1. Find the limit.

$$\lim_{x \to 0^+} \tan^{-1} \left( \frac{2}{x} \right)$$

2. Evaluate the limit.

$$\lim_{x \to -9} |x + 9|$$

3. Find the limit.

$$\lim_{x \to \frac{6}{x}} \tan^{-1} \left( \frac{3}{x} \right)$$

- 4. How close to -9 do we have to take x so that  $\frac{1}{(x+9)^4} > 10000$ ?
- 5. How close to 2 do we have to take x so that 5x + 3 is within a distance of 0.01 from 13?
- 6. If f and g are continuous functions with f(13) = 4 and  $\lim_{x \to 13} [2f(x) g(x)] = 13$ , find g(13).
- 7. Find the limit.

$$\lim_{y \to \infty} \frac{7 - 3y^2}{7y^2 + 3y}$$

8. Find the limit.

$$\lim_{y \to -\infty} \left( y + \sqrt{y^2 + 5y} \right)$$

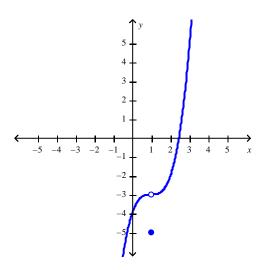
9. Find an equation of the tangent line to the curve.

$$y = \frac{\sqrt{x}}{x+6}$$
 at (4, 0.6)

10. Find the limit if  $g(x) = x^3$ .

$$\lim_{x\to 2} \frac{g(x) - g(2)}{x - 2}$$

11. Use the graph of the function to find each limit.



$$\lim_{x \to 1^-} f(x), \quad \lim_{x \to 1^+} f(x), \quad \lim_{x \to 1} f(x)$$

12. Complete the table by computing f(x) at the given values of x, accurate to five decimal places. Use the results to guess at the indicated limit, if it exists, to three decimal places.

$$\lim_{x \to 4} \frac{\sqrt{x+12} - 4}{x-4}$$

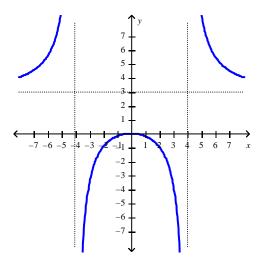
х	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

13. Use the precise definition of a limit to prove that  $\lim_{n \to 1} 8 = 8$ .

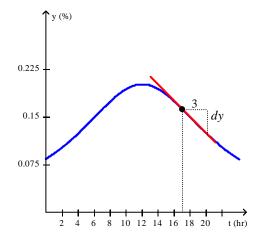
14. Find the limit  $\lim_{x \to 0^+} \frac{3 + \sqrt{x}}{\sqrt{x + 64}}$ .

15. Find the numbers, if any, where the function  $f(x) = \frac{x-1}{x^2-1}$  is discontinuous.

- 16. Find the interval(s) where  $f(x) = \frac{1}{x} + \frac{8\sqrt{x}}{(x-9)^2}$  is continuous.
- 17. You are given the graph of f. Find the horizontal and vertical asymptotes of the graph of f.



18. The graph shows the percentage of households in a certain city watching television during a 24-hr period on a weekday (t = 0 corresponds to 6 a.m.). By computing the slope of the respective tangent line, estimate the rate of change of the percentage of households watching television at a-12 p.m. Note that dy = 0.039



19. Find the derivative of the function and evaluate f'(x) at the given value of x.

$$f(x) = \left(\sqrt{x} + 2x\right)\left(x^{3/2} - x\right); \qquad x = 9$$

20. Let 
$$f(x) = x |x|$$
.

- a. Sketch the graph of f.b. For what values of x is f differentiable?
- **c.** Find a formula for f'(x).

## **Answer Key**

1. 
$$\frac{\pi}{2}$$

4. 
$$|x+9| < 0.1$$

5. 
$$|x-2| < 0.002$$

$$7. -3/7$$

$$8. -2.5$$

9. 
$$y = \frac{1}{200}(x-4) + 0.6$$

11. 
$$\lim_{x \to 1^{-}} f(x) = -3$$
,  $\lim_{x \to 1^{+}} f(x) = -3$ ,  $\lim_{x \to 1} f(x) = -3$ 

12.

Ī	х	3.9	3.99	3.999	4.001	4.01	4.1
	f(x)	0.1252	0.12502	0.125	0.125	0.12498	0.12481

0.125

13. The function is f(x) = 8 and L = 8.

Let  $\varepsilon > 0$  be given. Choose  $\delta$  to be any real number greater than 0.

We see that for any x such that  $|x-1| < \delta$  (actually for any x whatsoever), we have  $|f(x) - 8| = |8 - 8| = 0 < \varepsilon$ .

This proves the assertion.

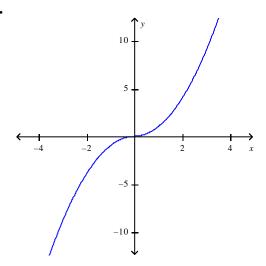
14. 
$$\frac{3}{8}$$

15. 
$$\pm 1$$

17. HA 
$$y = 3$$
, VA  $x = -4$ , 4

19. 
$$\frac{225}{2}$$

20. a.



**b.** 
$$x \in (-\infty, \infty)$$

**b.** 
$$x \in (-\infty, \infty)$$
  
**c.**  $f'(x) = \begin{cases} -2x & \text{if } x < 0 \\ 2x & \text{if } x \ge 0 \end{cases}$ 

1. The position of a car is given by the values in the following table.

t(seconds)	0	1	2	3	4	5
s(feet)	0	16	35	71	112	179

Estimate the instantaneous velocity when t = 2 by averaging the velocities for the periods [1, 2] and [2, 3].

2. Find the limit.

$$\lim_{x \to 0^+} \tan^{-1} \left( \frac{2}{x} \right)$$

3. Evaluate the limit.

$$\lim_{x \to \infty} \frac{10x^2 - 3x + 1}{7x^2 + 3x - 3}$$

4. Evaluate the limit.

$$\lim_{x \to 0} \frac{(6+x)^{-1} - 6^{-1}}{x}$$

5. Find the limit.

$$\lim_{x \to \frac{10}{x}} \tan^{-1} \left( \frac{5}{x} \right)$$

6. Consider the following function.

$$f(x) = \begin{cases} 5 - x & x < -1 \\ x & -1 \le x < 5 \\ (x - 5)^2 & x \ge 5 \end{cases}$$

Determine the values of a for which  $\lim_{x \to a} f(x)$  exists.

