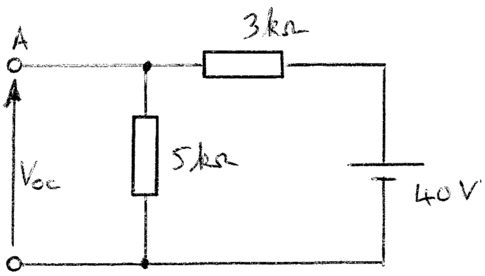
UNIT 64 PRACTICE EXAMINATION QUESTIONS 2

ANSWERS

MARKS

1. (a) **By Thevenin's theorem**

Removing all components in the 2 kΩ branch gives the circuit shown below.



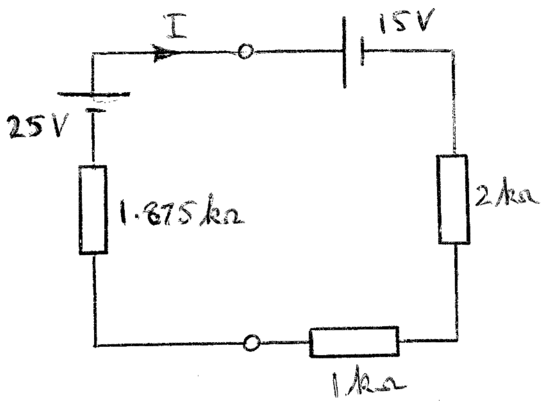
By voltage division, $\mathbf{V_{OC}} = \left(\frac{5}{3+5}\right)(40) = \mathbf{25\ V}$ 3

Removing the voltage source, the resistance 'looking-in' at AB,

$$\mathbf{r} = \frac{3\times 5}{3+5} = \mathbf{1.875\ k\Omega}$$ 2

The equivalent Thevenin circuit is shown below with the 2 kΩ branch

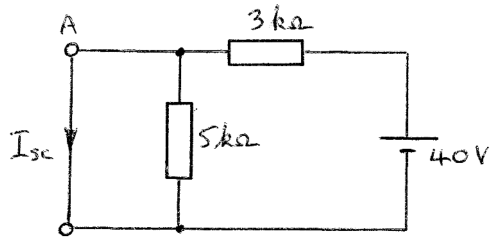
re-connected.

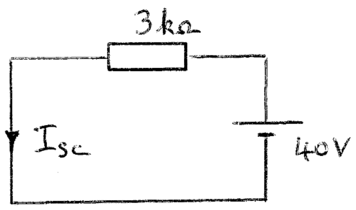


$$\mathbf{I} = \frac{25-15}{(1.875+2+1)\times 10^3} = \mathbf{2.05\ mA}$$ 3

(b) **By Norton's theorem**

Short-circuiting the branch containing the 2 kΩ resistor gives circuit (i) below, which simplifies to circuit (ii)

(i) 

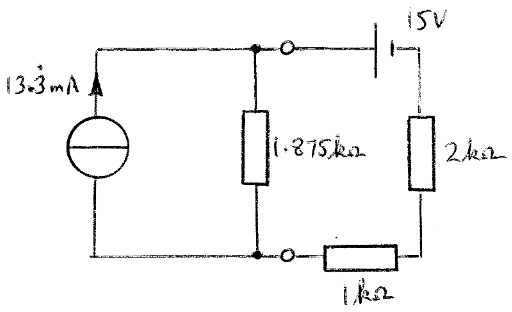
(ii) 

$$\mathbf{I}_{SC} = \frac{40}{3\times10^3} = \mathbf{13.333\ mA}$$ 3

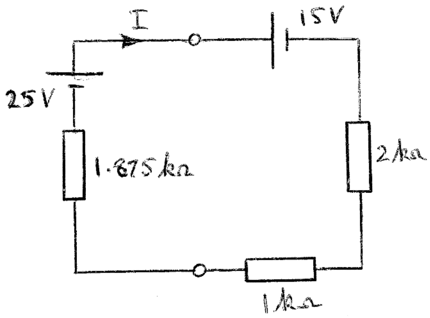
Resistance 'looking-in' at a break in AB with the voltage source removed,

$$\mathbf{r} = \frac{3\times5}{3+5} = \mathbf{1.875\ k\Omega}$$ 2

The equivalent Norton circuit is shown in circuit (i) below, with the 2 kΩ branch re-connected. Circuit (i) is modified to the equivalent Thevenin of circuit (ii).



