CHAPTER 1 FUNCTIONS

The opening chapter of the text focuses on functions: their properties, their graphs, and their use in applications. It can also be viewed as an overview of the prerequisite knowledge from algebra and trigonometry that is necessary for success in a calculus course. Additional review material appears in Appendix A. Some departments skip this material and begin the calculus curriculum with Chapter 2; if that describes your situation, you may refer your students to this chapter and to Appendix A when algebra and trigonometry difficulties arise. For those instructors who provide a review of basic skills, a daunting task lies ahead of you. In one or two weeks, you must race through enough material to fill a term-long course. Plan accordingly, and recognize that you won't likely be able to cover everything. Rely on your students to fill in the details.

Section 1.1 Review of Functions

Overview

A function is defined and its properties are developed.

Lecture Support Notes

It's important to set the right tone at the beginning of a semester. Experience shows that when you set the bar high, your class will jump higher as a result. Assign a good deal of homework from these initial sections to ensure that your students get a thorough review of the prerequisites necessary for calculus, and so that they get used to devoting plenty of time to the study of calculus outside of class.

This chapter is also an ideal time to think about an early diagnostic quiz, assuming your department does not have an entrance exam for the class already in place. Providing immediate feedback for your students helps them to determine whether they are ready for the rigors of calculus, or whether they need to drop the course before your institution's deadline. It's an old story, but one that calculus teachers share often: The main reason for failure in calculus usually stems from difficulties in algebra and trigonometry. Try to develop a quiz—which could be given in the first week—in order to gather data for correlations between your students' scores on the quiz and their final grade in the course. Over time, you will be able to spot insurmountable deficiencies, or problem areas where your students need help, and you can address the challenges accordingly.

- Cover the definition of a function, its geometric interpretation (the vertical line test), and the concepts of domain and range (both the domain of definition and the domain in the context of an application).
- Review composition of functions. Composite functions are featured prominently in calculus, so cover all the bases (Examples 4–7).
- Focus on the idea of a difference quotient (examples 8 and 9), and be sure your students can simplify expressions such as $\frac{f(x+h)-f(x)}{h}$ and $\frac{f(x)-f(a)}{x-a}$.
- Explain the notion of symmetry in graphs, and give definitions of even and odd functions.

Interactive Figures

- Figures 1.4–1.6 display the domain and range of three functions.
- Figure 1.7 illustrates the distinction between the vertical trajectory of a stone thrown upward and the graph of its height as a function of time.
- Figure 1.13 illustrates the ideas of symmetry about the y-axis, x-axis, and origin.
- Figures 1.14–1.16 display symmetric and non-symmetric functions.

Connections

- Exercises 57–70 and Exercises 89–92 ask students to compute the difference quotients $\frac{f(x)-f(a)}{x-a}$ and $\frac{f(x+h)-f(x)}{h}$ for various functions; these exercises will prepare students for upcoming limit and derivative computations.
- The notion of symmetry is used when graphing functions in Cartesian coordinates (Section 4.3) and polar coordinates (Section 12.2), and when evaluating definite integrals (Section 5.4).

Additional Activities

Suggested Guided Projects: *Problem-solving skills* and *Constant-rate problems*

- *Problem-solving skills* is a guided project that can be used in a variety of ways.
- *Constant-rate problems* continues the theme of problem solving. Assign a few of their brain teasers to develop critical thinking skills, and to introduce students to Pólya's four-step method to problem solving. The exercises in these guided projects could also be used as icebreakers in the initial days of class, as they are sure to generate discussion. Finally, you could help your students get to know one another by asking them to work on a handful of exercises in groups of 2–4 students.

Name: ______________________ Section: ______________ Date: ____________

Section 1.1 Quick Quiz

Answer the following multiple choice questions by circling the correct response.

1. The function $f(x)=x^3-x$ is defined for

(a) $\{x : x \geq 0\}$. (b) $\{x : x < 0\}$. (c) $\{x : -\infty < x < \infty\}$.

