Name: ______________________ Class: ______________________ Date: ______________

Chapter 1 - Systems of Linear Equations

1. Determine which of the points (2, –3), (2, 3), and (4, 2) lie on both of the lines $x+2y=8$ and $3x-2y=0$.

ANSWER: (2, 3)

2. Determine which of the points (1, 0, –1, 0), (0, 1, 2, 3), and (2, 1, –1, –1) satisfy the linear system

$$\begin{aligned} x_1 \quad\quad + 2x_3 - x_4 &= 1 \\ 2x_1 - x_2 \quad\quad + 2x_4 &= 5 \end{aligned}$$

ANSWER: (0, 1, 2, 3)

3. Determine which of (3, s_2, s_1, 2), (0, 0, 0 ,0), (2 – s_1 – s_2, 1 + s_1 + s_2, s_1, s_2), and (3 – s_1, s_1, s_2, 2 – s_2) form a solution to the following system for all choices of the free parameters s_1, s_2.

$$\begin{aligned} x_1 + x_2 \quad\quad\quad\quad &= 3 \\ x_1 \quad\quad + x_3 + x_4 &= 2 \end{aligned}$$

ANSWER: (2 – s_1 – s_2, 1 + s_1 + s_2, s_1, s_2)

4. Determine if the system is in echelon form, and if so, identify the leading variables and the free variables. If it is not in echelon form, explain why.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ x_2 - x_3 &= 0 \\ x_1 - 4x_2 \quad\quad &= 0 \end{aligned}$$

ANSWER: Not in echelon form since x_1 is the leading variable of two equations

5. Determine if the system is in echelon form, and if so, identify the leading variables and the free variables. If it is not in echelon form, explain why.

$$\begin{aligned} x_1 + 2x_2 + x_3 \quad\quad &= 1 \\ x_2 - x_3 + 2x_4 &= 0 \\ x_4 &= 3 \end{aligned}$$

ANSWER: Echelon form; leading variables: x_1, x_2, and x_4; free variable: x_3

6. Find all solutions to the system

$$\begin{aligned} 2x_1 + x_2 &= 1 \\ x_1 - x_2 &= 2 \end{aligned}$$

ANSWER: $(1, -1)$

7. Find all solutions to the system

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_2 - x_3 &= 2 \\ -4x_2 + x_3 &= -6 \end{aligned}$$

ANSWER: $(-3, 2, 2)$

Name: ______________________ Class: ______________________ Date: ______________

8. Find all solutions to the system

$$\begin{aligned} x_1 - 3x_2 + x_3 \quad\quad &= 6 \\ 2x_1 \quad\quad + x_3 + 2x_4 &= 2 \end{aligned}$$

ANSWER: $(1-s_1-s_2/2,\ -5/3-s_1/3+s_2/6,\ s_2,\ s_1)$

9. Reorder the equations to put the following system of three equations with four unknowns in echelon form:

$$\begin{aligned} x_2 - x_3 + 2x_4 &= 0 \\ x_3 - x_4 &= 1 \\ x_1 + 2x_2 + x_3 &= 3 \end{aligned}$$

ANSWER:

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ x_2 - x_3 + 2x_4 &= 0 \\ x_3 - x_4 &= 1 \end{aligned}$$

10. Determine the value(s) of k so that the following system is consistent.

$$\begin{aligned} x_1 - 2x_2 &= 1 \\ 2x_1 - 4x_2 &= k \end{aligned}$$

ANSWER: $k=-1/2$

11. Suppose a system of 12 equations has two free variables. How many leading variables are there?

ANSWER: 10

12. **True** or **False**: If a linear system has more variables than equations, then the system is inconsistent.

a. True

b. False

ANSWER: False

13. **True** or **False**: If a linear system has infinitely many solutions, then there are more variables than equations.

a. True

b. False

ANSWER: False

14. **True** or **False**: If a linear system has no free variables, then there exists at most one solution.

a. True

b. False

ANSWER: True

Chapter 1 - Systems of Linear Equations

15. Convert the augmented matrix to the equivalent linear system.

$$\begin{bmatrix} 3 & 2 & -4 \\ -1 & 6 & 5 \end{bmatrix}$$

ANSWER:

$$\begin{aligned} 3x_1 + 2x_2 &= -4 \\ -x_1 + 6x_2 &= 5 \end{aligned}$$

16. Convert the augmented matrix to the equivalent linear system.

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 3 & 1 & 0 \\ 2 & -2 & 0 & 0 & 1 \end{bmatrix}$$

