(3x^2 - 6\sqrt{x}\right)$$

A) $f'(x) = 9x^2 - 9\sqrt{x}$

B) $f'(x) = \dfrac{6x^{3/2} - 3}{\sqrt{x}}$

C) $f'(x) = 6x^2 - 3\sqrt{x}$

D) $f'(x) = 6x - 3\sqrt{x}$

Ans: A Difficulty: Moderate Section: 2.3

41. Find the third derivative of $f(x) = 2x^5 + 8x + \dfrac{3}{x}$.

A) $f'''(x) = 120x^2 + \dfrac{18}{x^4}$

B) $f'''(x) = 120x^2 + 8 - \dfrac{18}{x^4}$

C) $f'''(x) = 40x^3 + \dfrac{6}{x^3}$

D) $f'''(x) = 120x^2 - \dfrac{18}{x^4}$

Ans: D Difficulty: Moderate Section: 2.3

42. Find the second derivative of $y = -4x - \dfrac{6}{\sqrt{x}}$.

A) $\dfrac{d^2y}{dx^2} = -4 - \dfrac{9}{2\sqrt{x^5}}$

B) $\dfrac{d^2y}{dx^2} = -\dfrac{9}{2\sqrt{x^5}}$

C) $\dfrac{d^2y}{dx^2} = \dfrac{9}{2\sqrt{x^5}}$

D) $\dfrac{d^2y}{dx^2} = -\dfrac{9}{2\sqrt{x^3}}$

Ans: B Difficulty: Moderate Section: 2.3

43. Using the position function $s(t) = 3t^4 - 4t^3 + \dfrac{2}{t}$, find the velocity function.

A) $v(t) = 12t^3 - 12t^2 - \dfrac{2}{t^2}$

B) $v(t) = 9t^3 - 8t^2 - \dfrac{2}{t^2}$

C) $v(t) = 12t^3 - 12t^2 + \dfrac{2}{t^2}$

D) $v(t) = -12t^3 + 12t^2 - \dfrac{2}{t^2}$

Ans: A Difficulty: Moderate Section: 2.3

44. Using the position function $s(t) = -7t^3 - 6t - 8$, find the acceleration function.

A) $a(t) = -21t$ B) $a(t) = -14t$ C) $a(t) = -42t$ D) $a(t) = -42t - 6$

Ans: C Difficulty: Moderate Section: 2.3

45. Using the position function $s(t) = -\sqrt{t} + \frac{3}{t}$, find the velocity function.

A) $v(t) = \frac{1}{2\sqrt{t}} + \frac{3}{t^2}$
B) $v(t) = -\frac{1}{2\sqrt{t}} - \frac{3}{t^2}$
C) $v(t) = \frac{1}{2\sqrt{t}} - \frac{3}{t^2}$
D) $v(t) = -\frac{1}{2\sqrt{t}} - \frac{6}{t^2}$

Ans: B Difficulty: Moderate Section: 2.3

46. Using the position function $s(t) = -\frac{8}{\sqrt{t}} + 1$, find the acceleration function.

A) $a(t) = \frac{6}{\sqrt{t^5}}$ B) $a(t) = -\frac{2}{\sqrt{t^5}}$ C) $a(t) = \frac{4}{\sqrt{t^3}}$ D) $a(t) = -\frac{6}{\sqrt{t^5}}$
Ans: D Difficulty: Moderate Section: 2.3

47. The height of an object at time t is given by $h(t) = -16t^2 + 4t - 1$. Determine the object's velocity at $t = 2$.

A) 60 B) –59 C) –60 D) –28
Ans: C Difficulty: Easy Section: 2.3

48. The height of an object at time t is given by $h(t) = 8t^2 - 4t$. Determine the object's acceleration at $t = 3$.

A) 60 B) 16 C) 44 D) –16
Ans: B Difficulty: Easy Section: 2.3

49. Find an equation of the line tangent to $f(x) = x^2 + 5x - 8$ at $x = 2$.

A) $g(x) = 9x - 12$
B) $g(x) = 4x - 12$
C) $g(x) = 9x - 10$
D) $g(x) = 4x - 10$

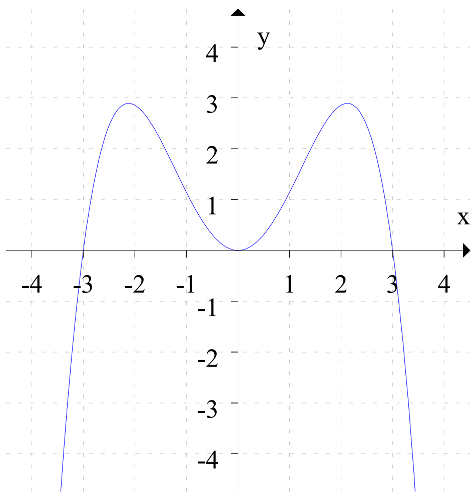
Ans: A Difficulty: Easy Section: 2.3

50. Find an equation of the line tangent to $f(x) = 7\sqrt{x} - 2x - 4$ at $x = 3$.

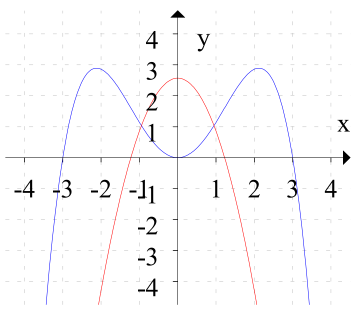
A) $g(x) = \left(\frac{-7\sqrt{3} + 12}{6}\right)x - \frac{7}{2}\sqrt{3} + 4$
B) $g(x) = \left(\frac{7\sqrt{3} - 4}{3}\right)x + \frac{7}{2}\sqrt{3} + 4$
C) $g(x) = \left(\frac{7\sqrt{3} - 6}{6}\right)x + \frac{7}{2}\sqrt{3}$
D) $g(x) = \left(\frac{7\sqrt{3} - 12}{6}\right)x + \frac{7}{2}\sqrt{3} - 4$

Ans: D Difficulty: Moderate Section: 2.3

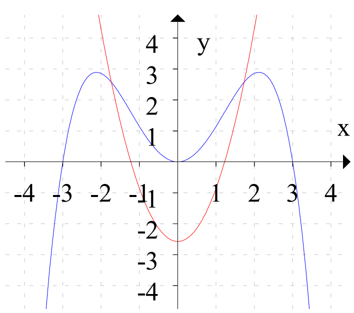
51. Use the graph of $f(x)$ below to sketch the graph of $f''(x)$ on the same axes. (Hint: sketch $f'(x)$ first.)