2. The range of $y = f(x) = x^3 - x$ is

(a) $\{y : y \geq 1\}$. (b) $\{y : -\infty < y < \infty\}$. (c) $\{y : y < 0\}$.

3. The function $f(x)=\sqrt{9-x^2}$ is defined for

(a) $\{x : |x| > 3\}$. (b) $\{x : |x| < 3\}$. (c) $\{x : |x| \leq 3\}$.

4. The graph of the function $f(x) = -3x+8$ is

(a) a line with slope 8 and y-intercept (0, –3).
(b) a line with slope 3 and y-intercept (0, 8).
(c) a line with slope –3 and y-intercept (0, 8).

5. Suppose the height of a soccer ball that is kicked from the ground at time $t = 0$ is $h(t) = -5t^2 + 60t$ (in feet). An appropriate domain for this problem is

(a) $\{t : 0 \leq t \leq 12\}$. (b) $\{t : 0 \leq t \leq 6\}$. (c) $\{t : -\infty < t < \infty\}$.

6. If $f(x)=\sqrt{x}$ and $g(x)=1/(x+1)$, then $f(g(x))$ is

(a) $\dfrac{1}{\sqrt{x+1}}$. (b) $\dfrac{1}{\sqrt{x}+1}$. (c) $\sqrt{x+1}$.

7. If $f(x)=x^3-x$ and $g(x)=x^{-2}$, then $g(f(x))$ is

(a) $x^{-6}-x^{-2}$. (b) $(x^3-x)^{-2}$. (c) x^6-x^2.

8. With $f(x)=\sqrt{x}$ and $g(x)=4-x^2$, the function $f \circ g$ is defined for

(a) all real numbers. (b) $\{x : |x| \geq 2\}$. (c) $\{x : |x| \leq 2\}$.

9. Suppose $f(x)=3x^2-2$. When simplified, the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ is equal to

(a) $6x+3h-4$. (b) 1. (c) $6x+3h$.

10. Suppose $f(x) = -\dfrac{3}{x}$. When simplified, the difference quotient $\dfrac{f(x)-f(a)}{x-a}$ is equal to

(a) 0. (b) $\dfrac{3}{ax}$. (c) –3.

11. The function $h(x)=x^4-3x+1$

(a) is even. (b) is odd. (c) has no symmetry.

12. The curve described by the equation $x^2 - y^4 = 1$ is symmetric about the

(a) x-axis only. (b) y-axis only. (c) origin.

Section 1.2 Representing Functions

Overview

We introduce the full catalog of functions that will be encountered in calculus, and present four ways to represent a function: through formulas, graphs, tables, and words.

Lecture Support Notes

- Run through the gamut of standard functions and provide representative graphs for each family of functions. This section covers only the graphs of polynomial, algebraic, and rational functions—trigonometric functions are featured in the following section—and we keep things simple at this stage (e.g. power functions instead of polynomial functions). Though we introduce exponential and logarithmic functions in this section, they are not encountered again until Chapter 7.
- Cover piecewise functions, which are used repeatedly in the next chapter (limits). Include a piecewise definition of the absolute value function, another fact that will be used frequently in upcoming material.
- Let your students know where you stand regarding the question of technology and graphing: To what degree will they need to be able to produce graphs by hand? Whatever your policy is, it's probably best to ask your students to become familiar with at least the basic shapes of the standard functions.
- Reviewing transformations of graphs is important, as this topic gives students the tools needed to quickly visualize more complicated functions.
- Examples 6 and 7 provide a preview of the derivative and integral. Although the main goal of the chapter is to review algebra and trigonometry, these examples give an early introduction to slope functions and area functions.

Interactive Figures

- Figures 1.24–1.25 display $y = x^n$ for n even and n odd, respectively.
- Figures 1.26–1.27 display $y = x^{1/n}$ for n even and n odd, respectively.
- Figure 1.28 zooms in and out on a particular rational function.
- Figure 1.32 illustrates how the graph of g, which is a slope function for the function f, is related to the graph of f.
- Figures 1.35–1.36 illustrate how an area function A is generated by a given function f.
- Figures 1.37–1.42 display shifts and scalings in the x- and y-directions. Figures 1.43 and 1.44 display similar shifts and scalings in a parabola and absolute value function, respectively.