- 7. How close to -8 do we have to take x so that  $\frac{1}{(x+8)^2} > 100$ ?
- 8. Use a graph to find a number  $\delta$  such that  $\left| \sin x \frac{1}{2} \right| < 0.1$  whenever  $\left| x \frac{\pi}{6} \right| < \delta$ .

Round your answer to the nearest thousandth.

9. Use the definition of the limit to find values of  $\delta$  that corresponds to  $\varepsilon = 0.75$ .

$$\lim_{x \to 1} \left( 4 + x - 3x^3 \right) = 2$$

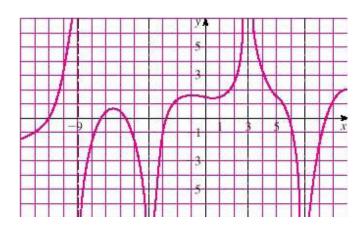
Round your answer to the nearest thousandth.

- 10. Use a graph to find a number  $\delta$  such that  $\left| \sqrt{4x+1} 3 \right| < 0.1$  whenever  $|x-2| < \delta$ .
- 11. How close to 2 do we have to take x so that 5x + 3 is within a distance of 0.01 from 13?
- 12. Use continuity to evaluate the limit.

$$\lim_{x \to 9} \frac{18 + \sqrt{x}}{\sqrt{16 + x}}$$

13. For the function f whose graph is shown, state the following.

$$\lim_{x \to -4} f(x)$$

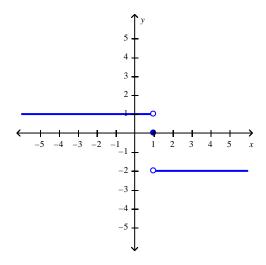


14. If a cylindrical tank holds 10000 gallons of water, which can be drained from the bottom of the tank in an hour, then Torricelli's Law gives the volume of water remaining in the tank after *t* minutes as

$$V(t) = 10000 \left( 1 - \frac{1}{60} t \right)^2, 0 \le t \le 60$$

Find the rate at which the water is flowing out of the tank (the instantaneous rate of change of V with respect to t) as a function of t.

- 15. Find an equation of the tangent line to the curve  $8x^2 + 7y^2 = 3$  at the point (1, 1).
- 16. Use the graph of the function to find each limit.

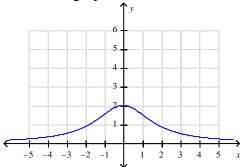


$$\lim_{x \to 1^{-}} f(x), \quad \lim_{x \to 1^{+}} f(x), \quad \lim_{x \to 1} f(x)$$

- 17. Use the precise definition of a limit to prove that  $\lim_{x\to 6} 5 = 5$ .
- 18. Find the limit.

$$\lim_{x \to \infty} \frac{\sin(7x)}{7x}$$

19. Sketch the graph of the derivative f' of the function f whose graph is given.



- 20. Let  $f(x) = x |x^3|$ .
  - **a.** Sketch the graph of f.
  - **b.** For what values of x is f differentiable?
  - **c.** Find a formula for f'(x).

## **Answer Key**

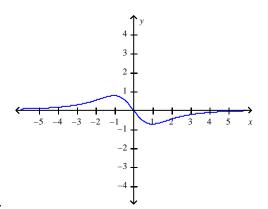
- 1. 27.5 ft/s
- 2.  $\frac{\pi}{2}$
- 3.  $\frac{10}{7}$
- 4.  $-\frac{1}{36}$
- 5. 0
- 6.  $(-\infty, -1) \cup (-1, 5) \cup (5, \infty)$
- 7. |x+8| < 0.1
- δ ≤ 0.114
- 9. δ≤0.085
- 10.  $\delta \le 0.15$
- 11. |x-2| < 0.002
- 12.  $\frac{21}{5}$
- 13. -∞
- 14.  $V'(t) = \frac{-1000}{3} + \frac{50t}{9}$
- 15.  $y = -\frac{8}{7}x + \frac{15}{7}$
- 16.  $\lim_{x \to 1^{-}} f(x) = 1$ ,  $\lim_{x \to 1^{+}} f(x) = -2$ ,  $\lim_{x \to 1} f(x)$  Does not exist
- 17. The function is f(x) = 5 and L = 5.

Let  $\varepsilon > 0$  be given. Choose  $\delta$  to be any real number greater than 0.

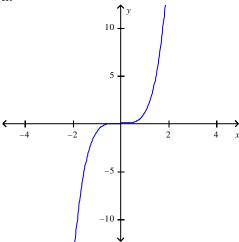
We see that for any x such that  $|x-6| < \delta$  (actually for any x whatsoever), we have  $|f(x)-5| = |5-5| = 0 < \varepsilon$ .

This proves the assertion.

18. 0



- 19.
- 20. **a.**



**b.** 
$$x \in (-\infty, \infty)$$
  
**c.**  $f'(x) = \begin{cases} -4x^3 & \text{if } x < 0 \\ 4x^3 & \text{if } x \ge 0 \end{cases}$ 

Select the correct answer for each question.

- 1. Suppose the distance s (in feet) covered by a car moving along a straight road after t sec is given by the function  $s = f(t) = 3t^2 + 13t$ . Calculate the (instantaneous) velocity of the car when t = 35.
  - a. 223 ft/sec
  - b. 16 ft/sec
  - c. 560 ft/sec
  - d. 4130 ft/sec
  - 2. If a rock is thrown upward on the planet Mars with a velocity of 12 m/s, its height in meters t seconds later is given by

$$y = 12t - 1.92t^2.$$

Find the average velocity over the time interval [2, 3].