(i)



(ii)

$$\mathbf{I} = \frac{25-15}{(1.875+2+1)\times10^3} = \mathbf{2.05\ mA}$$ 4

Total: 17

MARKS

2. (a) Current $I = \dfrac{E}{r + R_L}$ and power in the load, $P_L = I^2 R_L = \left(\dfrac{\mathbf{E}}{\mathbf{r + R_L}}\right)^2 \mathbf{R_L}$ 2

(b) $$P_L = \frac{E^2 R_L}{(r + R_L)^2} = \frac{E^2 R_L}{r^2 + 2rR_L + R_L^{\ 2}}$$

Rearranging gives:

$$P_L = \frac{E^2 R_L}{r^2 + 2rR_L + R_L^{\ 2}} = \frac{E^2}{\dfrac{r^2}{R_L} + \dfrac{2rR_L}{R_L} + \dfrac{R_L^{\ 2}}{R_L}} = \frac{E^2}{r^2 R_L^{\ -1} + 2r + R_L}$$

Let $y = r^2 R_L^{\ -1} + 2r + R_L$ then $\dfrac{dy}{dR_L} = -r^2 R_L^{\ -2} + 0 + 1 = 0$ for a turning point

i.e. $1 = r^2 R_L^{\ -2}$ or $1 = \dfrac{r^2}{R_L^{\ 2}}$

from which, $R_L^{\ 2} = r^2$ and $\mathbf{R_L = r}$ 5

$\dfrac{d^2y}{dR_L^{\ 2}} = 2r^2 R_L^{\ -3} = \dfrac{2r^2}{R_L^{\ 3}}$ which is positive, hence a minimum 1

An alternative method of determining the conditions for maximum power transfer is as follows:

Let, say, E = 10V, and r = 3 Ω in the circuit shown in the question.

r	R_L	$P_L = \left(\dfrac{E}{r + R_L}\right)^2 R_L$
3	1	6.25 W
3	2	8.0 W
3	3	8.33 W
3	4	8.16 W
3	5	7.81 W

When $R_L = r = 3\ \Omega$, maximum power is transferred, i.e. for maximum power,

$$\mathbf{R_L = r}$$ (6)

MARKS

(c)(i) Maximum power dissipated in load, $P_L = I_L^2 R_L$

Hence, $80 = I_L^2(5)$

from which, $I_L^2 = \frac{80}{5} = 16$ and $I_L = \sqrt{16} = \mathbf{4\ A}$ 2

(ii) For maximum power transfer, r = R_L = **5 Ω** 1

(iii) Voltage, E = I(r + R_L) = 4(5 + 5) = **40 V** 2

Total: 13

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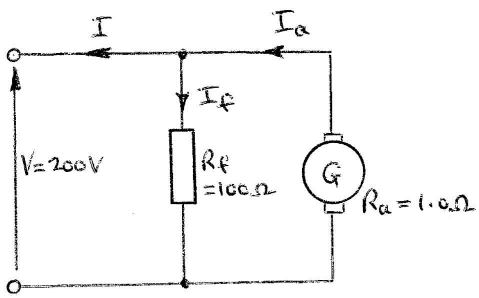
3. (a) See pages 380 4

(b) The circuit is shown below.