ANSWER:

$$\begin{aligned} x_1 + x_3 - x_4 &= 3 \\ 3x_3 + x_4 &= 0 \\ 2x_1 - 2x_2 &= 1 \end{aligned}$$

17. Determine whether the matrix is in reduced echelon form, echelon form only, or not in echelon form.

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

ANSWER: Not in echelon form

18. Determine whether the matrix is in reduced echelon form, echelon form only, or not in echelon form.

$$\begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 4 & 0 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

ANSWER: Echelon form only

19. Determine whether the matrix is in reduced echelon form, echelon form only, or not in echelon form.

$$\begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

ANSWER: Reduced echelon form

20. Identify the row operation which transforms the matrix on the left to the matrix on the right.

$$\begin{bmatrix} 2 & 6 & 1 \\ -4 & 4 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & 6 & 1 \\ 0 & 16 & 4 \end{bmatrix}$$

ANSWER: $2R_1 + R_2 \to R_2$

21. Identify the row operation which transforms the matrix on the left to the matrix on the right.

Name: ________________ Class: ________________ Date: ________________

Chapter 1 - Systems of Linear Equations

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 4 & 4 \\ 0 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

ANSWER: $-2R_3 + R_2 \to R_2$

22. Convert the given system to an augmented matrix, and find all solutions by reducing to echelon form and using back substitution.

$$\begin{aligned} x_1 + 2x_2 &= -4 \\ -x_1 + 2x_2 &= 1 \end{aligned}$$

ANSWER: $x_1 = -\frac{5}{2}$, $x_2 = -\frac{3}{4}$

23. Convert the given system to an augmented matrix, and find all solutions by reducing to echelon form and using back substitution.

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 1 \\ 4x_1 + x_2 &= 2 \end{aligned}$$

ANSWER: $x_1 = \frac{3}{7} - \frac{1}{7}s_1$, $x_2 = \frac{2}{7} + \frac{4}{7}s_1$, $x_3 = s_1$

24. Convert the given system to an augmented matrix, and find all solutions by reducing to echelon form and using back substitution , if needed.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ x_2 - x_3 &= 0 \\ 2x_1 + 4x_2 &= 0 \end{aligned}$$

ANSWER: $x_1 = -2$, $x_2 = 1$, $x_3 = 1$

25. Convert the given system to an augmented matrix, and find all solutions by transforming to reduced echelon form and using back substitution.

$$\begin{aligned} 3x_1 - 3x_2 + x_3 + 4x_4 &= 10 \\ 2x_1 - 2x_2 + x_3 + 3x_4 &= 8 \end{aligned}$$

ANSWER: $x_1 = 2 - s_1 + s_2$, $x_2 = s_2$, $x_3 = 4 - s_1$, $x_4 = s_1$

26. Convert the given system to an augmented matrix, and find all solutions by transforming to reduced echelon form and using back substitution.

$$\begin{aligned} 2x_2 + x_3 &= 1 \\ x_1 - x_2 - x_3 + x_4 &= 1 \\ 2x_1 + 4x_2 - x_4 &= 1 \end{aligned}$$

Chapter 1 - Systems of Linear Equations

ANSWER: $x_1=\frac{7}{2}-\frac{5}{2}s_1$, $x_2=-\frac{3}{2}+\frac{3}{2}s_1$, $x_3=4-3s_1$, $x_4=s_1$

27. **True** or **False**: Every homogeneous linear system has at least one solution.

a. True

b. False

ANSWER: True

28. **True** or **False**: A linear system with infinitely many solutions must have more variables than equations.

a. True

b. False

ANSWER: False

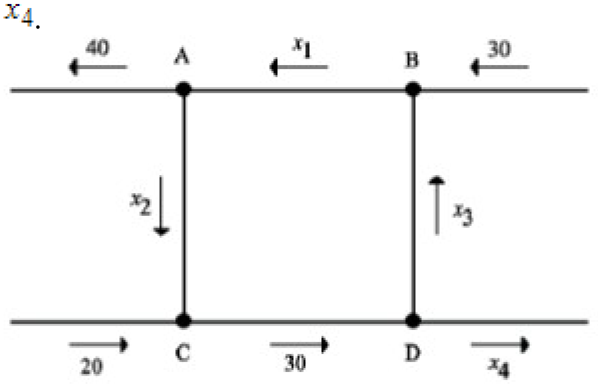
29. **True** or **False**: A linear system with a unique solution can not have more variables than equations.

