Note: This test bank was developed by Joshua Stangle (University of Wisconsin – Superior) to accompany the fourth edition of *Fundamentals of Probability: With Stochastic Processes* by Saeed Ghahramani.

# Chapter 1

(1) The waiting time (in seconds) between one song's end and the next song's beginning on a certain radio station is a random number between 3 and 6.8. Find the probability that the time between one song and another is at least 4.9 seconds.

Answer: $\frac{1.8}{3.8}=.47$

(2) Two fair 20-sided dice are rolled. How many total possible outcomes are there? What is the probability that the second number rolled is exactly 4 more than the first?

Answer: There are 400 total possible outcomes. The probability the second is 4 more than the first is 16/400.

(3) At the Internal Revenue Service, a social security number (9-digits) is selected randomly to receive an audit. (a) What is the event that the last two digits are odd? (b) What is the event that the last two digits form a number divisible by 5?

Answer: (a) $E=\{\sum_{i=0}^{8} a_i \cdot 10^i \mid a_i \in \{0,1,2,\ldots,9\} \text{ for } i=2,\ldots,8, a_j \in \{1,3,5,7,9\} \text{ for } j=0,1\}$.

(b) $E=\{\sum_{i=0}^{8} a_i \cdot 10^i \mid a_i \in \{0,1,2,\ldots,9\} \text{ for } i=1,\ldots,8, a_j \in \{0,5\} \text{ for } j=0\}$.

(4) Let $\Omega=\{a,b,c,d\}$. Choose two distinct 2-element subsets at random from $P(\Omega)$, the set of all subsets of $\Omega$. Find the size of the sample space of this experiment. Describe the following events explicitly:

Sample Space: Let $S$ denote the sample space, then $|S|=30$.

(a) The two sets are disjoint.

Answer: $E=\{\{\{a,b\},\{c,d\}\},\{\{a,c\},\{b,d\}\},\{\{a,d\},\{b,c\}\}\}$

(b) The intersection of the sets is $\{a\}$.

Answer: $F=\{\{\{a,b\},\{a,d\}\},\{\{a,c\},\{a,d\}\},\{\{a,b\},\{a,c\}\}\}$

(c) The complement of the union of the sets consists of only the element $d$.

Answer: $G=\{\{a,b\},\{b,c\}\}$

(5) At a given company there are 4 employees: Jim, Pam Dwight, and Michael. There is a breakroom and each employee can be inside or outside of the breakroom at any given time. Let $A_J$, $A_P$, $A_D$, and $A_M$ denote the event that Jim, Pam, Dwight, and Michael are in the

breakroom, respectively. In terms of the events $A_J, A_P, A_D$, and $A_M$, describe the event that at most two people are in the breakroom at a given time. What about the event that at least three people are in the breakroom?

Answer: The event that at most two people are in the room at the same time is

$$(A_J \cap A_P \cap A_D)^C \cup (A_P \cap A_D \cap A_M)^C \cup (A_J \cap A_P \cap A_M)^C \cup (A_J \cap A_D \cap A_M)^C$$

The event that at least 3 people are in the breakroom is

$$(A_J \cap A_P \cap A_D) \cup (A_P \cap A_D \cap A_M) \cup (A_J \cap A_P \cap A_M) \cup (A_J \cap A_D \cap A_M).$$

(6) George likes antiques. He purchases an antique chair for $200 and plans to refurbish it. Give an explicit sample space for the resale value of his chair after he refurbishes it. Define, in set notation, the event that he loses money on his chair. Suppose George will only charge whole dollar amounts for convenience.

Answer: The sample space for the value of his chair is $S = \{0,1,2,3,\ldots\}$. The event he loses money is $E = \{0,1,\ldots,199\}$.

(7) Suppose we have a sample space $\Omega$ and two events $A$ and $B$. Suppose $A \subset B$. Which statement is false? Show a counterexample for your answer.

(a) $B$ cannot occur unless $A$ has occurred.

(b) $A$ cannot occur unless $B$ has occurred.

Answer: (a) Is false. Let $\Omega = \{1,2,3,4,5\}$, $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$. Then if the experiment turns up 4, $B$ has occurred without $A$ occurring.