Output power = 5000 W = VI

from which, load current, $I = \frac{5000}{200} = 25$ A 1

Field current, $I_f = \frac{V}{R_f} = \frac{200}{100} = 2$ A 1

Armature current, $I_a = I_f + I = 2 + 25 = 27$ A 1

Efficiency, $\eta = \left(\frac{VI}{VI + I_a^2 R + I_f V + C}\right) \times 100\%$

$= \left(\frac{5000}{5000 + (27)^2(1.0) + (2)(200) + 121}\right) \times 100\% = \mathbf{80\%}$ 4

Total: 11

MARKS

4. (a) See page 384-386 4

(b)(i) Back e.m.f. at 100 A, $E_1 = V - I_aR_a = 400 - (100)(0.25)$

$= 400 - 25 = 375$ volts 2

When $I_a = 80$ A, $E_2 = 400 - (80)(0.25 + 0.65)$

$= 400 - (80)(0.90) = 400 - 72 = 328$ volts 2

Now $\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$ i.e. $\frac{375}{328} = \frac{\Phi_1(12)}{\Phi_1 n_2}$ since $\Phi_2 = \Phi_1$

from which, **speed n_2** $= \frac{(12)(328)}{375}$ = **10.50 rev/s** 2

(ii) Back e.m.f. when $I_a = 80$ A, $E_3 = 400 - (80)(0.25)$

$= 400 - 20 = 380$ volts 2

Now $\frac{E_1}{E_3} = \frac{\Phi_1 n_1}{\Phi_3 n_3}$ i.e. $\frac{375}{380} = \frac{\Phi_1(12)}{0.7\Phi_1 n_3}$ since $\Phi_3 = 0.7\,\Phi_1$

from which, **speed n_3** $= \frac{(12)(380)}{(0.7)(375)}$ = **17.37 rev/s** 3

Total: 15

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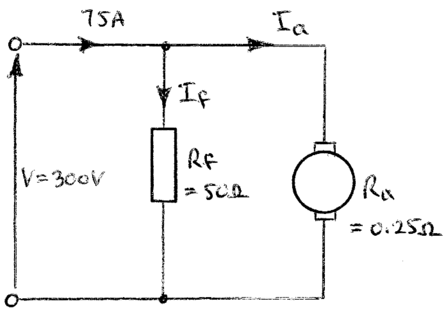
5. The circuit is shown below.



Field current, $I_f = \frac{V}{R_f} = \frac{300}{50} = 6$ A 1

Armature current $I_a = I - I_f = 75 - 6 = 69$ A 1

C = iron, friction and windage losses = 1200 W

MARKS

Efficiency, $\eta = \left(\dfrac{VI - I_a^2R_a - I_fV - C}{VI}\right) \times 100\%$

$= \left(\dfrac{(300)(75) - (69)^2(0.25) - (6)(300) - 1200}{(300)(75)}\right) \times 100\%$

$= \left(\dfrac{18309.75}{22500}\right) \times 100\% =$ **81.38%** 5

Total: 7

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6. (a)(i) $I_{m(charge)} = \dfrac{V}{R} = \dfrac{15}{0.5 \times 10^6} =$ **30 μA** 2

(ii) $V_C = V_m\left(1 - e^{-\frac{t}{\tau}}\right)$ where $\tau = CR = 2\times10^{-6} \times 0.5\times10^6 = 1$ s 1

$= 15\left(1 - e^{-\frac{1.5}{1}}\right) =$ **11.65 V** 2

(iii) $8 = 15\left(1 - e^{-\frac{t}{1}}\right)$ from which, $\dfrac{8}{15} = 1 - e^{-t}$

and $e^{-t} = 1 - \dfrac{8}{15}$ i.e. $\mathbf{t} = -\ln\left(1 - \dfrac{8}{15}\right) =$ **0.762 s** 4

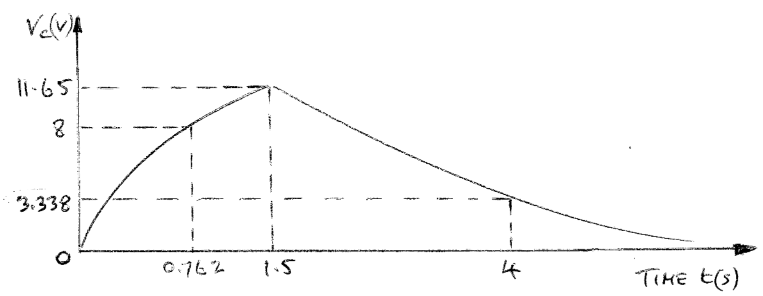
(b)(i) $I_{m(discharge)} = \dfrac{V}{R} = \dfrac{11.65}{1\times10^6} =$ **11.65 μA** 2