a. True

b. False

ANSWER: True

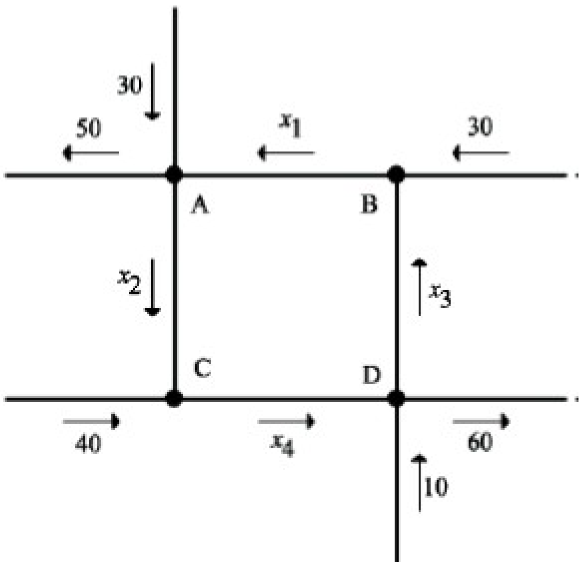
30. The volume of traffic for a collection of intersections is shown. Find all possible values for x_1, x_2, x_3, and x_4.



ANSWER: $x_1=50$, $x_2=10$, $x_3=20$, and $x_4=10$

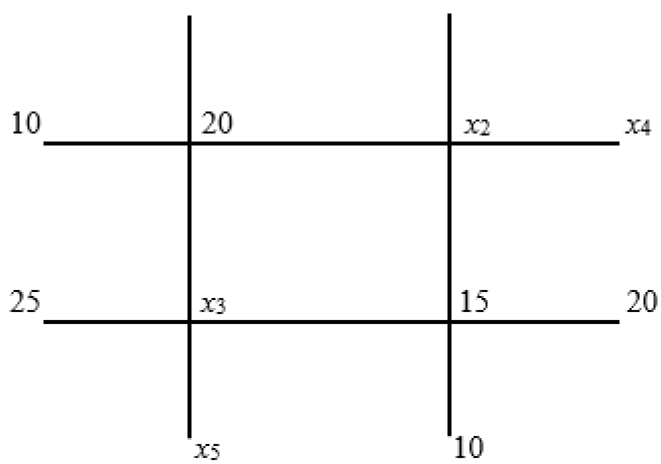
31. The volume of traffic for a collection of intersections is shown. Find the minimum volume of traffic from C to D.

Name: ____________________ Class: ____________________ Date: ____________



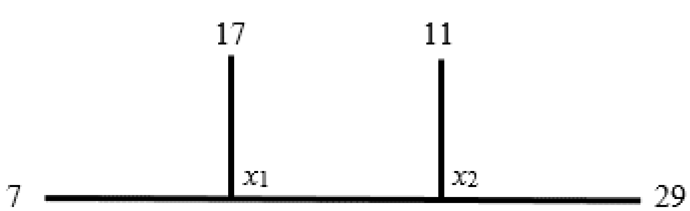
ANSWER: 50

32. Find all possible equilibrium and endpoint temperatures for the heavy wires with endpoints held at the indicated temperatures.



ANSWER: $x_1 = 40, x_2 = 15 - \frac{1}{4}s, x_3 = 15 + \frac{1}{4}s, x_4 = 10 - s, x_5 = s$

33. Find the equilibrium temperatures for the heavy wires with endpoints held at the given temperatures.



ANSWER: $x_1 = 14, x_2 = 18$

34. An economy has three industries: A, B, and C. These industries have annual consumer sales of 45, 37, and 64 (in millions of dollars), respectively. In addition, for every dollar of goods that A sells, A requires 25 cents from B and 15 cents from C. For each dollar of goods that B sells, B requires 35 cents from A and 25 cents from C. For each dollar of goods that C sells, C requires 20 cents from A and 45 cents from B. Let *a*, *b*, *c* be the total output from industries A, B, C, respectively. What values of *a*, *b*, *c* (rounded to the nearest thousand dollars) will satisfy both consumer and between-industry demand?

ANSWER: a = \$105,764,000; *b* = \$111,978,000; *c* = \$107,859,000

35. Balance the given chemical equation.

$CH_4 + O_2 \rightarrow CO_2 + H_2O$

ANSWER: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

36. Balance the given chemical equation.

$Al\,Br_3 + Cl_2 \rightarrow Al\,Cl_3 + Br_2$

ANSWER: $2Al\,Br_3 + 3Cl_2 \rightarrow 2Al\,Cl_3 + 3Br_2$

37. Use a system of linear equations to find the values A and B for the partial fraction decomposition

$$\frac{x-10}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

ANSWER: $A = -3$, $B = 4$

38. Use a system of linear equations to find the values A, B, and C for the partial fraction decomposition

$$\frac{3x^2+4x-6}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

ANSWER: $A = 2$, $B = -2$, $C = 1$

39. The points (–6, 0, –1), (3, 2, 0), and (0, 3, –1) lie on a plane *ax* + *by* + *cz* = 1. Use a system of linear equations to find the equation of this plane.