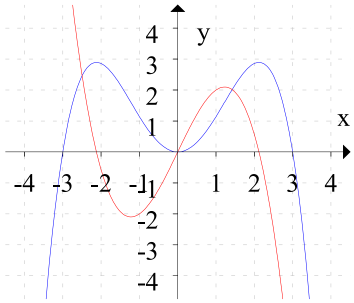
A)



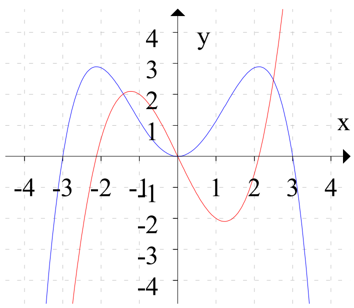
B)



C)



D)



Ans: A Difficulty: Difficult Section: 2.3

52. Determine the real value(s) of x for which the line tangent to $f(x) = 7x^2 + 9x - 4$ is horizontal.

A) $x = -\frac{9}{14}, x = 0$ B) $x = \frac{-9 \pm \sqrt{193}}{14}$ C) $x = -\frac{9}{14}$ D) $x = 0$

Ans: C Difficulty: Easy Section: 2.3

53. Determine the real value(s) of x for which the line tangent to $f(x) = 2x^4 - 4x^2 - 1$ is horizontal.

A) $x = -1, x = 1$ B) $x = 0, x = -1, x = 1$ C) $x = 0$ D) $x = 0, x = 1$

Ans: B Difficulty: Easy Section: 2.3

54. Determine the value(s) of x, if there are any, for which the slope of the tangent line to $f(x) = |x^2 + 3x - 54|$ does not exist.

A) $x = -1.5$

B) $x = -6,\ x = 9$

C) $x = -9,\ x = 6$

D) The slope exists for all values of x.

Ans: C Difficulty: Moderate Section: 2.3

55. Find the second-degree polynomial (of the form $ax^2 + bx + c$) such that $f(0) = 0$, $f'(0) = 5$, and $f''(0) = 1$.

A) $\frac{x^2}{2} + 5x$ B) $-\frac{x^2}{2} + 5x$ C) $\frac{x^2}{2} - 5x + 1$ D) $-\frac{x^2}{2} + 5x + 1$

Ans: A Difficulty: Moderate Section: 2.3

56. Find a formula for the nth derivative $f^{(n)}(x)$ of $f(x) = \frac{4}{x+8}$.

A) $f^{(n)}(x) = (-1)^{n+1}\frac{32n!}{(x+8)^{n+1}}$

C) $f^{(n)}(x) = (-1)^{n}\frac{32n!}{(x+8)^{n}}$

B) $f^{(n)}(x) = (-1)^{n+1}\frac{4n!}{(x+8)^{n}}$

D) $f^{(n)}(x) = (-1)^{n}\frac{4n!}{(x+8)^{n+1}}$

Ans: D Difficulty: Difficult Section: 2.3

57. Find a function with the given derivative.

$$f'(x) = 20x^4$$

A) $f(x) = 20x^5$ B) $f(x) = 4x^5$ C) $f(x) = 20x^3$ D) $f(x) = 80x^3$

Ans: B Difficulty: Moderate Section: 2.3

58. Let $f(t)$ equal the average monthly salary of families in a certain city in year t. Several values are given in the table below. Estimate and interpret $f''(2010)$.

t	1995	2000	2005	2010
$f(t)$	\$1700	\$2000	\$2100	\$2250

A) $f''(2010) \approx 2$; The rate at which the average monthly salary is increasing each year in 2010 is increasing by \$2 per year.

B) $f''(2010) \approx 2$; The average monthly salary is increasing by \$2 per year in 2010.

C) $f''(2010) \approx 30$; The rate at which the average monthly salary is increasing each year in 2010 is increasing by \$30 per year.

D) $f''(2010) \approx 30$; The average monthly salary is increasing by \$30 per year in 2010.

Ans: A Difficulty: Moderate Section: 2.3

59. Find the derivative of $f(x) = \left(9\sqrt{x} + 5x\right)\left(-3x^2 - \frac{1}{x}\right)$.

A) $f'(x) = -45x^2 + \frac{135}{2}x^{3/2} + \frac{9}{2x^{3/2}}$

B) $f'(x) = -45x^2 - \frac{135}{2}x^{3/2} + \frac{9}{2x^{3/2}}$

C) $f'(x) = 45x^2 - \frac{135}{2}x^{3/2} - \frac{9}{2x^{3/2}}$

D) $f'(x) = -45x^2 - \frac{135}{2}x^{3/2} - \frac{10}{x} + \frac{9}{2x^{3/2}}$

Ans: B Difficulty: Moderate Section: 2.4

60. Find the derivative of $f(x) = \frac{2x+2}{-3x+2}$.

A) $\frac{-10}{(-3x+2)^2}$ B) $-\frac{2}{3}$ C) $\frac{2}{3}$ D) $\frac{10}{(-3x+2)^2}$

Ans: D Difficulty: Moderate Section: 2.4

61. Find the derivative of $f(x) = \frac{4x}{-8x^2-3}$.

A) $\frac{32x^2-12}{(-8x^2-3)^2}$ B) $\frac{1}{2x^2}$ C) $\frac{-32x^2+12}{(-8x^2-3)^2}$ D) $-\frac{1}{2x^2}$

Ans: A Difficulty: Moderate Section: 2.4

62. Find the derivative of $f(x) = \left(-5\sqrt[3]{x}+6\right)x$.

A) $f'(x) = \frac{20}{3}\sqrt[3]{x} + 6$

C) $f'(x) = -\frac{20}{3}\sqrt[3]{x} + 6$

B) $f'(x) = -\frac{5}{3}\sqrt[3]{x} - 6$

D) $f'(x) = -\frac{10}{3}\sqrt[3]{x} + 12$

Ans: C Difficulty: Moderate Section: 2.4

63. Find an equation of the line tangent to $h(x) = f(x)g(x)$ at $x = -3$ if $f(-3) = 2$, $f'(-3) = 1$, $g(-3) = 3$, and $g'(-3) = 3$.

A) $y = 3x - 3$ B) $y = 3x + 33$ C) $y = 9x + 33$ D) $y = 9x - 21$

Ans: C Difficulty: Moderate Section: 2.4

64. Find an equation of the line tangent to $h(x)=\dfrac{f(x)}{g(x)}$ at $x=3$ if $f(3)=1$, $f'(3)=-1$, $g(3)=1$, and $g'(3)=-2$.

A) $y=-3x-2$ B) $y=x-2$ C) $y=-3x+10$ D) $y=x+4$
Ans: B Difficulty: Moderate Section: 2.4

65. A small company sold 1500 widgets this year at a price of $12 each. If the price increases at rate of $1.75 per year and the quantity sold increases at a rate of 200 widgets per year, at what rate will revenue increase?

A) $350/year B) $5025/year C) $225/year D) $5375/year
Ans: B Difficulty: Moderate Section: 2.4

66. The Dieterici equation of state, $Pe^{an/VRT}(V-nb)=nRT$, gives the relationship between pressure P, volume V, and temperature T for a liquid or gas. At the critical point, $P'(V)=0$ and $P''(V)=0$ with T constant. Using the result of the first derivative and substituting it into the second derivative, find the critical volume V_c in terms of the constants n, a, b, and R.