(ii) $V_C = V_m e^{-\frac{t}{\tau_{dis}}}$ where $\tau_{dis} = 2\times10^{-6} \times 1\times10^6 = 2$ s 1

$= 11.65\,e^{-\frac{2.5}{2}} = 3.338$ V 2

Hence, energy stored, $\mathbf{W} = \dfrac{1}{2}CV_C^2 = \dfrac{1}{2}\left(2\times10^{-6}\right)\left(3.338\right)^2 =$ **11.14 μJ** 2

(c)



3

Total: 19

MARKS

7. (a) Time constant, $\tau = \frac{L}{R} = \frac{8}{50}$ = **0.16 s** 1

(b) Current after 1 time constant, $i = I_m\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{V}{R}\left(1 - e^{-\frac{\tau}{\tau}}\right) = \frac{100}{50}\left(1 - e^{-1}\right)$

$= \mathbf{1.264\ A}$ 3

(c) Time to develop maximum current = 5 τ = 5 × 0.16 = **0.80 s** 2

(d) When i = 1.4 A, then $1.4 = 2\left(1 - e^{-\frac{t}{\tau}}\right) = 2\left(1 - e^{-\frac{t}{0.16}}\right)$

from which, $\frac{1.4}{2} = 1 - e^{-\frac{t}{0.16}}$ i.e. $e^{-\frac{t}{0.16}} = 1 - \frac{1.4}{2} = 0.3$

i.e. $-\frac{t}{0.16} = \ln 0.3$ and time, **t** = - 0.16 ln 0.3 = **0.193 s** 4

(e) Initial rate of increase = $\frac{I_m}{\tau} = \frac{2}{0.16}$ = **12.5 A/s** 2

Total: 12

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8. (a) **Capacitive reactance, X_C** = $\frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$ = **63.66 Ω** 2

(b) **Impedance, Z** = R - jX_C = 50 - j63.66 = **80.95∠ -51.85° Ω** 2

(c) **Current, I** = $\frac{V}{Z} = \frac{240\angle 0°}{80.95\angle -51.85°}$ = **2.965∠51.85° A** 2

(d) **ϕ = 51.85° leading** 1

(e) $V_R = IR = (2.965\angle 51.85°)(50\angle 0°)$ = **148.25∠51.85° V** 2

(f) $V_C = IX_C = (2.965\angle 51.85°)(63.66\angle -90°)$ = **188.75∠ − 38.15° V** 2

Total: 11

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9. (a) See page 270 4

(b)(i) As resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$

MARKS

$$(2\pi f_r)^2 = \frac{1}{LC} \quad \text{and capacitance, } C = \frac{1}{(2\pi f_r)^2 L}$$

$$= \frac{1}{(2\pi \times 10 \times 10^3)^2 \times 150 \times 10^{-6}} = \mathbf{1.689\ \mu F}$$ 3

(ii) At resonance, Z = R and current, $\mathbf{I} = \frac{V}{R} = \frac{10}{2} = \mathbf{5\ A}$ 2

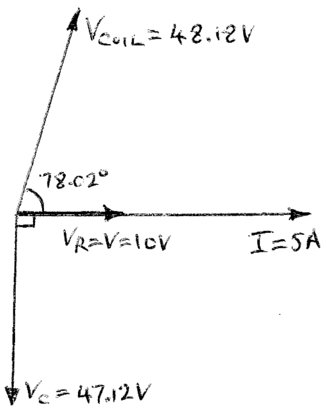
(iii) $\mathbf{V_C} = IX_C = (5)\left(\frac{1}{2\pi \times 10 \times 10^3 \times 1.689 \times 10^{-6}} \angle -90°\right) = \mathbf{47.12\angle{-90°}\ V}$ 2

(iv) $\mathbf{V_{COIL}} = IZ_L = I(R + jX_L) = (5)(2 + j(2\pi \times 10 \times 10^3 \times 150 \times 10^{-6})$