- a. -0.6 m/s
- b. 4.4 m/s
- c. 2.4 m/s
- d. 3.4 m/s
- e. 1.4 m/s
- 3. The point P(16, 4) lies on the curve  $y = \sqrt{x}$ . If is the point  $Q(x, \sqrt{x})$ , use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the value x = 3.89.
  - a.  $m_{PQ} = -0.044439$
  - b.  $m_{PQ} = -0.167439$
  - c.  $m_{PO} = -0.307439$
  - d.  $m_{PQ} = 0.377439$
  - e.  $m_{PQ} = 0.167439$

4. Use the graph of the function to state the value of  $\lim_{x\to 0} f(x)$ , if it exists.

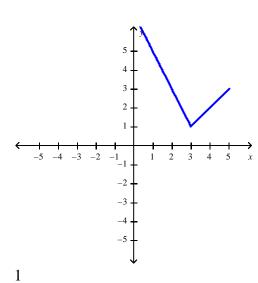
$$f(x) = \frac{1}{1 + 4^{1/x}}$$

- a.  $\frac{1}{3}$
- b.  $\frac{1}{2}$
- c. does not exist
- d.
- e. 00

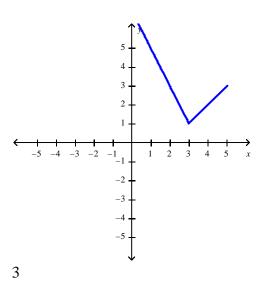
5. Sketch the graph of the function f and evaluate  $\lim_{x\to 3} f(x)$ .

$$f(x) = \begin{cases} x - 2, & \text{if } x \le 3\\ -2x + 7, & \text{if } x > 3 \end{cases}$$

a.



c.



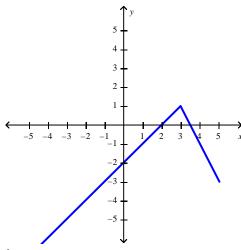
b.







d.



1

6. Evaluate the limit.

$$\lim_{x \to 1} (x+5)^3 \left(x^2 - 6\right)$$

- a. -448
- b. -1070
- c. -1090
- d. -1080
- e. 320

7. Find the limit  $\lim_{x\to 1} \frac{x^2+x-2}{x-1}$ , if it exists.

- a. 1
- b. 3
- c. 2
- d. Does not exist

8. Find the limit by evaluating the derivative of a suitable function at an appropriate value of *x*. (Hint: Use the definition of the derivative.)

$$\lim_{h \to 0} \frac{2(5+h)^2 + 5(5+h) - 75}{h}$$

- a. 25
- b. 75
- c. 0
- d. oo

9. You are given that  $\lim_{x \to a} f(x) = -3$ ,  $\lim_{x \to a} g(x) = -4$ , and  $\lim_{x \to a} h(x) = 2$ . Find the limit

$$\lim_{x \to a} \left\{ \left[ h(x) \right]^2 - f(x)g(x) \right\}.$$

- a. 17
- b. -8
- c. 0
- d. 22

10	Find	41	11
1()	Fina	rne	11m1r

$$\lim_{x \to 2} \sqrt{\frac{4x^2 + 1}{3x - 2}}$$

- a. 0
- b.  $-\frac{4}{3}$
- c.  $\frac{\sqrt{17}}{2}$
- d.  $\frac{4}{3}$
- e. does not exist

11. A machinist is required to manufacture a circular metal disk with area 1000 cm<sup>2</sup>. If the machinist is allowed an error tolerance of ±15 cm<sup>2</sup> in the area of the disk, how close to the ideal radius must the machinist control the radius?

Round your answer to the nearest hundred thousandth.

- a.  $\delta \le 0.13131 \, \text{cm}$
- b.  $\delta \le 0.13281 \, \text{cm}$
- c.  $\delta \leq 0.13231 \, \mathrm{cm}$
- d. δ≤0.13331cm
- e.  $\delta \le 0.13431 \, \text{cm}$

12. If f and g are continuous functions with 
$$f(9) = 6$$
 and  $\lim_{x \to 9} [2f(x) - g(x)] = 9$ , find  $g(9)$ .

a. 
$$g(9) = 21$$

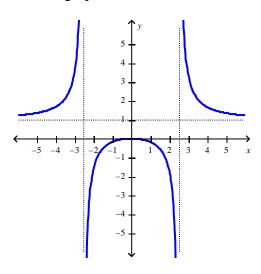
b. 
$$g(9) = 15$$

c. 
$$g(9) = 12$$

d. 
$$g(9) = 24$$

e. 
$$g(9) = 3$$

13. Use the graph to determine where the function is discontinuous.



- a. At 0
- b. On the interval (0, 1)
- c. At  $\pm 2.5$
- d. At 1

\_\_\_\_ 14. Find the numbers, if any, where the function 
$$f(x) = \begin{cases} 3x - 2 & \text{if } x \le 1 \\ 0 & \text{if } x > 1 \end{cases}$$
 is discontinuous.

$$f(x) = \begin{cases} x - 4 & \text{if } x \le 5\\ kx^2 - 24x + 46 & \text{if } x > 5 \end{cases}$$

Find the value of k that will make f continuous on  $(-\infty, \infty)$ .

- a. 5
- b. 46
- c. -4
- d. 3

#### 16. Find the limit.

$$\lim_{x \to -\infty} \frac{-5x^2 - 9}{4x^2 + 5}$$

- a.  $-\frac{9}{5}$
- b. (
- c.  $\frac{5}{4}$
- d.  $-\frac{5}{4}$

#### 17. Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{9x^3}{\sqrt{4x^6 + 1}}.$$

a. HA 
$$y = \frac{9}{2}$$
, VA  $x = -\frac{1}{2}$ ,  $\frac{1}{2}$ 

b. HA 
$$y = -\frac{9}{2}, \frac{9}{2}, VA$$
 none

c. HA 
$$y = \frac{9}{2}$$
, VA none

d. HA 
$$y = -\frac{9}{2}, \frac{9}{2}, \text{ VA } x = -\frac{1}{2}, \frac{1}{2}$$



$$y = \frac{4x^2 + 1}{3x - 4x^2}$$

a. 
$$x = -\frac{4}{3}$$

b. 
$$x = \frac{4}{3}$$

c. 
$$x = 3$$

d. 
$$x = 0, \frac{4}{3}$$

e. None of these

19. If  $g(x) = \sqrt{2-3x}$ , use the definition of derivative to find g'(x).

a. 
$$g'(x) = -\frac{1}{2}(2-3x)^{-1/2}$$

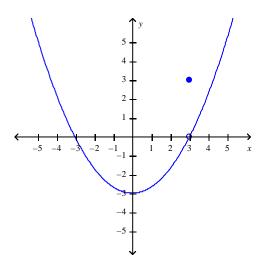
b. 
$$g'(x) = -(2-3x)^{-1/2}$$

c. 
$$g'(x) = -\frac{3}{2}(2-3x)^{1/2}$$

d. 
$$g'(x) = -\frac{3}{2}(2-3x)^{-1/2}$$

e. None of these

20. Use the graph of the function f to find the value(s) of x at which f is not differentiable.



- a. -3
- b. ±6
- c. -6
- d. 3

## **Answer Key**

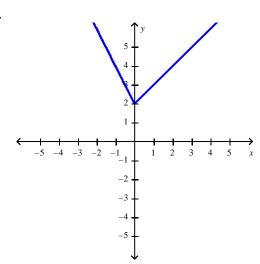
- 1. A
- 2. C
- 3. E
- 4. C
- 5. D
- 6. D
- 7. B
- 8. A
- 9. B
- 10. C
- 11. D
- 12. E
- 13. C
- 14. C
- 11. C
- 15. D 16. D
- 17. B
- 18. E
- 19. D
- 20. D

Select the correct answer for each question.