Ans: $P'(V)=\left(\dfrac{an^2}{V^2}-\dfrac{nRT}{(V-nb)}\right)\left(\dfrac{1}{V-nb}\right)e^{-an/VRT}=0$ gives the result that

$RT=\dfrac{an(V-nb)}{V^2}$.

$$P''(V)=\left(\frac{-2an^2}{V^3(V-nb)}+\frac{2nRT}{(V-nb)^3}-\frac{2an^2}{V^2(V-nb)^2}+\frac{a^2n^3}{V^4(V-nb)^2RT}\right)e^{-an/VRT}=0.$$

When the result of the first derivative is substituted for RT in the parentheses, the result is that $V_c = 2nb$.
Difficulty: Difficult Section: 2.4

67. Find the derivative of $f(x)=\dfrac{(x^2+2)^4}{6}$.

A) $f'(x)=\frac{2}{3}x(x^2+2)^3$
B) $f'(x)=\frac{1}{3}x(x^2+2)^3$
C) $f'(x)=\frac{4}{3}x(x^2+2)^3$
D) $f'(x)=\frac{1}{6}x(x^2+2)^3$

Ans: C Difficulty: Moderate Section: 2.5

68. Find the derivative of $f(x)=\sqrt{x^2-2}$.

A) $f'(x)=\dfrac{2x}{\sqrt{x^2-2}}$

B) $f'(x)=\dfrac{4x}{\sqrt{x^2-2}}$

C) $f'(x)=\dfrac{-x}{\sqrt{x^2-2}}$

D) $f'(x)=\dfrac{x}{\sqrt{x^2-2}}$

Ans: D Difficulty: Moderate Section: 2.5

69. Differentiate the function.

$$f(t)=t^6\sqrt{t^3-5}$$

A) $f'(t)=\dfrac{13t^6-60t^5}{2\sqrt{t^3-5}}$

B) $f'(t)=\dfrac{6t^5}{2\sqrt{t^3-5}}$

C) $f'(t)=\dfrac{15t^8-60t^5}{2\sqrt{t^3-5}}$

D) $f'(t)=\dfrac{9t^7}{\sqrt{t^3-5}}$

Ans: C Difficulty: Difficult Section: 2.5

70. Find the derivative of $f(x)=\sqrt{\dfrac{x}{x^2+9}}$.

A) $\dfrac{1}{2}\left[\dfrac{1}{\sqrt{x\left(x^2+9\right)}}-\sqrt{\dfrac{x}{\left(x^2+9\right)^3}}\right]$

B) $\dfrac{1}{2\sqrt{x\left(x^2+9\right)}}-\sqrt{\left(\dfrac{x}{x^2+9}\right)^3}$

C) $\dfrac{1}{2}\left[\dfrac{1}{\sqrt{x\left(x^2+9\right)}}-\sqrt{x\left(x^2+9\right)}\right]$

D) $\sqrt{\dfrac{1}{x^2+9}-\dfrac{2x^2}{\left(x^2+9\right)^2}}$

Ans: B Difficulty: Moderate Section: 2.5

71. Find the derivative of $f(x)=\dfrac{-3}{\sqrt{8x^2-9}}$.

A) $f'(x)=\dfrac{24x}{\sqrt{(8x^2-9)^3}}$

B) $f'(x)=\dfrac{-48x}{\sqrt{(8x^2-9)^3}}$

C) $f'(x)=\dfrac{-24x}{\sqrt{(8x^2-9)^3}}$

D) $f'(x)=\dfrac{-6x}{\sqrt{(8x^2-9)^3}}$

Ans: A Difficulty: Moderate Section: 2.5

72. Differentiate the function.

$$f(x)=\left(\sqrt{x^3-4}+3x\right)^{-2}$$

A) $$f'(x)=-\frac{6\sqrt{x^3-4}+3x^2}{\left(\sqrt{x^3-4}\right)\left(\sqrt{x^3-4}+3x\right)^3}$$

B) $$f'(x)=-\frac{12\sqrt{x^3-4}+3x^2}{\left(2\sqrt{x^3-4}\right)\left(\sqrt{x^3-4}+3x\right)^2}$$

C) $$f'(x)=-\frac{2\sqrt{x^3-4}+6}{\left(\sqrt{x^3-4}+3x\right)^3}$$

D) $$f'(x)=-\frac{2\sqrt{x^3-4}+6}{\left(\sqrt{x^3-4}+3x\right)^2}$$

Ans: A Difficulty: Difficult Section: 2.5

73. $f(x)=-5x^3-6x+6$ has an inverse $g(x)$. Compute $g'(17)$.

A) $g'(17)=\frac{1}{21}$ B) $g'(17)=-\frac{1}{9}$ C) $g'(17)=-\frac{1}{21}$ D) $g'(17)=\frac{1}{9}$

Ans: C Difficulty: Moderate Section: 2.5

74. $f(x)=2x^5+3x^3+2x$ has an inverse $g(x)$. Compute $g'(7)$.

A) $g'(7)=\frac{1}{24453}$ B) $g'(7)=\frac{1}{21}$ C) $g'(7)=-\frac{1}{7}$ D) $g'(7)=\frac{1}{7}$

Ans: B Difficulty: Moderate Section: 2.5

75. The function $f(x)=\sqrt{x^3+5x+36}$ has an inverse $g(x)$. Find $g'(6)$.

A) $g'(6)=\frac{12}{5}$ B) $g'(6)=\frac{5}{12}$ C) $g'(6)=6$ D) $g'(6)=\frac{1}{6}$

Ans: A Difficulty: Moderate Section: 2.5

76. Find an equation of the line tangent to $f(x)=\frac{1}{\sqrt{x^2-24}}$ at $x = 5$.

A) $y=-5x+24$ B) $y=-5x$ C) $y=5x+6$ D) $y=-5x+26$

Ans: D Difficulty: Moderate Section: 2.5

77. Use the position function $s(t) = \sqrt{t^2 + 48}$ meters to find the velocity at $t = 4$ seconds.

A) 8 m/s B) $\frac{1}{2}$ m/s C) $\frac{1}{8}$ m/s D) $\frac{1}{4}$ m/s
Ans: B Difficulty: Moderate Section: 2.5

78. Compute the derivative of $h(x) = f\left(g(x)\right)$ at $x = 9$ where
$f(9) = -5$, $g(9) = -8$, $f'(9) = -2$, $f'(-8) = -4$, $g'(9) = 6$, and $g'(-8) = -7$.

A) $h'(9) = -12$ B) $h'(9) = -30$ C) $h'(9) = -24$ D) $h'(9) = 40$
Ans: C Difficulty: Moderate Section: 2.5

79. Find the derivative where f is an unspecified differentiable function.

$$f(3x^7)$$

A) $21x^6 f'(3x^7)$ B) $(21x^6 + 3x^7)f'(3x^7)$ C) $f'(21x^6)$ D) $f'(21x^6 + 3x^7)$
Ans: A Difficulty: Moderate Section: 2.5

80. Find the second derivative of the function.

$$f(x) = \sqrt{9 - x^2}$$

A) $f''(x) = \frac{9x}{(9 - x^2)^{3/2}}$

B) $f''(x) = \frac{x^2 + 9}{(9 - x^2)^{3/2}}$

C) $f''(x) = -\frac{9}{(9 - x^2)^{3/2}}$

D) $f''(x) = -\frac{9x}{(9 - x^2)^{3/2}}$

Ans: C Difficulty: Moderate Section: 2.5

81. Find a function $g(x)$ such that $g'(x) = f(x)$.

$$f(x) = \left(x^2 - 9\right)^8 (2x)$$

A) $\left(\frac{x^3}{3} - 9x\right)^9 \frac{x^2}{9}$

B) $g(x) = \left(x^2 - 9\right)^7 (32x)$

C) $g(x) = \left(x^2 - 9\right)^9$

D) $g(x) = \frac{\left(x^2 - 9\right)^9}{9}$

Ans: D Difficulty: Moderate Section: 2.5

82. Use the table of values to estimate the derivative of $h(x) = f\left(g(x)\right)$ at $x = 6$.

x	–1	0	1	2	3	4	5	6	7
$f(x)$	–5	–4	–3	–4	–5	–6	–5	–3	–1
$g(x)$	6	4	2	2	4	6	4	2	1

A) $h'(6) \approx 2$ B) $h'(6) \approx -3$ C) $h'(6) \approx -2$ D) $h'(6) \approx 3$
Ans: A Difficulty: Moderate Section: 2.5