Connections

- Examples 6 and 7, and their related Exercises 35–42, are included as prelude to big ideas on the horizon. Exercises 78–79 and 83–85 are also designed with future chapters in mind.
- The theme of investigating the properties of the standard families of functions (polynomials, rational functions, algebraic functions, etc.) continues throughout the text. We first learn how to differentiate each family of functions (Chapter 3), and then later learn how to integrate each family (Section 4.9, Chapter 5, and Chapter 8). This program is repeated in multivariable calculus (in fact, several times over, as we learn how to differentiate and integrate vector-valued functions, followed by multivariable functions, and finally vector fields).

Additional Activities

Suggested Guided Projects: *Functions in action I, Functions in action II* and *Supply and demand*

- *Functions in action I and II* are guided projects that explore the behavior of functions in an applied setting. Each mini-project provides a detailed and compelling look into how functions are used to model various phenomena in a variety of disciplines (biology, meteorology, physics, economics, and general interest—see *Chasing a dog*—to name a few).
- *Supply and demand* is another guided project along these lines that is devoted to illustrating principles in economics with graphs.

Name: ______________________ Section: ______________ Date: ____________

Section 1.2 Quick Quiz

Answer the following multiple choice questions by circling the correct response.

1. The function $f(x)=\dfrac{x+1}{x-2}$ is a

(a) polynomial. (b) rational function. (c) transcendental function.

2. The function $f(x)=2x^{10}-3x^2$ is a

(a) polynomial. (b) trigonometric function. (c) transcendental function.

3. The function $f(x)=\sqrt{x-1}-3x^{-2}$ is

(a) a polynomial. (b) an algebraic function. (c) a rational function.

4. The graph of $f(x)=2x^5$

(a) lies in the first and second quadrants.

(b) lies in the second and fourth quadrants.

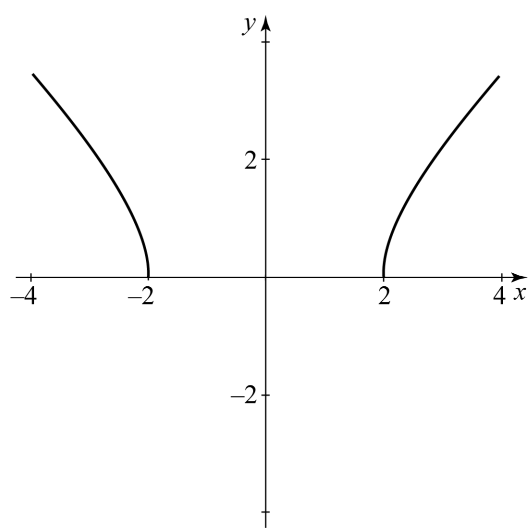
(c) has a point for every real number *x*.

5. The graph to the right best represents the function

(a) $f(x)=\sqrt{4-x^2}$.

(b) $f(x)=\dfrac{1}{x^2-4}$.

(c) $f(x)=\sqrt{x^2-4}$.



6. The data in the table below is best represented by the function

(a) $f(x)=\sqrt{x}+1$. (b) $f(x)=x^2-3$. (c) $f(x)=x^2-x$.

x	–2	–1	1	3	4	6
f(*x*)	1	–2	–2	6	13	33

7. Suppose your car gets exactly 30 miles per gallon of gasoline and you start driving with 15 gallons in the tank. The number of gallons of gasoline left in the tank after driving x miles is given by the function

(a) $g(x) = 30 - 15x$.

(b) $g(x) = 15 - 30x$.

(c) $g(x) = 15 - \frac{x}{30}$.

8. The value of $f(0)$ for the piecewise linear function $f(x) = \begin{cases} 2x+1 & \text{if } x \le 0 \\ -x & \text{if } x > 0 \end{cases}$ is

(a) 1.

