

Discrete Mathematics: An Introduction to Mathematical Reasoning
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Test Bank Questions

Chapter 1

1. Fill in the blanks to rewrite the following statement with variables: Is there an integer with a remainder of 1 when it is divided by 4 and a remainder of 3 when it is divided by 7?
 - (a) Is there an integer n such that n has _____?
 - (b) Does there exist _____ such that if n is divided by 4 the remainder is 1 and if _____?
2. Fill in the blanks to rewrite the following statement with variables:
Given any positive real number, there is a positive real number that is smaller.
 - (a) Given any positive real number r , there is _____ s such that s is _____.
 - (b) For any _____, _____ such that $s < r$.
3. Rewrite the following statement less formally, without using variables:
There is an integer n such that $1/n$ is also an integer.
4. Fill in the blanks to rewrite the following statement:
For all objects T , if T is a triangle then T has three sides.
 - (a) All triangles _____.
 - (b) Every triangle _____.
 - (c) If an object is a triangle, then it _____.
 - (d) If T _____, then T _____.
 - (e) For all triangles T , _____.
5. Fill in the blanks to rewrite the following statement:
Every real number has an additive inverse.
 - (a) All real numbers _____.
 - (b) For any real number x , there is _____ for x .
 - (c) For all real numbers x , there is real number y such that _____.
6. Fill in the blanks to rewrite the following statement:
There is a positive integer that is less than or equal to every positive integer.
 - (a) There is a positive integer m such that m is _____.
 - (b) There is a _____ such that _____ every positive integer.
 - (c) There is a positive integer m which satisfies the property that given any positive integer n , m is _____.
7. (a) Write in words how to read the following out loud $\{n \in \mathbf{Z} \mid n \text{ is a factor of } 9\}$.
(b) Use the set-roster notation to indicate the elements in the set.

8. (a) Is $\{5\} \in \{1, 3, 5\}$?
 (b) Is $\{5\} \subseteq \{1, 3, 5\}$?
 (c) Is $\{5\} \in \{\{1\}, \{3\}, \{5\}\}$?
 (d) Is $\{5\} \subseteq \{\{1\}, \{3\}, \{5\}\}$?
9. Let $A = \{a, b, c\}$ and $B = \{u, v\}$. Write *a.* $A \times B$ and *b.* $B \times A$.
10. Let $A = \{3, 5, 7\}$ and $B = \{15, 16, 17, 18\}$, and define a relation R from A to B as follows: For all $(x, y) \in A \times B$,
- $$(x, y) \in R \iff \frac{y}{x} \text{ is an integer.}$$
- (a) Is $3 R 15$? Is $3 R 16$? Is $(7, 17) \in R$? Is $(3, 18) \in R$?
 (b) Write R as a set of ordered pairs.
 (c) Write the domain and co-domain of R .
 (d) Draw an arrow diagram for R .
 (e) Is R a function from A to B ? Explain.
11. Define a relation R from $\mathbf{R}$ to $\mathbf{R}$ as follows: For all $(x, y) \in \mathbf{R} \times \mathbf{R}$, $(x, y) \in R$ if, and only if, $x = y^2 + 1$.
- (a) Is $(2, 5) \in R$? Is $(5, 2) \in R$? Is $(-3) R 10$? Is $10 R (-3)$?]
 (b) Draw the graph of R in the Cartesian plane.
 (c) Is R a function from A to B ? Explain.
12. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Define a function $G: A \rightarrow B$ as follows:
- $$G = \{(1, b), (2, c), (3, b), (4, c)\}.$$
- (a) Find $G(2)$.
 (b) Draw an arrow diagram for G .
13. Define functions F and G from $\mathbf{R}$ to $\mathbf{R}$ by the following formulas:
- $$F(x) = (x + 1)(x - 3) \quad \text{and} \quad G(x) = (x - 2)^2 - 7.$$
- Does $F = G$? Explain.

Chapter 2

1. Which of the following is a negation for “Jim is inside and Jan is at the pool.”
- (a) Jim is inside or Jan is not at the pool.
 (b) Jim is inside or Jan is at the pool.
 (c) Jim is not inside or Jan is at the pool.
 (d) Jim is not inside and Jan is not at the pool.
 (e) Jim is not inside or Jan is not at the pool.

2. Which of the following is a negation for “Jim has grown or Joan has shrunk.”
 - (a) Jim has grown or Joan has shrunk.
 - (b) Jim has grown or Joan has not shrunk.
 - (c) Jim has not grown or Joan has not shrunk.
 - (d) Jim has grown and Joan has shrunk.
 - (e) Jim has not grown and Joan has not shrunk.
 - (f) Jim has grown and Joan has not shrunk.
3. Write a negation for each of the following statements:
 - (a) The variable S is undeclared and the data are out of order.
 - (b) The variable S is undeclared or the data are out of order.
 - (c) If Al was with Bob on the first, then Al is innocent.
 - (d) $-5 \leq x < 2$ (where x is a particular real number)
4. Are the following statement forms logically equivalent: $p \vee q \rightarrow p$ and $p \vee (\sim p \wedge q)$? Include a truth table and a few words explaining how the truth table supports your answer.
5. State precisely (but concisely) what it means for two statement forms to be logically equivalent.
6. Write the following two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words explaining how the truth table supports your answer.

If Sam bought it at Crown Books, then Sam didn't pay full price.

Sam bought it at Crown Books or Sam paid full price.

7. Write the following two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words explaining how the truth table supports your answer.

If Sam is out of Schlitz, then Sam is out of beer.

Sam is not out of beer or Sam is not out of Schlitz.
8. Write the converse, inverse, and contrapositive of “If Ann is Jan's mother, then Jose is Jan's cousin.”
9. Write the converse, inverse, and contrapositive of “If Ed is Sue's father, then Liu is Sue's cousin.”
10. Write the converse, inverse, and contrapositive of “If Al is Tom's cousin, then Jim is Tom's grandfather.”
11. Rewrite the following statement in if-then form without using the word “necessary”: Getting an answer of 10 for problem 16 is a necessary condition for solving problem 16 correctly.
12. State precisely (but concisely) what it means for a form of argument to be valid.