83. Find the derivative of $f(x) = -4\sin(x) + 9\cos(3x) - x$.

A) $f'(x) = -4\cos x - 27\sin 3x - 1$
B) $f'(x) = -4\cos x - 9\sin 3x - 1$
C) $f'(x) = 4\cos x + 27\sin 3x - 1$
D) $f'(x) = \cos x - 3\sin 3x - 1$

Ans: A Difficulty: Easy Section: 2.6

84. Find the derivative of $f(x) = 4\sin^2 x - 3x^2$.

A) $f'(x) = -8\sin x\cos x - 6x$
B) $f'(x) = 8\sin x\cos x - 3x$
C) $f'(x) = 8\sin x - 6x$
D) $f'(x) = 8\sin x\cos x - 6x$

Ans: D Difficulty: Easy Section: 2.6

85. Find the derivative of $f(x) = \dfrac{-6\cos x^2}{x^2}$.

A) $f'(x) = \dfrac{-12(x^2\sin x^2 + \cos x^2)}{x^3}$
B) $f'(x) = \dfrac{12(x\sin x^2 + \cos x^2)}{x^3}$
C) $f'(x) = \dfrac{12(x^2\sin x^2 + \cos x^2)}{x^3}$
D) $f'(x) = \dfrac{12(x^2\sin x^2 + \cos x^2)}{x^4}$

Ans: C Difficulty: Moderate Section: 2.6

86. Find the derivative of $f(x) = \sqrt{-\sin x\sec x}$.

A) $f'(x) = -\dfrac{\sec x}{2\sqrt{-\tan x}}$
B) $f'(x) = -\dfrac{\sec^2 x}{2\sqrt{-\tan x}}$
C) $f'(x) = -\dfrac{\sec^2 x}{\sqrt{-\tan x}}$
D) $f'(x) = -\dfrac{\sec x\tan x}{2\sqrt{-\tan x}}$

Ans: B Difficulty: Moderate Section: 2.6

87. Find the derivative of the function.
$$f(w) = w^2 \sec^2 10w$$

A) $f'(w) = 20w\sec^2(10w)\tan(10w)$

B) $f'(w) = 2w\sec^2(10w) + 20w^2\sec^2(10w)\tan(10w)$

C) $f'(w) = 2w\sec^2(10w) + 20w^2\sec(10w)$

D) $f'(w) = 2w\sec^2(10w) + 20w^2\sec^2(10w)\tan^2(10w)$

Ans: B Difficulty: Moderate Section: 2.6

88. Find the derivative of the function.
$$f(x) = \cos^3\left(\sin\left(\left(x^5 + 7x^4\right)^2\right)\right)$$

Ans: $f'(x) = -6\cos^2\left(\sin\left(\left(x^5 + 7x^4\right)^2\right)\right)\cdot\sin\left(\sin\left(\left(x^5 + 7x^4\right)^2\right)\right)\cdot\cos\left(\left(x^5 + 7x^4\right)^2\right)\cdot\left(x^5 + 7x\right.$

Difficulty: Difficult Section: 2.6

89. Find an equation of the line tangent to $f(x) = x\sin 10x$ at $x = \pi$.

A) $y = -10(x - \pi)$

B) $y = 10(x - \pi)$

C) $y = -10\pi(x - \pi)$

D) $y = 10\pi(x - \pi)$

Ans: D Difficulty: Moderate Section: 2.6

90. Find an equation of the line tangent to $f(x) = \tan 4x$ at $x = -1$. (Round coefficients to 3 decimal places.)

A) $y = -6.12x + 8.204$

B) $y = -9.362x - 13.993$

C) $y = 9.362x + 8.204$

D) $y = 9.362x - 10.751$

Ans: C Difficulty: Moderate Section: 2.6

91. Find an equation of the line tangent to $f(x) = x\cos x$ at $x = -4$. (Round coefficients to 3 decimal places.)

A) $y = 3.681x + 12.109$

B) $y = 2.374x + 12.109$

C) $y = 2.374x - 12.109$

D) $y = 3.681x - 12.109$

Ans: B Difficulty: Moderate Section: 2.6

92. Use the position function $s(t) = \cos 2t - t^2$ feet to find the velocity at $t = 3$ seconds. (Round answer to 2 decimal places.)

A) $v(3) = -5.44$ ft/s
B) $v(3) = -6.56$ ft/s
C) $v(3) = 6.56$ ft/s
D) $v(3) = -7.92$ ft/s

Ans: A Difficulty: Moderate Section: 2.6

93. Use the position function $s(t) = 7\sin(2t) + 6$ meters to find the velocity at $t = 4$ seconds. (Round answer to 2 decimal places.)

A) $v(4) = 13.85$ m/s
B) $v(4) = -9.15$ m/s
C) $v(4) = -1.02$ m/s
D) $v(4) = -2.04$ m/s

Ans: D Difficulty: Moderate Section: 2.6

94. Use the position function to find the velocity at time $t = t_0$. Assume units of feet and seconds.

$$s(t) = \frac{\sin 10t}{t}, \quad t = \pi$$

A) $v(\pi) = 0$ ft/sec
B) $v(\pi) = -\dfrac{10}{\pi^2}$ ft/sec
C) $v(\pi) = \dfrac{10}{\pi}$ ft/sec
D) $v(\pi) = \dfrac{1}{\pi^2}$ ft/sec

Ans: C Difficulty: Moderate Section: 2.6

95. A weight hanging by a spring from the ceiling vibrates up and down. Its vertical position is given by $s(t) = 9\sin(7t)$. Find the maximum speed of the weight and its position when it reaches maximum speed.