(b) 0.

(c) undefined.

9. The function that gives the slope of $f(x) = \begin{cases} 2x+1 & \text{if } x \le 0 \\ -x & \text{if } x > 0 \end{cases}$ is

(a) $g(x) = \begin{cases} 2 & \text{if } x < 0 \\ 1 & \text{if } x > 0. \end{cases}$

(b) $g(x) = \begin{cases} 2 & \text{if } x < 0 \\ -1 & \text{if } x > 0. \end{cases}$

(c) $g(x) = \begin{cases} 1 & \text{if } x < 0 \\ 0 & \text{if } x > 0. \end{cases}$

10. The graph of $g(x) = (x-2)^2 + 3$ is obtained by shifting the graph of $f(x) = x^2$

(a) left 2 units and up 3 units.

(b) right 2 units and up 3 units.

(c) right 3 units and up 2 units.

11. The graph of $y = f(3x)$ is

(a) the graph of $y = f(x)$ compressed horizontally by a factor of 3.

(b) the graph of $y = f(x)$ stretched horizontally by a factor of 3.

(c) the graph of $y = f(x)$ shifted horizontally by 3 units.

12. The graph of $y = 3(x-2)^4$ is the graph of $y = x^4$

(a) stretched vertically by a factor of 3 and stretched horizontally by a factor of 2.

(b) stretched vertically by a factor of 3 and shifted horizontally 2 units to the right.

(c) stretched vertically by a factor of 2 and compressed horizontally by a factor of 2.

Section 1.3 Trigonometric Functions

Overview

The trigonometric functions are defined, and the graphs and properties of these functions are examined.

Lecture Support Notes

This section includes a lot of information, and some of it will be used only peripherally for the first several chapters of the text. We made the decision to collect the necessary prerequisite facts from trigonometry in this section so that students know where to look when confronted with difficulties in trigonometry; all of it will be used at some point in the book. However, you needn't feel obligated to cover every detail the first time around. Cherry-pick from the major ideas in the following bullet list to construct your lecture.

- Define the six trigonometric functions, using both the right-triangle definition and by treating them as circular functions (both are helpful interpretations in subsequent chapters).
- Ask students to memorize Figure 1.48—unless your teaching style does not require it—and explain how it is used to evaluate the trigonometric functions at standard angles.
- Cover the trigonometric identities assembled on page 29. These are the most frequently used identities in calculus. Additional important trigonometric identities are listed on the front papers of the text.
- Review the graphs of the trigonometric functions, and explain how a trigonometric graph can be translated using the methods of Section 1.2.

Interactive Figures

- Figure 1.45 illustrates how changing the radius *r* and the angle θ impacts the arc length *s*.
- Figure 1.47 illustrates the relationship between the right triangle and circular definitions of the trigonometric functions.
- Figure 1.48 shows the coordinates of the points on the unit circle for all the standard angles.
- Figures 1.49–1.50 show how to evaluate trigonometric functions on angles that lie outside the interval $[0, 2\pi]$.
- Figures 1.51–1.52 display $\sin\theta$, $\csc\theta$, and $\cos\theta$, $\sec\theta$, and $\tan\theta$, $\cot\theta$.
- Figure 1.53 illustrates how the graph of $y = A\sin\left(B(\theta - C)\right) + D$ changes when the parameters *A*, *B*, *C*, and *D* are varied.
- Figure 1.54 illustrates how a daylight function varies over one year.

Connections

- Use Figure 1.48 in later chapters anytime you want to refer to the unit circle for the purposes of evaluating a trigonometric function.
- As noted previously, all the material reviewed in this section (indeed, in the entire chapter) will appear in subsequent chapters of the book, so some level of coverage is essential.

Additional Activities

Suggested Guided Project: *Phase and amplitude* focuses on properties of the sine and cosine functions; in particular, it explains how the sum of two sine or cosine waves of different amplitude combine to produce a single sine (or cosine) wave, which can be expressed in phase-amplitude form.