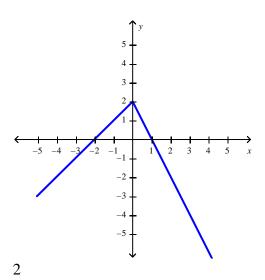
1. Sketch the graph of the function f and evaluate  $\lim_{x\to 0} f(x)$ .

$$f(x) = \begin{cases} x+2, & \text{if } x \le 0 \\ -2x+2, & \text{if } x > 0 \end{cases}$$

a.

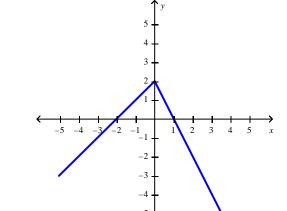


c.

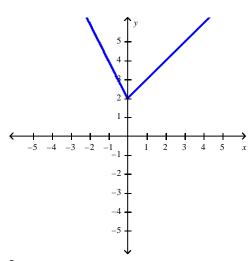


0

b.

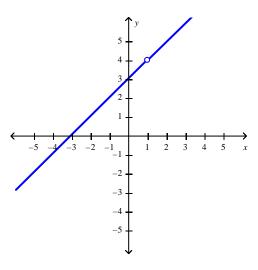


d.



0

2. Use the graph of  $f(x) = \frac{x^2 + 2x - 3}{x - 1}$  to guess at the limit  $\lim_{x \to 1} \frac{x^2 + 2x - 3}{x - 1}$ , if it exists.



- a. 1
- b. 4
- c. 3
- d. Does not exist
- 3. Find the vertical asymptotes of the function.

$$y = \frac{4x^2 + 1}{5x - 4x^2}$$

- a. x = 4
- b.  $x = \frac{1}{5}$
- c.  $x = 0, x = \frac{1}{4}$
- d.  $x = -\frac{2}{5}$
- e. none of these

#### 4. Find the limit.

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 - 25}}{6x - 18}$$

- a.  $\frac{1}{6}$
- b. 5
- c. -5
- d. 18
- e. does not exist

## 5. If $1 \le f(x) \le x^2 + 5x + 5$ , for all x, find $\lim_{x \to -1} f(x)$ .

- a. 1
- b. 8
- c.  $-\frac{1}{16}$
- d.  $-\frac{1}{8}$
- e. does not exist

6. Find the limit 
$$\lim_{x \to 3} \frac{x^2 - 5x + 6}{x - 3}$$
, if it exists.

- a. -2
- b. Does not exist
- c. 3
- d. 1

7. Evaluate 
$$\lim_{h \to 0} \frac{\cot\left(\frac{\pi}{4} + h\right) - 1}{h}$$
.

- a. 0
- b. -2
- c. 2
- d. Undefined
- 8. Evaluate the limit, if it exists.

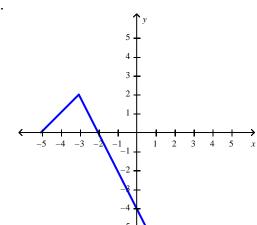
$$\lim_{h \to 0} \frac{(x-h)^7 - x^7}{h}$$

- a. -7
- b. 1 c.  $-7x^6$ d.  $7x^6$
- e. does not exist

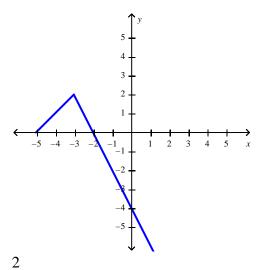
9. Sketch the graph of the function f and evaluate  $\lim_{x \to -3} f(x)$ .

$$f(x) = \begin{cases} x+5, & \text{if } x \le -3\\ -2x-4, & \text{if } x > -3 \end{cases}$$

a.

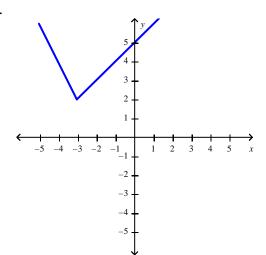


c.

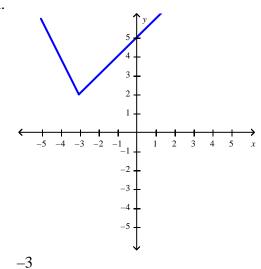


-3

b.



d.



2

10. For what value of the constant c is the function f continuous on  $(-\infty, \infty)$ ?

$$f(x) = \begin{cases} cx + 5 & \text{for } x \le 2\\ cx^2 - 7 & \text{for } x > 2 \end{cases}$$

a. 
$$c = 2$$

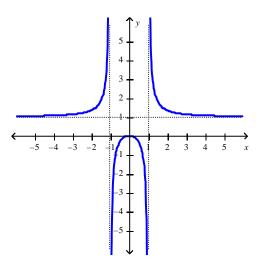
b. 
$$c = 1$$

c. 
$$c = 6$$

d. 
$$c = -2$$

e. 
$$c = -6$$

11. Use the graph to determine where the function is discontinuous.



d. At 
$$\pm 1$$

12. Find the limit.

$$\lim_{x \to \infty} \frac{x^7 - 4}{x^6 + 8}$$

a. 
$$-1/4$$

13. Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{7x^3}{\sqrt{9x^6 + 1}}.$$

a. HA 
$$y = \frac{7}{3}$$
, VA  $x = -\frac{1}{3}$ ,  $\frac{1}{3}$ 

b. HA 
$$y = -\frac{7}{3}, \frac{7}{3}, VA$$
 none

c. HA 
$$y = \frac{7}{3}$$
, VA none

d. HA 
$$y = -\frac{7}{3}$$
,  $\frac{7}{3}$ , VA  $x = -\frac{1}{3}$ ,  $\frac{1}{3}$ 

\_ 14. Let  $F(x) = \frac{x^2 - 1}{|x - 1|}$ . Find the following limits.

$$\lim_{x \to 1^+} F(x), \quad \lim_{x \to 1^-} F(x)$$

a. 
$$2$$
 and  $-2$ 

b. 
$$2 \text{ and } -1$$

15. The cost (in dollars) of producing x units of a certain commodity is

$$C(x) = 4,571 + 17x + 0.05x^2$$

Find the instantaneous rate of change with respect to x when x = 103. (This is called the *marginal cost*.)