A) speed = 9, position = 63
B) speed = 63, position = 0
C) speed = 7, position = 9
D) speed = 63, position = 7

Ans: B Difficulty: Moderate Section: 2.6

96. Given that $\lim_{x\to 0}\dfrac{\sin x}{x}=1$, find $\lim_{t\to 0}\dfrac{\sin(7t)}{-8t}$.

A) $-\dfrac{1}{8}$ B) –56 C) $-\dfrac{7}{8}$ D) $\dfrac{1}{7}$
Ans: C Difficulty: Easy Section: 2.6

97. Given that $\lim_{x\to 0}\dfrac{\cos x-1}{x}=0$, find $\lim_{t\to 0}\dfrac{\cos t-1}{2t}$.

A) 0 B) $\dfrac{1}{2}$ C) 2 D) $\dfrac{1}{2}$
Ans: A Difficulty: Easy Section: 2.6

98. Given that $\lim_{x\to 0}\dfrac{\sin x}{x}=1$, find $\lim_{t\to 0}\dfrac{6t}{\sin(7t)}$.

A) 42 B) $\dfrac{1}{6}$ C) $\dfrac{7}{6}$ D) $\dfrac{6}{7}$
Ans: D Difficulty: Easy Section: 2.6

99. Given that $\lim_{x\to 0}\dfrac{\sin x}{x}=1$, find $\lim_{t\to 0}\dfrac{\tan(7t)}{8t}$.

A) $\dfrac{1}{7}$ B) $\dfrac{7}{8}$ C) $\dfrac{8}{7}$ D) $\dfrac{1}{8}$
Ans: B Difficulty: Moderate Section: 2.6

100. For $f(x)=\sin x$, find $f^{(22)}(x)$.

A) $\cos x$ B) $-\cos x$ C) $\sin x$ D) $-\sin x$
Ans: D Difficulty: Easy Section: 2.6

101. The total charge in an electrical circuit is given by $Q(t)=3\sin(3t)+t+2$. The current is the rate of change of the charge, $i(t)=\dfrac{dQ}{dt}$. Determine the current at $t=0$ (Round answer to 2 decimal places.)

A) $i(0)=4$ B) $i(0)=10$ C) $i(0)=12$ D) $i(0)=1$
Ans: B Difficulty: Moderate Section: 2.6

102. Find the derivative of $f(x)=x^{-9}e^{-2x}$.

A) $f'(x)=\left(-9x^{-8}+2x^{-9}\right)e^{-2x}$

B) $f'(x)=-9x^{-10}e^{-2}-2x^{-9}e^{-2x-1}$

C) $f'(x)=\left(-9x^{-10}-2x^{-9}\right)e^{-2x}$

D) $f'(x)=-9x^{-10}-2e^{-2x}$

Ans: C Difficulty: Easy Section: 2.7

103. Differentiate the function.

$$f(x)=e^{3x}\cos 4x$$

A) $f'(x)=-12e^{3x}\sin 4x$

B) $f'(x)=3e^{3x}\cos 4x+4e^{3x}\sin 4x$

C) $f'(x)=12e^{3x}\sin 4x$

D) $f'(x)=3e^{3x}\cos 4x-4e^{3x}\sin 4x$

Ans: D Difficulty: Moderate Section: 2.7

104. Find the derivative of $f(x)=9^{3x+8}$.

A) $f'(x)=9^{3x+8}(3\ln 9)$

B) $f'(x)=(3)9^{3x+8}$

C) $f'(x)=9^{3x+8}\ln 9$

D) $f'(x)=9^{3x+8}(3x+8)\ln 9$

Ans: A Difficulty: Easy Section: 2.7

105. Differentiate the function.

$$f(w)=\frac{w}{e^{3w}}$$

A) $f'(w)=\dfrac{1-3w}{e^{3w}}$ B) $f'(w)=\dfrac{1}{3e^{3w}}$ C) $f'(w)=\dfrac{3}{e^{3w}}$ D) $f'(w)=\dfrac{3w-1}{e^{3w}}$

Ans: A Difficulty: Moderate Section: 2.7

106. Find the derivative of $f(x)=\ln(2x)$.

A) $f'(x)=\dfrac{1}{x}+\dfrac{1}{2}$ B) $f'(x)=\dfrac{2}{x}$ C) $f'(x)=\dfrac{1}{2x}$ D) $f'(x)=\dfrac{1}{x}$

Ans: D Difficulty: Easy Section: 2.7

107. Find the derivative of $f(x)=\ln\left(\sqrt{3x}\right)$.

A) $f'(x)=\dfrac{1}{6x}$ B) $f'(x)=\dfrac{2}{3x}$ C) $f'(x)=\dfrac{1}{2x}$ D) $f'(x)=\dfrac{1}{2}\left[\dfrac{1}{x}+\dfrac{1}{3}\right]$
Ans: C Difficulty: Easy Section: 2.7

108. Differentiate the function.
$f(t)=\ln(t^5+8t)$

A) $f'(t)=\dfrac{1}{t^5+8t}$
B) $f'(t)=\dfrac{1}{5t^4+8}$
C) $f'(t)=(5t^4+8)\ln(t^5+8t)$
D) $f'(t)=\dfrac{5t^4+8}{t^5+8t}$
Ans: D Difficulty: Moderate Section: 2.7

109. Differentiate the function.
$g(x)=\sin x\ \ln(x^5+3)$

A) $g'(x)=\cos x\ \ln(x^5+3)+\dfrac{5x^4\sin x}{x^5+3}$
B) $g'(x)=-\cos x\ \ln(x^5+3)+\dfrac{\sin x}{x^5+3}$
C) $g'(x)=\dfrac{5x^4\cos x}{x^5+3}$
D) $g'(x)=\dfrac{\cos x}{x^5+3}$
Ans: A Difficulty: Moderate Section: 2.7

110. Differentiate the function.
$h(x)=7^{e^x}$

A) $h'(x)=7^{e^x}$ B) $h'(x)=7^{e^x}\ln 7$ C) $h'(x)=e^x7^{e^x}\ln 7$ D) $h'(x)=e^x7^{e^x}$
Ans: C Difficulty: Moderate Section: 2.7

111. Find an equation of the line tangent to $f(x)=3^x$ at $x=3$.

A) $y=27\left(x\ln 3-(1+3\ln 3)\right)$
B) $y=x\ln 3+(1-3\ln 3)$
C) $y=27\left(x\ln 3+(1-3\ln 3)\right)$
D) $y=x\ln 3+(3\ln 3-1)$

Ans: C Difficulty: Moderate Section: 2.7

112. Find an equation of the line tangent to $f(x)=3\ln(x^4)$ at $x=2$.

A) $y=\dfrac{x}{2}+(\ln 2-1)$
C) $y=12\left(\dfrac{x}{2}+(1-\ln 2)\right)$

B) $y = 12\left(\frac{x}{2} + (\ln 2 - 1)\right)$

D) $y = \frac{x}{2} + (1 - \ln 2)$

Ans: B Difficulty: Moderate Section: 2.7

113. Find all values of x for which the tangent line to $f(x) = x^2 e^{-4x}$ is horizontal.

A) $x = 0$ B) $x = 0,\ x = -4$ C) $x = 0,\ x = 8$ D) $x = 0,\ x = \frac{1}{2}$

Ans: D Difficulty: Moderate Section: 2.7

114. The value of an investment is given by $v(t) = (600)4^t$. Find the instantaneous percentage rate of change. (Round to 2 decimal places.)

A) 1.39 % per year

C) 138.63 % per year

B) 33.27 % per year

D) 17.31 % per year

Ans: C Difficulty: Moderate Section: 2.7

115. A bacterial population starts at 300 and quadruples every day. Calculate the percent rate of change rounded to 2 decimal places.