Name: ______________________________ Section: __________________ Date: _______________

Section 1.3 Quick Quiz

Answer the following multiple choice questions by circling the correct response.

1. The radian measure of an angle corresponding to one-eighth of a circle is

(a) $\pi/8$. (b) $\pi/4$. (c) $\pi/6$.

2. The value of tan $3\pi/4$ is

(a) $-\sqrt{3}$. (b)1. (c) -1.

3. Among the zeros of the function $y = \cos 2\theta$ are

(a) 0, $\pm\pi/2$. (b) $\pm\pi$. (c) $\pm\pi/4$.

4. The function $y = \sec x/2$ is undefined at

(a) $x = \pm\pi$. (b) $x = 0,\ \pi/2$. (c) $x = 0,\ \pm 2\pi$.

5. Among the solutions of $\sin 2x = \cos 2x$ are

(a) $\pi/4$. (b) $\pi/8$. (c) $\pi/3$.

6. The function $f(x) = \cos(\pi x/4)$ has a period of

(a) 4. (b) 8. (c) 16.

7. The maximum value of the function $f(x) = 4\cos x + 1$ is

(a) 4. (b) 6. (c) 5.

8. The function $f(x) = \tan x$ has a period of

(a) 2π. (b) π. (c) $\tan x$ is not a periodic function.

9. The range of $f(x) = \sin x$ is

(a) $[-\pi/2, \pi/2]$. (b) all real numbers. (c) $[-1, 1]$.

10. The domain of $f(x) = \tan x$ is

(a) $\{x : x \neq 2n\pi\}$. (b) all real numbers. (c) $\left\{x : x \neq \dfrac{(2n+1)\pi}{2}\right\}$.

Chapter 1 Key Terms and Concepts

Chapter 1 Review Questions

1. Express the set $\{x : -2 < x \le 8\}$ using interval notation and draw it on the number line.

2. Describe the set $\{x : |x-3| < 4\}$ in words and draw it on the number line.

3. Describe the set $\{(x, y) : x^2 + (y-3)^2 < 4\}$ in words.

4. Use the terms *domain*, *range*, *independent variable*, and *dependent variable* to explain the concept of a function.

5. If $f(x) = x^3$ and $g(x) = \dfrac{1}{x}$, find $f(g(x))$, $g(f(x))$, and $g(g(x))$.

6. Give an example of a polynomial, a rational function, and an algebraic function.

7. What is the domain of the function $f(x) = \sqrt{4-x^2}$?

8. Describe the vertical line test. How does it detect functions?

9. What is a secant line? How is it related to a difference quotient?

10. What is the equation of the line that passes through $P(x_1, y_1)$ with slope m?

11. Explain what is meant by a piecewise linear function and explain how one might arise.

12. Why can't an entire circle be described by a single function?

13. What are the possible solution sets of the equation $x^2 + y^2 + Cx + Dy + E = 0$?

14. Draw a picture to show how a composite function takes a value of x and produces a value of $y = f(g(x))$.

15. Sketch an even function and a curve that is symmetric about the origin.

16. Explain in words how you would modify the graph of $y = f(x)$ to obtain the graph of $y = 3f(x-4) - 4$.

17. Explain how the six trigonometric functions are defined in terms of a point $P(x, y)$ on the unit circle.

18. How is the Pythagorean property for sine and cosine obtained from the Pythagorean theorem for right triangles?

19. What is the period and amplitude of the function $y = 3\sin(\theta / 3)$?

20. What are the domain and range of the cosine function?

21. State the reciprocal identities for the trigonometric functions $\cot\theta$, $\csc\theta$, and $\sec\theta$.

Chapter 1 Test Bank Exercises

1–2. Algebra review Simplify or evaluate the following expressions without a calculator.

1. $(-27)^{4/3}$

2. $16^{-3/4}$

3–4. Solving equations

3. Solve $\dfrac{x^2-1}{x-1}=0$.

4. Solve $2x+3y=4$ and $4x-6y=20$ for x and y .

5–6. Composite functions and the difference quotient Let $f(x)=x^2-4$ and $F(x)=\dfrac{1}{x-3}$. Simplify or evaluate the following expressions.