16	Find on oo	uation of th	a tangant	ling to the	o ourwo	r <sup>3</sup> _	- 6x at	thor	oint (	۲. A	o١	
10.	rina an eq	uanon or u	ie tangent	me to me	curve .	y = x -	-ox ai	me p	) mio	Ο,	0)	•

a. 
$$y = 8 + 30x$$

b. 
$$y = 6 + 30x$$

c. 
$$y = 6 - 30x$$

d. 
$$y = x - 8$$

17. Find the tangent line to the parabola 
$$y = 5x - x^2$$
 at the point (2, 4).

a. 
$$y = \sqrt{3}x - 4$$

b. 
$$y = -\sqrt{6}x - 3$$

$$c. \quad y = \frac{\sqrt{3}}{3} x - 4$$

d. 
$$y = x + 2$$

18. The cost (in dollars) of producing 
$$x$$
 units of a certain commodity is

$$C(x) = 4,280 + 13x + 0.03x^{2}$$
.

Find the average rate of change with respect to x when the production level is changed from x = 100 to x = 120.

19. At what point is the function 
$$f(x) = |3 - x|$$
 not differentiable.

a. 
$$-3$$

d. 
$$-1$$

20. The symbol  $\lfloor \rfloor$  can be used to denote the greatest integer function, which is defined by  $\lfloor x \rfloor =$  the greatest integer n such that  $n \leq x$ . Use the graph of the function to find the indicated limit, if it exists.

$$\lim_{x \to -0.3} \lfloor x \rfloor$$

- a. 0
- b. −1
- c. -0.3
- d. 0.3

## **Answer Key**

- 1. C
- 2. B
- 3. E
- 4. A
- 5. A
- 6. D
- 7. C
- 8. C
- 9. C
- 10. C
- 11. D
- 12. E
- 13. B
- 14. A
- 15. E
- 16. E
- 17. D
- 18. C
- 19. E
- 20. B

1. The displacement (in feet) of a certain particle moving in a straight line is given by

$$s = \frac{t^3}{8}$$

where t is measured in seconds. Find the average velocity over the interval [1, 1.19].

Round your answer to three decimal places.

2. If a ball is thrown into the air with a velocity of 58 ft/s, its height (in feet) after t seconds is given by

$$H = 58t - 9t^2$$
.

Find the velocity when t = 9. Select the correct answer.

a. 
$$-101$$
 ft/s

b. 
$$-104$$
 ft/s

c. 
$$-106$$
 ft/s

e. 
$$-99$$
 ft/s

3. Find the value of the limit.

$$\lim_{x \to 0} 3 \frac{\tan 4x - 4x}{x^3}$$

4. Find the limit.

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 - 25}}{2x - 6}$$

5. Find the limit  $\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1}$ , if it exists. Select the correct answer.

$$c = 2$$

d. Does not exist

6. Evaluate the limit, if it exists. Select the correct answer.

$$\lim_{h \to 0} \frac{(x-h)^6 - x^6}{h}$$

d. 
$$-6x^5$$

7. Sketch the graph of the function f and evaluate  $\lim_{x \to -1^+} f(x)$ .

$$f(x) = \begin{cases} x + 1, & \text{if } x \le -1 \\ -2x + 1, & \text{if } x > -1 \end{cases}$$

8. You are given that  $\lim_{x \to a} f(x) = -3$ ,  $\lim_{x \to a} g(x) = -4$ , and  $\lim_{x \to a} h(x) = 2$ . Find the limit  $\lim_{x \to a} \left\{ \left[ h(x) \right]^2 - f(x)g(x) \right\}$ . Select the correct answer.

9. Let  $F(x) = \frac{x-5}{|x-5|}$ . Find the following limits.

$$\lim_{x \to 5^+} F(x), \lim_{x \to 5^-} F(x)$$

10. Find the limit. Select the correct answer.

$$\lim_{x \to 2^{-}} \frac{x^2 - 2x}{x^2 - 4x + 4}$$

- a. oo
- b. 0
- c. 00
- d. -2
- e. 2

\_\_ 11. Find the numbers, if any, where the function  $f(x) = \begin{cases} 3x - 2 & \text{if } x \le 1 \\ 0 & \text{if } x > 1 \end{cases}$  is discontinuous.

Select the correct answer.

- a. -2
- b. 2
- c. 1
- d. 0

12. Let

$$f(x) = \begin{cases} x - 4 & \text{if } x \le 5\\ kx^2 - 24x + 46 & \text{if } x > 5 \end{cases}$$

Find the value of k that will make f continuous on  $(-\infty, \infty)$ .

- 13. Find the interval(s) where  $f(x) = \sqrt{x^2 2x + 3}$  is continuous.
- 14. Find the limit.

$$\lim_{x \to -\infty} \frac{-5x^2 - 9}{4x^2 + 5}$$

Select the correct answer.

- a.  $-\frac{9}{5}$
- b. (
- c.  $\frac{5}{4}$
- d.  $-\frac{5}{4}$

#### 15. Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{9x^3}{\sqrt{4x^6 + 1}}.$$

Select the correct answer.

a. HA 
$$y = \frac{9}{2}$$
, VA  $x = -\frac{1}{2}$ ,  $\frac{1}{2}$ 

b. HA 
$$y = -\frac{9}{2}, \frac{9}{2}, VA$$
 none

c. HA 
$$y = \frac{9}{2}$$
, VA none

d. HA 
$$y = -\frac{9}{2}$$
,  $\frac{9}{2}$ , VA  $x = -\frac{1}{2}$ ,  $\frac{1}{2}$ 

#### 16. Evaluate the limit. Select the correct answer.

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2 + x}$$

- a. 1
- b. 1
- C. OD
- d. π
- e. ()

#### 17. The cost (in dollars) of producing x units of a certain commodity is

$$C(x) = 4,571 + 17x + 0.02x^2.$$

Find the instantaneous rate of change with respect to x when x = 103. (This is called the *marginal cost.*)

# 18. Find the instantaneous rate of change of the function $f(x) = x^3 + 8x$ when x = -2. Select the correct answer.

- a. -16
- b. 20
- c. -22
- d. 8

19. The cost (in dollars) of producing x units of a certain commodity is

$$C(x) = 4,280 + 13x + 0.03x^2$$
.