(8) At a certain ice cream parlor, 40% of patrons get hot fudge, 25% do not get whipped cream, and 30% of patrons get both hot fudge and whipped cream. How many get hot fudge or whipped cream.

Answer: 35%

(9) A fair (6-sided) die is rolled until the first time two even numbers are rolled in a row. Give 10 elements of the sample space.

Answer: The following is an example: let each tuple represent the result of the rolls beginning at roll one. Then, ten elements of the sample space are

$$(2,4),(2,6),(4,6),(1,2,4),(1,2,6),(1,4,6),(3,2,6),(3,4,6),(3,2,4),(1,3,2,4)$$

(10) A lion, who hunts only gazelle, buffalo, and giraffe, wakes up and hunts her first meal. If she is twice as likely to hunt a gazelle as a buffalo, and three times as likely to hunt a buffalo as a giraffe, find the respective probabilities of the lion hunting a gazelle, buffalo, and giraffe.

Answer: Let $Z, B, G$ denote the events that the lion hunts a gazelle, buffalo, and giraffe, respectively. Then $P(Z) = 2/3$, $P(B) = 2/9$ and $P(G) = 1/9$.

(11) In order to be crowned "Best in Town," John, a tennis player from Newton, MA must beat both Melissa and Jeffrey at tennis. The probability that he beats Melissa is 50% and the probability he beats Jeffrey is 61%. If the probability that he beats at least one of them is 93%, find the probability that John is crowned "Best in Town."

Answer: 18%

(12) Let $S=\{\omega_1,\omega_2,\dots\}$ be the sample space of some experiment. Let $m:S\to R$ be given by $m(\omega_n)=\frac{1}{3^n}$. Can we define a probability distribution $P$ on $S$ so that $P(\{\omega_n\})=m(\omega_n)$?

Answer: No, since for a probability distribution we need $\sum_{\omega\in\Omega}P(\omega)=1$ and

$$\sum_{\omega\in\Omega}P(\omega)=\sum_{n=1}^{\infty}P(\omega_n)=\sum_{n=1}^{\infty}\frac{1}{3^n}=\frac{1/3}{1-1/3}=\frac{1}{2}.$$

(13) Jeff has 3 pens, which he uses to do professional illustrations. He has a red pen, a blue pen, and a green pen. He often works on illustrations outside of home, and it is important that all pens are at least half-full with ink. Suppose a pen can be full, half-full or empty and that the level of ink in a pen is independent of the level of ink in other pens. How large is the sample space of this experiment? Describe the event that all three pens are at least half-full.

Answer: The sample space contains 27 elements. The event that all three pens are at least half-full is

$$E=\{(redhalf, bluehalf, greenhalf),(redfull, bluehalf, greenhalf),(redhalf, bluefull, greenhalf),$$
$$(redhalf, bluehalf, greenfull),(redfull, bluefull, greenhalf),(redfull, bluehalf, greenfull),$$
$$(redhalf, bluefull, greenfull),(redfull, bluefull, greenfull)\}.$$

(14) For two events $A$ and $B$, explain why the following are impossible:

i. $P(AB)=2P(A)$

ii. $P(A\cup B)=P(A)-P(B)$

Answer: (1) is impossible because $AB\subset of\ A$, therefore $P(AB)\le P(A)$. (2) is impossible because $A\subset A\cup B$ therefore $P(A)\le P(A\cup B)$.

(15) A certain auto shop receives 7 autos, each on a random day, in the next seven days. The Smiths drop off all three of their vehicles while the Jones and the Masoods each drop off two vehicles. Write a sample space for the number of days between the day the Smiths drop off a second vehicle and the day they drop off a third vehicle (for instance if they drop off their second vehicle on the third day and their third on the fourth day, one day is between these drop off times.) From this sample space, describe the event that the Jones drop off their two vehicles on the last two days of this period.

Answer: The sample space is given by $S=\{1,2,3,4,5\}$. The event the Jones drop their vehicles off on the last two days is $E=\{1,2,3\}$.