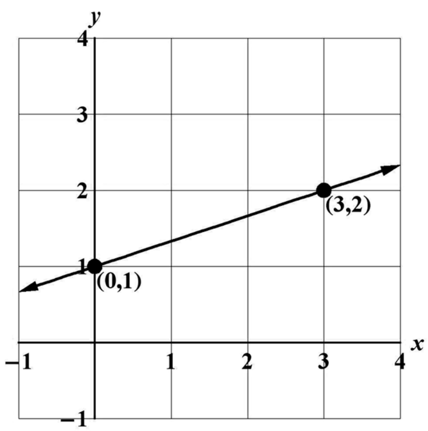
5. $\dfrac{F(x+h)-F(x)}{h}$

6. $F(f(x^2))$

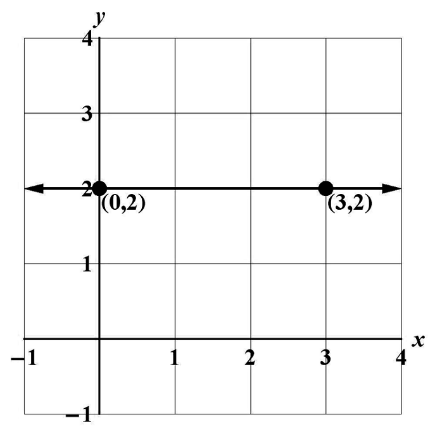
7. More composite functions Let $g(x)=x^2-4$ and $F(x)=\sqrt{x}$. Determine $g \circ F$ and give its domain.

8–9. Graphs of functions Find the linear functions that correspond to the following graphs.

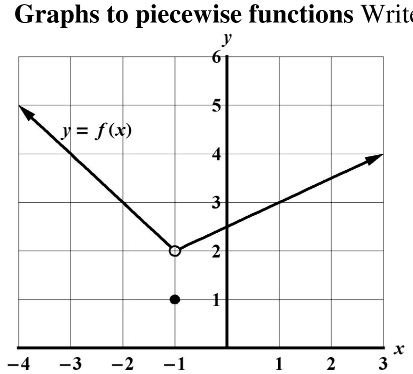
8.



9.



10. Graphs to piecewise functions Write a definition of the following function.



11. Intersection problem Find the point(s) of intersection between the parabola $y = x^2 + 3x - 4$ and the line $y + 4x + 16 = 0$.

12. Shifting and scaling Use analytical methods to graph $v(x) = 2x^2 - 4x + 6$. Then check your work with a graphing utility. Be sure to identify the original function on which the shifts and scalings are performed.

13–15. Symmetry Identify the symmetry in the graphs of the following equations.

13. $y = x^2 - 4$

14. $y = \sin 2x$

15. $x^2 + y^2 = 9$

16–21. Evaluating trigonometric functions Evaluate the following expressions by drawing the unit circle and the appropriate right triangle. All angles are in radians.

16. $\sin \frac{7\pi}{4}$

17. $\cot \frac{7\pi}{3}$

18. $\sin \frac{\pi}{4}$

19. $\cot \frac{\pi}{2}$

20. $\sec \frac{\pi}{4}$

21. $\tan \frac{\pi}{3}$

22. Evaluating trigonometric functions Suppose $\sin\theta = -\frac{2}{3}$, where $\pi < \theta < \frac{3\pi}{2}$. Find $\cos\theta$ and $\tan\theta$. Be sure to sketch a picture of the angle with the terminal side in the proper quadrant.

23–24. Solving trigonometric equations Solve the following equations.

23. $2\cos x + 1 = 0,\ \ 0 \le x < 2\pi$

24. $\sqrt{3}\csc(2\theta) = 2,\ \ 0 \le \theta < 2\pi$

25. Trigonometric identity Find the exact value of $\sin(\pi/8)$.