A) 160.94 % B) 138.63 % C) 1.39 % D) 88.63 %
Ans: B Difficulty: Moderate Section: 2.7

116. Use logarithmic differentiation to find the derivative of $f(x) = x^{\cos 2x}$.

A) $f'(x) = x^{\cos 2x}\left[\frac{\cos 2x}{x} - 2(\sin 2x)\ln x\right]$

B) $f'(x) = (-2\sin 2x)x^{\cos 2x}$

C) $f'(x) = (\cos 2x)x^{\cos 2x - 1}$

D) $f'(x) = x^{\cos 2x}(\ln x - 2\sin 2x)$

Ans: A Difficulty: Moderate Section: 2.7

117. Find the derivative of $f(x) = (x^3)^{3x}$.

A) $f'(x) = x^{9x}(\ln x + 9)$

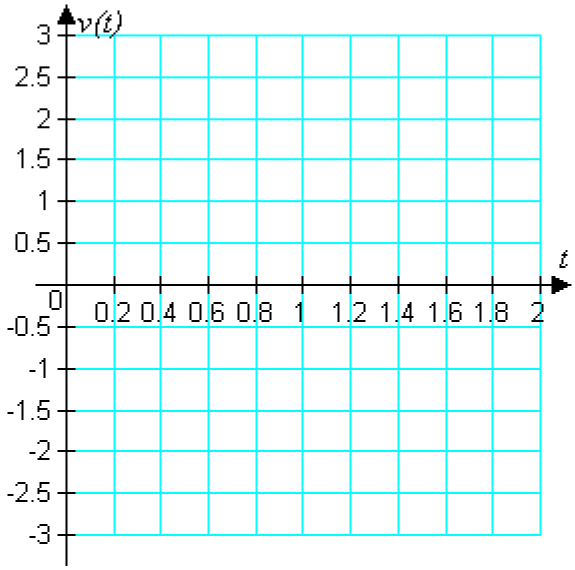
C) $f'(x) = 9x^{9x}$

B) $f'(x) = 9x^{9x-1}$

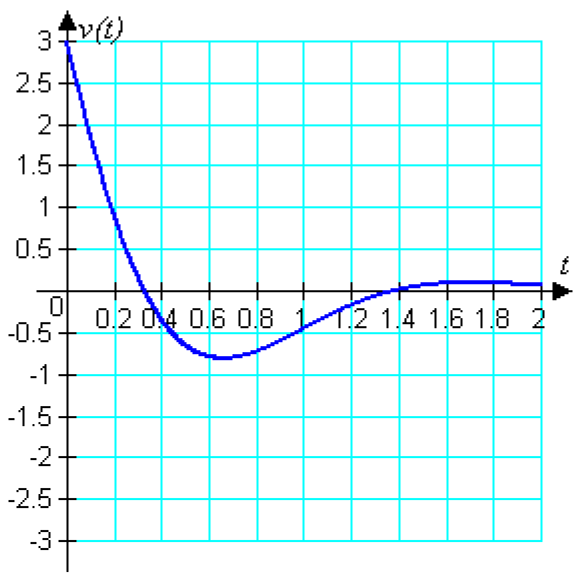
D) $f'(x) = 9x^{9x}(\ln x + 1)$

Ans: D Difficulty: Easy Section: 2.7

118. The position of a weight attached to a spring is described by $s(t) = e^{-2t}\sin 3t$. Determine and graph the velocity function for positive values of t and find the approximate first time when the velocity is zero. Find the approximate position of the weight the first time the velocity is zero. Round answers to tenths.



Ans: $v(t) = e^{-2t}(3\cos 3t - 2\sin 3t)$. The velocity is first zero at about 0.3 and its position is about 0.4.



Difficulty: Moderate Section: 2.7

119. An investment compounded continuously will be worth $f(t) = Ae^{rt}$, where A is the investment in dollars, r is the annual interest rate, and t is the time in years. APY can be defined as $(f(1) - A)/A$, the relative increase of worth in one year. Find the APY for an interest rate of 5%. Express the APY as a percent rounded to 2 decimal places.

A) $APY = 105.13\%$

B) $APY = 4.13\%$

C) $APY = 5.13\%$

D) $APY = 6.13\%$

Ans: C Difficulty: Moderate Section: 2.7

120. Compute the slope of the line tangent to $3x^2 + 3xy + 7y^2 = 34$ at $(2, -1)$.

A) slope = $\frac{15}{8}$ B) slope = $\frac{9}{8}$ C) slope = $\frac{8}{9}$ D) slope = $\frac{15}{14}$

Ans: B Difficulty: Moderate Section: 2.8

121. Find the derivative $y'(x)$ implicitly.

$$x^2y^2 - 7y = 5x$$

A) $y'(x) = \frac{5}{4xy + 7}$

B) $y'(x) = \frac{5 - 2xy^2}{2x^2y - 7}$

C) $y'(x) = -\frac{4xy - 5}{7}$

D) $y'(x) = \frac{2xy^2 + 12}{2x^2y}$

Ans: B Difficulty: Moderate Section: 2.8

122. Find the derivative $y'(x)$ implicitly if $2y^2 - \sqrt{xy} = -6$.

A) $y'(x) = -\frac{y}{4y\sqrt{xy} + x}$

B) $y'(x) = \frac{y\sqrt{xy}}{8y - x}$

C) $y'(x) = \frac{y}{8y\sqrt{xy} - x}$

D) $y'(x) = \frac{y}{8y - x\sqrt{xy}}$

Ans: C Difficulty: Moderate Section: 2.8

123. Find the derivative $y'(x)$ implicitly if $4\sin xy + 5x = -5$.

A) $y'(x) = \frac{5}{4x\cos xy} + \frac{y}{x}$

B) $y'(x) = -\frac{5}{4x} - \frac{y}{x\cos xy}$

C) $y'(x) = -\frac{5\cos xy}{4x} - \frac{y}{x}$

D) $y'(x) = -\frac{5}{4x\cos xy} - \frac{y}{x}$

Ans: D Difficulty: Moderate Section: 2.8

124. Find the derivative $y'(x)$ implicitly.

$$xe^y - 9y\cos x = 2$$

A) $y'(x) = -\dfrac{e^y}{9\sin x + xe^y}$

B) $y'(x) = -\dfrac{e^y}{9\sin x}$

C) $y'(x) = -\dfrac{9\sin x}{e^y}$

D) $y'(x) = \dfrac{e^y + 9y\sin x}{9\cos x - xe^y}$

Ans: D Difficulty: Difficult Section: 2.8

125. Find the derivative $y'(x)$ implicitly.

$$e^{5y} - \ln(y^2 - 1) = 3x$$

A) $y'(x) = \dfrac{3(y^2-1)}{5(y^2-1)e^{5y} - 2y}$

B) $y'(x) = \dfrac{(3-5e^{5x})(y^2-1)}{2y}$

C) $y'(x) = \dfrac{3(y^2-1)}{5(y^2-1)e^{5y} - 1}$

D) $y'(x) = \dfrac{3(y^2-1)+2y}{5(y^2-1)e^{5y}}$

Ans: A Difficulty: Difficult Section: 2.8

126. Find an equation of the tangent line at the given point.

$$x^2 - 16y^3 = 0 \text{ at } (4, 1)$$

A) $y = -\dfrac{1}{6}x + \dfrac{4}{3}$ B) $y = -\dfrac{1}{12}x + \dfrac{4}{3}$ C) $y = \dfrac{1}{6}x + \dfrac{1}{3}$ D) $y = \dfrac{1}{12}x + \dfrac{1}{3}$