$= (5)(2 + j9.425) = (5)(9.635\angle 78.02°)$

$= \mathbf{48.18\angle 78.02°\ V}$ 3

(v) $\mathbf{P} = VI \cos\phi = (10)(5)\cos 0 = \mathbf{50\ W}$ 2



3

Total: 19

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10. Left-hand branch 1 $\quad X_L = 2\pi(50)(90 \times 10^{-3}) = 28.27\ \Omega$ 1

$Z_1 = (20 + j28.27)\ \Omega = 34.63\angle 54.72°\ \Omega$ 1

$\mathbf{I_1} = \frac{100\angle 0°}{34.63\angle 54.72°} = \mathbf{2.888\angle{-54.72°}\ A}$ 1

Middle branch 2 $\quad \mathbf{I_2} = \frac{V_S}{R} = \frac{100\angle 0°}{20\angle 0°} = \mathbf{5\angle 0°\ A}$ 1

MARKS

Right-hand branch 3 $X_C = \dfrac{1}{2\pi(50)(70\times10^{-6})} = 45.47\ \Omega$ 1

$Z_3 = (30 - j45.47)\Omega = 54.47\angle\text{-}56.58°\ \Omega$ 1

$I_3 = \dfrac{100\angle0°}{54.47\angle\text{-}56.58°}$ = **$1.836\angle56.58°$ A** 1

Total supply current, I_T $= I_1 + I_2 + I_3 = 2.888\angle\text{-}54.72° + 5\angle0° + 1.836\angle56.58°$

$= 7.679 - j0.825 =$ **$7.723\angle\text{-}6.13°$ A** 4

Total circuit impedance, Z_T $= \dfrac{V_S}{I_T} = \dfrac{100\angle0°}{7.723\angle\text{-}6.13°} =$ **$12.95\angle6.13°\ \Omega$** 2

Power, P = VI cos ϕ = (100)(7.723) cos 6.13° = **767.9 W** 2

Total: 15

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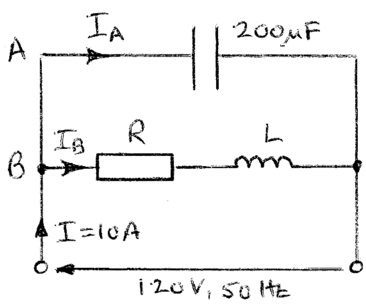
11. (a) $I_A = \dfrac{V}{X_C} = \dfrac{V}{\dfrac{1}{2\pi fC}\angle-90°} = 2\pi fCV\angle90° = 2\pi(50)(200\times10^{-6})(120)\angle90°$

$= 7.540\angle90°$ 2



(i) $\mathbf{I_B} = I - I_A = 10\angle0° - 7.540\angle90° = (10 + j0) - (0 + j7.540) = 10 - j7.540$

= **$12.524\angle\text{-}37.02°$A** 3

(ii) **Power factor** = cos ϕ = cos 37.02° = **0.798 lagging** 1

(iii) **Impedance, Z_B** $= \dfrac{V}{I_B} = \dfrac{120\angle0°}{12.524\angle-37.02°}$ = **$9.582\angle37.02°\ \Omega$** 2

(iv) $Z_B = 9.582\angle37.02°\ \Omega = (7.65 + j5.77)\Omega$ i.e. **resistance, R = 7.65 Ω** 2

MARKS

(b)(i) **Q-factor** $= \frac{X_L}{R} = \frac{5.77}{7.65} = \mathbf{0.754}$ — 2

(ii) $X_L = 2\pi fL = 5.77$ hence, $L = \frac{X_L}{2\pi f} = \frac{5.77}{2\pi(50)} = 18.366$ mH — 1

Dynamic impedance, $\mathbf{Z_D} = \frac{L}{CR} = \frac{18.366\times 10^{-3}}{\left(200\times 10^{-6}\right)(7.65)} = \mathbf{12\ \Omega}$ — 2

Total: 15

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12. (a) **Inductive reactance,** $\mathbf{X_L} = 2\pi fL = 2\pi(50)(0.2) = \mathbf{62.83\ \Omega}$ — 1