# Chapter 2

(1) An urn contains balls numbered one through 12. A person selects five balls at random and without replacement. What is the probability that they form a consecutive sequence of balls?

Answer: $7/\binom{12}{5}=\frac{7}{792}$

(2) Three fair twenty-sided die are rolled (these have faces labelled 1-20). What is the probability they each show a prime number after rolled?

Answer: $(8)^3/(20)^3$

(3) A hostess wants to seat a party of 9 around a circular table. John and Jennifer cannot sit next to each other. How many ways can she arrange the table?

Answer: 6*(7!)=30240

(4) 15 students in Mrs. Studebacher's class will be arranged randomly into three rows of 5 for a class picture. What is the probability the tallest 5 students occupy the top row? (Assume all the students are distinct heights.)

Answer: $\frac{5!}{15!}$

(5) A deli has 15 turkey and 7 ham sandwiches wrapped in aluminum foil unmarked. If they select 5 sandwiches at random, what is the probability they are all turkey?

Answer: $\left(\frac{15}{22}\right)\left(\frac{14}{21}\right)\left(\frac{13}{20}\right)\left(\frac{12}{19}\right)\left(\frac{11}{18}\right)=11.4\%$.

(6) A certain band consists of 20 players all of whom can play any instrument. They want to divide the players into a trio (group of 3), three quartets (group of 4), and a quintet (group of 5). How many ways can they do this?

Answer: $\frac{20!}{3!(4!)^3 5!}=244432188000$

(7) In a Bridge game, each of the 4 players is dealt 13 cards from a standard deck of cards at random. What is the probability two of the four players each get exactly two aces?

Answer: $\frac{\binom{4}{2}\binom{4}{2}\binom{13}{2}\binom{13}{2}}{\binom{52}{4}}=.81$.

(8) You play a game against a computer, which guesses your favorite Marvel comic book character. You answer a series of 15 yes or no questions, and the computer guesses your character based on the answers. If the computer is always correct, and for each sequence of yes's and no's there is a character, how many possible distinct characters are there?

Answer: $2^{15} = 32768$

(9) Gregory is new to a city and is speed dating. He can choose to go or not to go on a date with any number of 14 different suitors. If he goes on a date, he takes his date either to dinner or to see a movie, but not both. How many different options does Gregory have?

Answer: $3^{14} = 4{,}782{,}969$

(10) A valet who is completely color-blind is rearranging the cars of a parking lot. He has 5 Honda Civics, 4 Ford Fusions, 3 Land Rover LR1s and 3 Mercedes E500s. If the cars of the same model are indistinguishable how many ways can the valet park these in 3 rows of 5?

Answer: $\frac{15!}{5!4!3!3!} = 12{,}612{,}600$

(11) Prove the following binomial relationship:

$$\binom{2n}{n} = \sum_{j=0}^{n} \binom{n}{j}^2 .$$

Hint: Consider an urn containing $n$ red balls and $n$ white balls. Consider the number of ways to select $n$ balls from this urn at one time.

Answer: For the left-hand side this is obviously the number of ways to select $n$ balls from an urn containing $2n$. For the right-hand side we consider the case we select $j$ red balls for $0 \le j \le n$ (which means we must select $n - j$ white balls. We can do this in $\binom{n}{j}\binom{n}{n-j} = \binom{n}{j}^2$ ways. To account for all values of $j$ we sum over $j = 1, \ldots, n$, which is exactly the quantity on the right-hand side.

(12) For a certain setlist, a band wants to pick 3 songs from each of their 3 albums. If one of the albums contains 10 songs, one contains 11 songs, and one contains 12 songs, how many possible setlists are possible?

Answer: $\binom{9}{3}\binom{10}{3}\binom{12}{3}(9!) = 804722688000$.

(13) A company makes 50 deposits to their bank in one month. Four of the deposits are mistakenly written in the company's ledger as \$2 more than actually deposited and three of the deposits are mistakenly recorded in the ledger as \$2 less than was actually deposited. If the CEO checks the book keeping by comparing the sum of 6 random deposit receipts to the sum of their records in the ledger, what is the probability he thinks the ledger is accurate?