Ans: C Difficulty: Moderate Section: 2.8

127. Find an equation of the tangent line at the given point.

$$x^2y^2 = 3y + 1 \text{ at } (2, 1)$$

Ans: $y = -\dfrac{4}{5}x + \dfrac{13}{5}$

Difficulty: Moderate Section: 2.8

128. Find the second derivative, $y''(x)$, of $-2\sqrt{x^3} + 4\sqrt{y^3} = -3$.

A) $y''(x) = \dfrac{1}{4\sqrt{xy}} - \dfrac{y'}{2y}$

B) $y''(x) = \dfrac{1}{4\sqrt{xy}} - \dfrac{(y')^2}{2y}$

C) $y''(x) = -\dfrac{1}{2\sqrt{xy}} - \dfrac{(y')^2}{2y}$

D) $y''(x) = -\dfrac{1}{4\sqrt{xy}} + \dfrac{(y')^2}{2y}$

Ans: B Difficulty: Moderate Section: 2.8

129. Find the second derivative, $y''(x)$, of $-3y^2 = -2x^3 + x - \cos y$.

A) $y''(x) = \dfrac{-4x + (-\cos y - 3)(y')^2}{-3y + \sin y}$

C) $y''(x) = \dfrac{-12x + (\cos y - 3)y^2}{-6y^2 - \sin y}$

B) $y''(x) = \dfrac{-2x + (\cos y - 6)y'}{-6y - \cos y}$

D) $y''(x) = \dfrac{-12x + (\cos y + 6)(y')^2}{-6y - \sin y}$

Ans: D Difficulty: Moderate Section: 2.8

130. Find the derivative of $f(x) = \cos^{-1}(x^5 - 2)$.

A) $f'(x) = \dfrac{5x^4 \sin(x^5 - 2)}{\cos^2(x^5 - 2)}$

C) $f'(x) = \dfrac{5x^4}{\sqrt{1-(x^5 - 2)^2}}$

B) $f'(x) = \dfrac{5x^4}{\cos^2(x^5 - 2)}$

D) $f'(x) = -\dfrac{5x^4}{\sqrt{1-(x^5 - 2)^2}}$

Ans: D Difficulty: Moderate Section: 2.8

131. Find the derivative of $f(x) = \tan^{-1}(3/x)$.

A) $f'(x) = -\dfrac{3}{9 + x^2}$

C) $f'(x) = -\dfrac{3}{1 + 9x^2}$

B) $f'(x) = -\dfrac{3}{3 + x^2}$

D) $f'(x) = -\dfrac{3}{1 + 3x^2}$

Ans: A Difficulty: Moderate Section: 2.8

132. Find the derivative of $f(x) = 5e^{3\tan^{-1}x}$.

A) $f'(x) = \dfrac{30}{1 - x^2} e^{3\tan^{-1}x}$

C) $f'(x) = \dfrac{15}{1 + x^2} e^{3\tan^{-1}x}$

B) $f'(x) = \dfrac{5}{1 + x^2} e^{3\tan^{-1}x}$

D) $f'(x) = \dfrac{3}{1 - x^2} e^{3\tan^{-1}x}$

Ans: C Difficulty: Moderate Section: 2.8

133. Find the derivative of $f(x) = 4\sec^{-1}(x^5)$.

A) $f'(x) = \dfrac{20x^4}{|x^5|\sqrt{x^{10} - 1}}$

C) $f'(x) = \dfrac{4x^4}{|x|\sqrt{x^2 + 1}}$

B) $f'(x) = \dfrac{-20x^5}{|x|\sqrt{x^2 - 1}}$

D) $f'(x) = \dfrac{5x^4}{|x^4|\sqrt{x^8 - 1}}$

Ans: A Difficulty: Moderate Section: 2.8

134. Find the location of all horizontal and vertical tangents for $x^2 - xy^2 = 49$.

A) horizontal: none; vertical: (–7, 0), (7, 0)

B) horizontal: (7, 0); vertical: (–7, 0), (7, 0)

C) horizontal: (–7, 0), (7, 0); vertical: none

D) horizontal: none; vertical: (7, 0)

Ans: A Difficulty: Moderate Section: 2.8

135. Find the location of all horizontal and vertical tangents for $x^2 + xy^2 + 81 = 0$.

A) horizontal: $\left(-9, -3\sqrt{2}\right), \left(-9, 3\sqrt{2}\right)$; vertical: (–81, 0)

B) horizontal: $\left(-9, -3\sqrt{2}\right), \left(-9, 3\sqrt{2}\right)$; vertical: (0, 0)

C) horizontal: $\left(-9, -3\sqrt{2}\right), \left(-9, 3\sqrt{2}\right)$; vertical: none

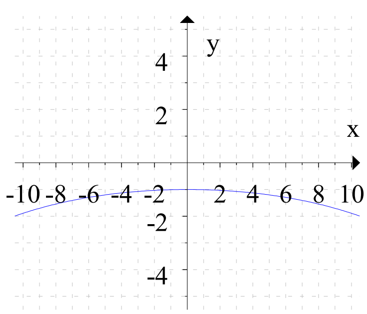
D) horizontal: $\left(9, -3\sqrt{2}\right), \left(9, 3\sqrt{2}\right)$; vertical: (–81, 0)

Ans: C Difficulty: Moderate Section: 2.8

136. Sketch the graph of the function.

$f(x) = \cosh(x/8)$

A)



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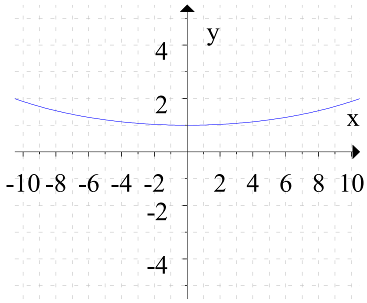

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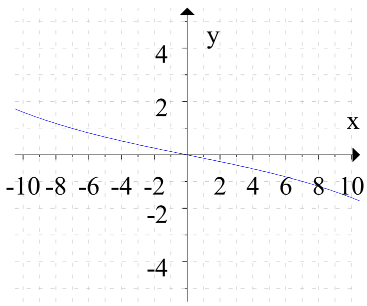
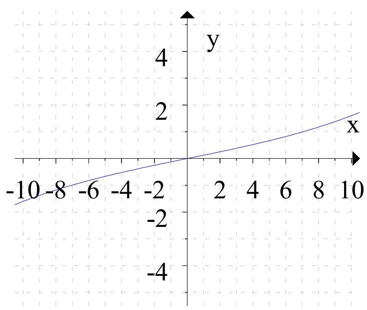


D)