(b) **Impedance, Z** $= R + jX_L = (40 + j62.83)\ \Omega$ or $\mathbf{74.48\angle 57.52°\ \Omega}$ — 2

(c) **Current,** $\mathbf{I_{LR}} = \frac{V}{Z} = \frac{500\angle 0°}{74.48\angle 57.52°} = \mathbf{6.71\angle{-57.52°}\ A}$ — 2

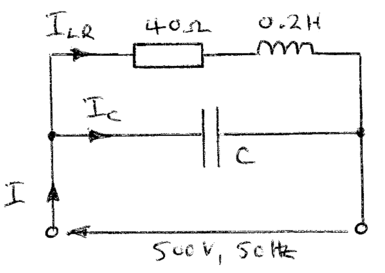
(d) $\boldsymbol{\phi} = \mathbf{57.52°}$ **lagging** — 1

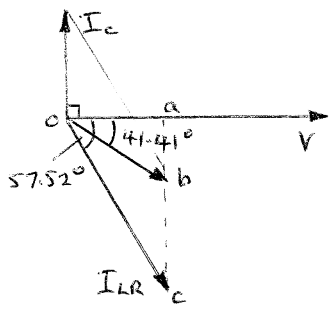
(e) A power factor of 0.75 means $\cos\phi = 0.75$, hence $\phi = \cos^{-1} 0.75 = 41.41°$ — 1

In the phasor diagram below, $\cos 57.52° = \frac{oa}{6.71}$ from which,

$oa = 6.71 \cos 57.52° = 3.603$ A — 1

and $\cos 41.41° = \frac{oa}{ob} = \frac{3.603}{I}$ from which, $I = \frac{3.603}{\cos 41.41°} = 4.804$ A — 1





$I = I_{LR} + I_C$ hence, $I_C = I - I_{LR} = 4.804\angle{-41.41°} - 6.71\angle{-57.52°}$

$= (0 + j2.483)$ A or $2.483\angle 90°$ — 3

$I_C = \frac{V}{X_C} = 2\pi fCV$ from which, $\mathbf{C} = \frac{I_C}{2\pi fV} = \frac{2.483}{2\pi(50)(500)} = \mathbf{15.81\mu F}$ — 3

Total: 15

	MARKS
13. (a) See page 698	2
(b) Comparing with the high-pass section of Figure 17.16(b), page 271, shows that:	
2L = 300 μH, i.e. inductance, L = 150 μH, and capacitance, C = 5 nF = 5×10^{-9}	2
Cut-off frequency, $f_C = \frac{1}{4\pi\sqrt{LC}} = \frac{1}{4\pi\sqrt{(150\times10^{-6}\times5\times10^{-9})}}$ = **91.89 kHz**	3
Nominal impedance, $R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{150\times10^{-6}}{5\times10^{-9}}}$ = **173.2 Ω**	2

Total: 9

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14. (a) See page 700/713	4
(b) The characteristic impedance at zero frequency is the nominal impedance R_0,	
i.e. R_0 = 600 Ω; cut-off frequency f_c = 4 MHz = 4×10^6 Hz.	
Capacitance, $C = \frac{1}{\pi R_0 f_c} = \frac{1}{\pi(600)(4\times10^6)}$ F = **132.6 pF**	2
Inductance, $L = \frac{R_0}{\pi f_c} = \frac{600}{\pi(4\times10^6)}$ H = **47.75 μH**	2
(i) For a **low-pass T-section filter** the **series arm inductances** are each $\frac{L}{2}$	
i.e. $\frac{47.75}{2}$ = **23.87 μH** and the **shunt arm capacitance** is **132.6 pF**	2
(ii) For a **low-pass π-section filter** the **shunt arm capacitances** are each $\frac{C}{2}$	
i.e. $\frac{132.6}{2}$ = **66.3 pF** and the **series arm inductance** is **47.75 μH**	2