Find the average rate of change with respect to x when the production level is changed from x = 102 to x = 122.

- 20. If an equation of the tangent line to the curve y = f(x) at the point where a = 2 is y = 7x 3, find f(5) and f'(5). Select the correct answer.
  - a. f(5) = 7

$$f'(5) = 37$$

b. f(5) = 7

$$f'(5) = 32$$

c. f(5) = 37

$$f'(5) = 7$$

d. f(5) = 32

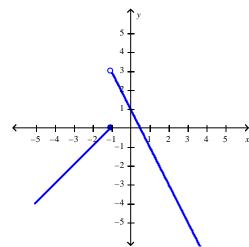
$$f'(5) = 7$$

e. f(5) = 32

$$f'(5) = 32$$

# **Answer Key**

- 1. 0.451
- 2. B
- 3. 64
- 4.  $\frac{1}{2}$
- 5. B
- 6. D
- 7.



- 3
- 8. B
- 9. 1 and -1
- 10. C
- 11. C
- 12. 3
- 13. (-∞, ∞)
- 14. D
- 15. B
- 16. C
- 17. 21.12
- 18. B
- 19. 19.72
- 20. D

1. If a ball is thrown into the air with a velocity of 59 ft/s, its height (in feet) after t seconds is given by

$$H = 59t - 8t^2$$
.

Find the velocity when t = 8.

2. Use the graph of the function to state the value of  $\lim_{x\to 0} f(x)$ , if it exists. Select the correct answer.

$$f(x) = \frac{x^6 + x}{2\sqrt{x^3 + x^6}}$$

a. 
$$-\frac{1}{2}$$

d. 
$$\frac{1}{2}$$

3. Sketch the graph of the function f and evaluate  $\lim_{x\to 1} f(x)$ .

$$f(x) = \begin{cases} x+1, & \text{if } x \le 1 \\ -2x+4, & \text{if } x > 1 \end{cases}$$

4. Find the limit.

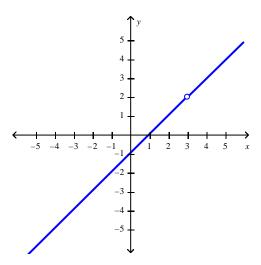
$$\lim_{x \to -\infty} \frac{\sqrt{x^2 - 49}}{5x - 15}$$

- 5. If  $1 \le f(x) \le x^2 + 4x + 4$ , for all x, find  $\lim_{x \to -1} f(x)$ . Select the correct answer.
  - a. 1
  - b.  $-\frac{1}{8}$
  - c.  $-\frac{1}{16}$
  - d. 8
  - e. does not exist
  - 6. Find the limit  $\lim_{x \to 0} \frac{\sqrt{x+6} \sqrt{6}}{x}$ , if it exists.
- 7. Evaluate the limit, if it exists. Select the correct answer.

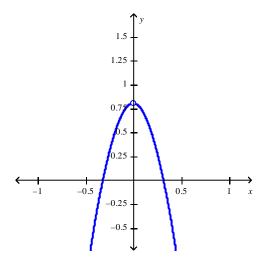
$$\lim_{h \to 0} \frac{(x-h)^3 - x^3}{h}$$

- a. 1
- b.  $3x^2$ c.  $-3x^2$ d. -3
- e. does not exist

8. Use the graph of  $f(x) = \frac{x^2 - 4x + 3}{x - 3}$  to guess at the limit  $\lim_{x \to 3} \frac{x^2 - 4x + 3}{x - 3}$ , if it exists.



9. Use the graph of  $f(x) = \frac{\sin 4x}{\tan 5x}$  to guess at the limit  $\lim_{x \to 0} \frac{\sin 4x}{\tan 5x}$ , if it exists. Select the correct answer.



- a. Does not exist
- b. 0.8
- c. 1.3
- d. 0.2

10. Estimate the value of the limit by graphing the function  $f(x) = \frac{2 \sin x}{\sin \pi x}$ . State your answer correct to two decimal places.

$$\lim_{x \to 0} \frac{2 \sin x}{\sin \pi x}$$

11. Find the limit.

$$\lim_{x\to 2} \sqrt{\frac{4x^2+1}{9x-2}}$$

\_ 12. Which of the given functions is discontinuous? Select the correct answer.

a. 
$$f(x) = \begin{cases} \frac{1}{x-2}, & x \ge 13 \\ \frac{1}{11}, & x < 13 \end{cases}$$

b. 
$$f(x) = \begin{cases} \frac{1}{x - 13}, & x \neq 13 \\ 11, & x = 13 \end{cases}$$

13. For what value of the constant c is the function f continuous on  $(-\infty, \infty)$ ?

$$f(x) = \begin{cases} cx + 9 & \text{for } x \le 2\\ cx^2 - 5 & \text{for } x > 2 \end{cases}$$

14. Find the limit. Select the correct answer.

$$\lim_{x \to \infty} \frac{x^7 - 2}{x^6 + 4}$$

- a. -1/2
- b. 00
- c. 0
- d. 1/2
- e. -oo
- 15. Find the limit.

$$\lim_{x \to -\infty} \frac{4x^2 - 5}{7x^2 + 4}$$

16. Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{5x^3}{\sqrt{9x^6 + 1}}.$$

17. Use the definition of the derivative to find f'(4), where  $f(x) = x^3 - 3x$ .

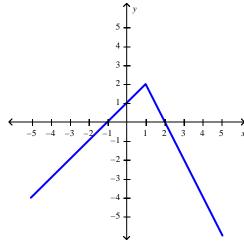
Select the correct answer.

- a. 45
- b. 32
- c. -45
- d. 48
- e. does not exist
- 18. Find an equation of the tangent line to the curve  $y = x^3 6x$  at the point (6, 4).
- 19. The slope of the tangent line to the graph of the exponential function  $y = 8^x$  at the point (0, 1) is  $\lim_{x \to 0} \frac{8^x 1}{x}$ . Estimate the slope to three decimal places.

- \_\_\_\_ 20. Find the tangent line to the parabola  $y = 5x x^2$  at the point (2, 4). Select the correct answer.
  - a. y = x + 2
  - b.  $y = -\sqrt{6}x 3$
  - c.  $y = \frac{\sqrt{3}}{3}x 4$
  - d.  $y = \sqrt{3}x 4$
  - e. None of these

### **Answer Key**

- 1. -69 ft/s
- 2. C
- 3.