Ans: B Difficulty: Moderate Section: 2.9

137. Find the derivative of $f(x) = \cosh\sqrt{2x}$.

A) $f'(x) = -\dfrac{\sqrt{2}\cosh\sqrt{2x}}{2\sqrt{x}}$

B) $f'(x) = \dfrac{\sqrt{2}\cosh\sqrt{2x}}{2\sqrt{x}}$

C) $f'(x) = -\dfrac{\sqrt{2}\sinh\sqrt{2x}}{2\sqrt{x}}$

D) $f'(x) = \dfrac{\sqrt{2}\sinh\sqrt{2x}}{2\sqrt{x}}$

Ans: D Difficulty: Moderate Section: 2.9

138. Find the derivative of $f(x)=(\tanh x)^3$.

A) $f'(x)=3(\tanh x)^2$

B) $f'(x)=3(\tanh x)^2 \operatorname{sech}^2 x$

C) $f'(x)=\operatorname{sech}^6 x$

D) $f'(x)=3\operatorname{sech}^5 x$

Ans: B Difficulty: Moderate Section: 2.9

139. Find the derivative of $f(x)=\operatorname{sech} 4x$.

A) $f'(x)=-4\operatorname{sech} 4x \tanh 4x$

B) $f'(x)=4\operatorname{sech} 4x \tanh 4x$

C) $f'(x)=4\operatorname{sech}^2 4x$

D) $f'(x)=\operatorname{sech}^2 4x$

Ans: A Difficulty: Moderate Section: 2.9

140. Find the derivative of $f(x)=x^4 \sinh 10x$.

A) $f'(x)=40x^3 \cosh 10x$

B) $f'(x)=4x^3 \cosh 10x$

C) $f'(x)=4x^3 \sinh 10x+10x^4 \cosh 10x$

D) $f'(x)=4x^3 \sinh 10x+x^4 \cosh 10x$

Ans: C Difficulty: Moderate Section: 2.9

141. Find the derivative of $f(x)=\dfrac{\cosh 4x}{x-2}$.

A) $f'(x)=\dfrac{4(x-2)\sinh 4x-\cosh 4x}{(x-2)^2}$

B) $f'(x)=\dfrac{(x-2)\sinh 4x-4\cosh 4x}{(x-2)^2}$

C) $f'(x)=\dfrac{4\sinh 4x}{x-2}$

D) $f'(x)=\dfrac{4\sinh 4x}{(x-2)^2}$

Ans: A Difficulty: Moderate Section: 2.9

142. Find the derivative of $f(x)=\cosh^{-1} 8x$.

A) $f'(x)=\dfrac{8}{\sqrt{64-x^2}}$

B) $f'(x)=\dfrac{8}{\sqrt{x^2-64}}$

C) $f'(x)=\dfrac{8}{\sqrt{64x^2-1}}$

D) $f'(x)=\dfrac{8}{\sqrt{1-64x^2}}$

Ans: C Difficulty: Moderate Section: 2.9

143. A general equation for a catenary is $y = a\cosh(x/b)$. Find a and b to match the following characteristics of a hanging cable. The ends are 20 m apart and have a height of $y = 20$ m. The height in the middle is $y = 10$ m.

Ans: $a = 10,\ b = \dfrac{10}{\ln(\sqrt{3}+2)},\ y = 10\cosh\left(\dfrac{\ln(\sqrt{3}+2)}{10}x\right)$

Difficulty: Moderate Section: 2.9

144. Suppose that the vertical velocity $v(t)$ of a falling object of mass $m = 30$ kg subject to gravity and air drag is given by

$$v(t) = -\sqrt{\frac{9.8m}{k}}\tanh\left(\sqrt{\frac{9.8k}{m}}\,t\right)$$

for some positive constant k. Suppose $k = 0.5$ and find the terminal velocity v_T by computing $\lim_{t\to\infty} v(t)$.

A) $v_T \approx -96.8$ m/sec

B) $v_T \approx -48.4$ m/sec

C) $v_T \approx -24.2$ m/sec

D) $v_T \approx -12.1$ m/sec

Ans: C Difficulty: Moderate Section: 2.9

145. Determine if the function satisfies Rolle's Theorem on the given interval. If so, find all values of c that make the conclusion of the theorem true.

$f(x) = 36 - x^2,\ [-9, 9]$

A) $x = 0$ B) $x = 36$ C) $x = -6,\ x = 6$ D) Rolle's Theorem not satisfied

Ans: A Difficulty: Easy Section: 2.10

146. Using the Mean Value Theorem, find a value of c that makes the conclusion true for $f(x) = 4x^3 + 5x^2$, in the interval $[-1,1]$.

A) $c \approx -1.129$ B) One or more hypotheses fail C) $c \approx 0.295$ D) $c = 0$

Ans: C Difficulty: Easy Section: 2.10

147. Using the Mean Value Theorem, find a value of c that makes the conclusion true for
$f(x) = \cos x, \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

A) One or more hypotheses fail B) $c = 0$ C) $c = \frac{\pi}{4}$ D) $c \approx .881$

Ans: B Difficulty: Easy Section: 2.10

148. Prove that $9x^3 + 9x - 9 = 0$ has exactly one solution.

Ans: Let $f(x) = 9x^3 + 9x - 9$. The function $f(x)$ is continuous and differentiable everywhere. Since $f(0) < 0$ and $f(1) > 0$, $f(x)$ must have at least one zero. The derivative of $f(x) = 9x^3 + 9x - 9$ is $f'(x) = 27x^2 + 9$, which is always greater than zero. Therefore $f(x)$ can only have one zero.
Difficulty: Moderate Section: 2.10

149. Find all functions g such that $g'(x) = f(x)$.
$f(x) = 6x^4$

A) $g(x) = 24x^3$

B) $g(x) = \frac{6}{5}x^5$

C) $g(x) = 24x^3 + C,$ for some constant C

D) $g(x) = \frac{6}{5}x^5 + C,$ for some constant C

Ans: D Difficulty: Easy Section: 2.10

150. Find all the functions $g(x)$ such that $g'(x) = \frac{6}{x^9}$.

A) $g(x) = -\frac{3}{4x^8}$

B) $g(x) = -\frac{3}{5x^{10}} + c$

C) $g(x) = \frac{12}{25x^8}$

D) $g(x) = -\frac{3}{4x^8} + c$

Ans: D Difficulty: Moderate Section: 2.10

151. Find all the functions $g(x)$ such that $g'(x) = -\sin x$.

A) $g(x) = -\cos x + c$

C) $g(x) = \cos x$

B) $g(x) = \cos x + c$ D) $g(x) = \sin x + c$

Ans: B Difficulty: Moderate Section: 2.10

152. Determine if the function $f(x) = 4x^3 + 5x + 2$ is increasing, decreasing, or neither.

A) Increasing B) Decreasing C) Neither
Ans: A Difficulty: Easy Section: 2.10

153. Determine if the function $f(x) = -5x^4 - 4x^2 + 9$ is increasing, decreasing, or neither.

A) Increasing B) Decreasing C) Neither
Ans: C Difficulty: Easy Section: 2.10

154. Explain why it is not valid to use the Mean Value Theorem for the given function on the specified interval. Show that there is no value of *c* that makes the conclusion of the theorem true.

$$f(x) = \frac{1}{x-4},\ [3, 5]$$

Ans: The function is not continuous on the specified interval, so the Mean Value Theorem does not apply. Note that $f(3) = -1$ and $f(5) = 1$, so that

$$\frac{f(5) - f(3)}{(5) - (3)} = \frac{1 - (-1)}{2} = 1.$$

Also, $f'(x) = -\dfrac{1}{(x-4)^2}$.

Since $f'(x) < 0$ for all *x* in the domain of *f*, there is no value of *c* such that $f'(c) = 1$. That is, there is no value of *c* such that $f'(c) = \dfrac{f(5) - f(3)}{(5) - (3)}$.

Difficulty: Moderate Section: 2.10