Total: 12

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15. (a) See page 348	3

MARKS

(b) $X_L = 2\pi fL = 2\pi(50)(95.49\times10^{-3}) = 30\ \Omega$ 1

$Z_P = \sqrt{(R^2 + X_L^2)} = \sqrt{40^2 + 30^3} = 50\ \Omega$ and $V_p = V_L$ in delta 2

(i) Phase current, $I_p = \frac{V_p}{Z_p} = \frac{V_L}{Z_p} = \frac{400}{50} = \mathbf{8\ A}$ 2

(ii) For a delta connection, $I_L = \sqrt{3}\ I_p = \sqrt{3}\,(8) = \mathbf{13.86\ A}$ 2

Total: 10

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16. (a)(i) Efficiency = $\frac{\text{power output}}{\text{power input}}$, hence $\frac{80}{100} = \frac{14400}{\text{power input}}$

from which, **power input** = $\frac{14400\times100}{80}$ = **18000 W** or **18 kW** 2

(ii) Power, $P = \sqrt{3}\ V_L\, I_L \cos\phi$

hence **line current**, $I_L = \frac{P}{\sqrt{3}(415)(0.75)} = \frac{18000}{\sqrt{3}(415)(0.75)} = \mathbf{33.39\ A}$ 2

(iii) For a delta connection, $I_L = \sqrt{3}\, I_p$

hence **phase current**, $I_p = \frac{I_L}{\sqrt{3}} = \frac{33.39}{\sqrt{3}} = \mathbf{19.28\ A}$ 2

(b)(i) **Total input power, P** = $P_1 + P_2 = 15 + (-4) =$ **11 kW** 2

(ii) $\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right) = \sqrt{3}\left(\frac{15-(-4)}{15+(-4)}\right) = \sqrt{3}\left(\frac{19}{11}\right) = 2.9917$ 2

Angle $\phi = \tan^{-1} 2.9917 = 71.52°$ 1

Power factor = $\cos\phi = \cos 71.52° =$ **0.317** 1

Total: 12

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17. (a) See page 397/398 4

MARKS

(b) Synchronous speed, $n_s = 1500$ rev/min $= \frac{1500}{60}$ rev/s = 25 rev/s | 1

Since $n_s = \frac{f}{p}$ then $p = \frac{f}{n_s} = \frac{50}{25} = 2$ | 1

Hence, **the number of pole pairs is 2** and thus **the number of poles is 4** | 1

Total: 7

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18. (a) See page 407 | 3

(b)(i) f = 60 Hz and $p = \frac{4}{2} = 2$ | 1

Hence, **synchronous speed,** $\mathbf{n_s} = \frac{f}{p} = \frac{60}{2} =$ **30 rev/s** = 60 × 30 = **1800 rev/min** | 2

(ii) Since slip, $s = \left(\frac{n_s - n_r}{n_s}\right) \times 100\%$ i.e. $3 = \left(\frac{30 - n_r}{30}\right) \times 100$

Hence $\frac{3 \times 30}{100} = 30 - n_r$ i.e. $n_r = 30 - \frac{3 \times 30}{100} = 29.1$ rev/s

i.e. the rotor runs at 29.1 × 60 = **1746 rev/min** | 4

Total: 10

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19. (a) See page 407 | 3

(b)(i) Slip, $s = \left(\frac{n_s - n_r}{n_s}\right) \times 100\% = \left(\frac{n_s - 0.30n_s}{n_s}\right) \times 100\% = (0.70)(100) = 70\%$

Input power to rotor = 40 kW

Since $s = \frac{\text{rotor copper loss}}{\text{rotor input}}$ then **rotor copper loss** = (s)(rotor input)

$= \left(\frac{70}{100}\right)(40) =$ **28 kW** | 4

(ii) Power developed by rotor = input power to rotor - rotor copper loss

MARKS

$= 40 - 28 = 12$ kW | 1

Output power of motor = power developed by rotor - friction and windage losses

$= 12 - 1.5 = 10.5$ kW | 1

Efficiency, $\eta = \left(\frac{\text{output power}}{\text{input power}}\right) \times 100\% = \left(\frac{10.5}{42}\right) \times 100\% = \mathbf{25\%}$ | 2

Total: 11

TOTAL MARKS: 240