- 2
- 4.  $\frac{1}{4}$
- 5. A
- 6.  $\frac{\sqrt{6}}{12}$
- 7. C
- 8. Does not exist
- 9. B
- 10. 0.64
- 11.  $\frac{\sqrt{17}}{4}$
- 12. B
- 13. c = 7
- 14. B
- 15.  $\frac{4}{7}$
- 16. HA  $y = -\frac{5}{3}$ ,  $\frac{5}{3}$ , VA none
- 17. A
- 18. None of these
- 19. 2.079
- 20. A

1. Estimate the value of the following limit by graphing the function  $f(x) = \frac{(5 \sin x)}{(\sin \pi x)}$ .

$$\lim_{x \to 0} \frac{5 \sin x}{\sin \pi x}$$

Round your answer correct to two decimal places.

2. Use the graph of the function to state the value of  $\lim_{x\to 0} f(x)$ , if it exists.

$$f(x) = \frac{1}{1 + 4^{1/x}}$$

\_ 3. Use the graph of the function to state the value of  $\lim_{x \to 0} f(x)$ , if it exists. Select the correct answer.

$$f(x) = \frac{x^2 + x}{2\sqrt{x^3 + x^2}}$$

- a.  $\frac{1}{2}$
- b. does not exist
- C. -00
- d. oo
- e.  $-\frac{1}{2}$
- 4. Find the value of  $\lim_{x\to 0^+} f(x)$ . Select the correct answer.

$$f(x) = \frac{1}{1 + 6^{1/x}}.$$

- a. 0
- b. -0.7
- c. -0.7
- d. -0.6
- e. 0.16

5. Find the limit by evaluating the derivative of a suitable function at an appropriate value of *x*. (Hint: Use the definition of the derivative.)

$$\lim_{h \to 0} \frac{2(5+h)^2 + 5(5+h) - 75}{h}$$

- 6. Is there a number a such that  $\lim_{x \to -3} \frac{3x^2 + ax + a + 3}{x^2 + x 6}$  exists? If so, find the value of a and the value of the limit.
- 7. Sketch the graph of the function f and evaluate  $\lim_{x\to 3} f(x)$ .

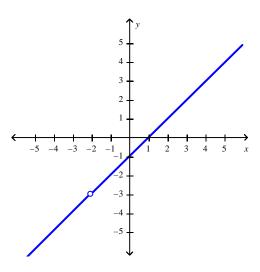
$$f(x) = \begin{cases} x - 4, & \text{if } x \le 3\\ -2x + 5, & \text{if } x > 3 \end{cases}$$

8. Sketch the graph of the function f and evaluate  $\lim_{x \to -1^+} f(x)$ .

$$f(x) = \begin{cases} x+1, & \text{if } x \le -1 \\ -2x+1, & \text{if } x > -1 \end{cases}$$

9. Use the graph of  $f(x) = \frac{x^2 + x - 2}{x + 2}$  to guess at the limit  $\lim_{x \to -2} \frac{x^2 + x - 2}{x + 2}$ , if it exists.

Select the correct answer.



- a. -2
- b. -1
- c. Does not exist
- d. -3
- 10. If f and g are continuous functions with f(9) = 6 and  $\lim_{x \to 9} [2f(x) g(x)] = 9$ , find g(9).
- 11. Let P(x) and Q(x) be polynomials.

Find  $\lim_{x \to \infty} \frac{P(x)}{Q(x)}$  if the degree of P(x) is 6 and the degree of Q(x) is 8.

12. A skydiver leaps from a helicopter hovering high above the ground. Her velocity *t* sec later and before deploying her parachute is given by

$$v(t) = 48 \left[ 1 - (0.82)^t \right]$$

where v(t) is measured in meters per second. What is her terminal velocity? Hint: Evaluate  $\lim_{t \to \infty} v(t)$ .

13. Evaluate the limit. Select the correct answer.

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2 + x}$$

- a. 1
- b. 1
- C. O
- d.  $\pi$
- e. 0
- 14. Let  $F(x) = \frac{x^3 1}{|x 1|}$ . Find the following limits.

$$\lim_{x \to 1^+} F(x), \quad \lim_{x \to 1^-} F(x)$$

- 15. Use the definition of the derivative to find f'(3), where  $f(x) = x^3 2x$ .
- 16. Find an equation of the tangent line to the parabola  $y = 5x^3$  at the point (-5, -145).

Select the correct answer.

a. 
$$y = 375x - 1730$$

b. 
$$y = 395x - 1730$$

c. 
$$y = 375x + 1730$$

d. 
$$y = 395x + 1730$$

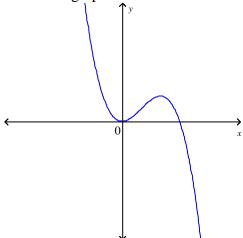
e. 
$$y = 395x - 1730$$

- 17. If an equation of the tangent line to the curve y = f(x) at the point where a = 2 is y = 7x 3, find f(5) and f'(5).
- 18. At what point is the function f(x) = |3 x| not differentiable. Select the correct answer.

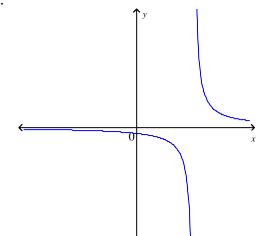
d. 
$$-1$$

19. If  $g(x) = \sqrt{2-3x}$ , use the definition of derivative to find g'(x).

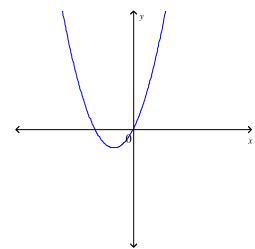
20. Match the graph of the function with the graph of its derivative.



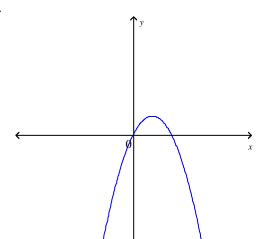
a.



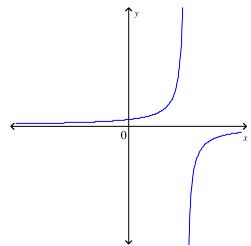
c.



b.

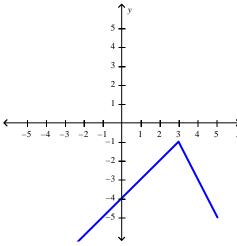


d.

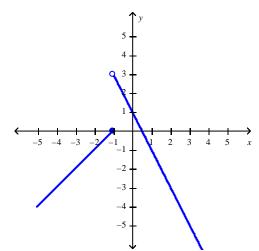


## **Answer Key**

- 1. 1.59
- 2. does not exist
- 3. B
- 4. A
- 5. 25
- 6. a = 15, limit equals 0.6
- 7.



- -1
- 8.



- 3
- 9. C
- 10. g(9) = 3
- 11. 0
- 12. 48 m/sec
- 13. C
- 14. 3 and -3
- 15. 25

17. 
$$f(5) = 32$$

$$f'(5) = 7$$

19. 
$$g'(x) = -\frac{3}{2}(2-3x)^{-1/2}$$

1. If a rock is thrown upward on the planet Mars with a velocity of 12 m/s, its height in meters t seconds later is given by

$$y = 12t - 1.92t^2$$
.

Find the average velocity over the time interval [2, 3].

2. Use the graph of the function to state the value of  $\lim_{x\to 0} f(x)$ , if it exists.

$$f(x) = \frac{1}{1 + 4^{1/x}}$$

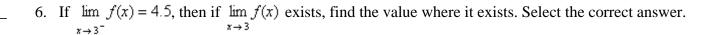
3. Sketch the graph of the function f and evaluate  $\lim_{x \to -3^+} f(x)$ .

$$f(x) = \begin{cases} x+5, & \text{if } x \le -3\\ -2x-1, & \text{if } x > -3 \end{cases}$$

- 4. Find the limit  $\lim_{h \to 3} (h^4 3h^3 4h + 5)$ . Select the correct answer.
  - a. 7
  - b. 3
  - c. 5
  - d. -7
- 5. Find the limit by evaluating the derivative of a suitable function at an appropriate value of *x*. (Hint: Use the definition of the derivative.) Select the correct answer.

$$\lim_{h \to 0} \frac{2(5+h)^2 + 5(5+h) - 75}{h}$$

- a. 25
- b. 75
- c. 0
- d. oo



7. Sketch the graph of the function 
$$f$$
 and evaluate  $\lim_{x \to 3} f(x)$ .

$$f(x) = \begin{cases} x - 4, & \text{if } x \le 3 \\ -2x + 5, & \text{if } x > 3 \end{cases}$$

8. Sketch the graph of the function 
$$f$$
 and evaluate  $\lim_{x \to -1^+} f(x)$ .

$$f(x) = \begin{cases} x+1, & \text{if } x \le -1 \\ -2x+1, & \text{if } x > -1 \end{cases}$$

9. The symbol 
$$\lfloor \rfloor$$
 can be used to denote the greatest integer function, which is defined by  $\lfloor x \rfloor =$  the greatest integer  $n$  such that  $n \leq x$ . Use the graph of the function to find the indicated limit, if it exists. Select the correct answer.

$$\lim_{x \to 0.7} \lfloor x \rfloor$$

c. 
$$-0.7$$

10. You are given that 
$$\lim_{x \to a} f(x) = -3$$
,  $\lim_{x \to a} g(x) = -4$ , and  $\lim_{x \to a} h(x) = 2$ . Find the limit

$$\lim_{x \to a} \left\{ \left[ h(x) \right]^2 - f(x)g(x) \right\}.$$
 Select the correct answer.

11. You are given  $\lim_{x \to a} f(x) = L$  and a tolerance  $\varepsilon$ . Find a number  $\delta$  such that  $|f(x) - L| < \varepsilon$ whenever  $0 < |x - a| < \delta$ .

$$\lim_{x \to 3} 4x = 12; \quad \varepsilon = 0.01$$

12. How would you define f(7) in order to make f continuous at 7? Select the correct answer.

$$f(x) = \frac{x^2 - 2x - 3}{x - 7}$$

- a. f(3) = -8

- b. f(7) = 12c. f(7) = 0d. f(7) = -12
- e. None of these
- 13. Find the numbers, if any, where the function  $f(x) = \begin{cases} 3x 2 & \text{if } x \le 1 \\ 0 & \text{if } x > 1 \end{cases}$  is discontinuous.

Select the correct answer.

- a. -2
- b. 2
- c. 1
- d. 0
- 14. Define the function  $f(x) = \frac{6x^3 + x}{4x}$  at 0 so as to make it continuous at 0.
- 15. Find the limit. Select the correct answer.

$$\lim_{x \to \infty} \frac{x^7 - 6}{x^6 + 12}$$

- a. -00
- b. 0
- C. 00
- d. 1/6
- e. -1/6

16. Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{9x^3}{\sqrt{4x^6 + 1}}.$$

- 17. Use the definition of the derivative to find f'(3), where  $f(x) = x^3 2x$ .
- 18. If the tangent line to y = f(x) at (8, 4) passes through the point (4, -32), find f'(8).

Select the correct answer.

a. 
$$f'(8) = 29$$

b. 
$$f'(8) = 19$$

c. 
$$f'(8) = 9$$

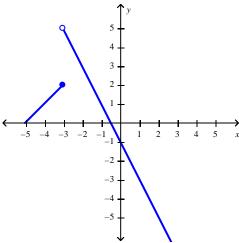
d. 
$$f'(8) = 34$$

e. 
$$f'(8) = -9$$

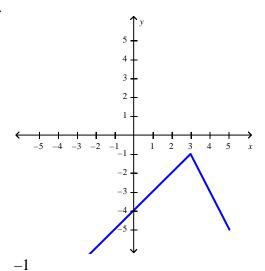
- 19. Find the slope of the tangent line to the curve  $y = 5x^2$  at the point (-4, 22).
- 20. Find the instantaneous rate of change of the function  $f(x) = x^3 + 8x$  when x = -2.

## **Answer Key**

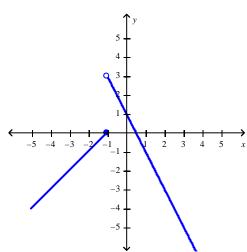
- 1. 2.4 m/s
- 2. does not exist
- 3.



- 5
- 4. D
- 5. A
- 6. C
- 7.



8.



- 3
- 9. A
- 10. B
- 11. 0.0025
- 12. B
- 13. C

14. 
$$f(0) = \frac{1}{4}$$

15. C

16. HA 
$$y = -\frac{9}{2}$$
,  $\frac{9}{2}$ , VA none

- 17. 25
- 18. C
- 19. -40
- 20. 20