ANSWER: $-x + 2y + 5z = 1$

40. Use a system of linear equations to find the equation $y = ax^2 + bx + c$ of the parabola which passes through the points (1, 2), (2, 0), and (3, –4).

ANSWER: $y = -x^2 + x + 2$

41. Use a system of linear equations to find a function of the form $f(x) = ax^3 + bx^2 + cx + d$ such that $f(-1) = 3$, $f(0) = -1$, $f(1) = 1$, and $f(2) = -3$.

ANSWER: $f(x) = -2x^3 + 3x^2 + x - 1$

42. Use a system of linear equations to find the values of the coefficients a, b, c if $f(x) = ae^{-x} + be^{2x} + ce^{-2x}$ with $f(0) = 4$, $f'(0) = -9$, and $f''(0) = 7$.

ANSWER: $a = 3$, $b = -1$, and $c = 2$

43. Use partial pivoting with Gaussian elimination to find the solutions to the system.

$$\begin{aligned} x_1 + 4x_2 &= 1 \\ 8x_1 + 4x_2 &= 15 \end{aligned}$$

ANSWER: $x_1 = 2$, $x_2 = -\frac{1}{4}$

44. Use partial pivoting with Gaussian elimination to find the solutions to the system.

$$\begin{aligned} 2x_1 + 4x_2 - x_3 &= 1 \\ 10x_1 + 4x_2 + x_3 &= 0 \\ x_1 \qquad\quad + x_3 &= 1 \end{aligned}$$

ANSWER: $x_1 = -\frac{1}{2}$, $x_2 = \frac{7}{8}$, $x_3 = \frac{3}{2}$

45. Solve the given system using Gaussian elimination with three significant digits of accuracy. Then solve the system again, incorporating partial pivoting.

$$\begin{aligned} x_1 + 250x_2 &= 2 \\ 100x_1 + 2x_2 &= 1 \end{aligned}$$

ANSWER: Gaussian elimination: $x_1 = 0.010$, $x_2 = 0.00796$; partial pivoting: $x_1 = 0.00984$, $x_2 = 0.00796$

46. Solve the given system using Gaussian elimination with three significant digits of accuracy. Then solve the system again, incorporating partial pivoting.

$$\begin{aligned} 2x_1 + 4x_2 - 40x_3 &= 1 \\ x_1 + 4x_2 + 120x_3 &= -1 \\ 180x_1 - 50x_2 + x_3 &= 1 \end{aligned}$$

ANSWER: Gaussian elimination: $x_1 = 0.0320$, $x_2 = 0.111$, $x_3 = -0.0123$;
partial pivoting: $x_1 = 0.0359$, $x_2 = 0.109$, $x_3 = -0.0123$

47. Compute the first three Jacobi iterations, using 0 as the initial value for each variable. Then find the exact solution and compare.

$$\begin{aligned} 6x_1 - x_2 &= 3 \\ x_1 + 4x_2 &= -1 \end{aligned}$$

ANSWER:

n	x_1	x_2
0	0	0
1	0.5	−.025
2	0.458	−0.375
3	0.438	−0.365

Exact solution: $x_1 = \frac{11}{25} = 0.44$, $x_2 = -\frac{9}{25} = -0.36$

48. Compute the first three Jacobi iterations, using 0 as the initial value for each variable. Then find the exact solution and compare.

$$\begin{aligned} 12x_1 + 2x_2 - 3x_3 &= 0 \\ x_1 - 8x_2 + x_3 &= 1 \\ -x_1 + x_2 + 10x_3 &= -1 \end{aligned}$$

ANSWER:

n	x_1	x_2	x_3
0	0	0	0
1	0	−0.125	−0.1
2	−0.00417	−0.138	−0.0875
3	0.00104	−0.136	−0.0867

Exact solution: $x_1 = \frac{1}{973} \approx 0.00103$, $x_2 = -\frac{132}{973} \approx -0.136$, $x_3 = -\frac{12}{139} \approx -0.0863$

49. Compute the first three Gauss-Seidel iterations for the system in question 5, using 0 as the initial value for each variable. Then find the exact solution and compare.

ANSWER:

n	x_1	x_2
0	0	0
1	0.5	−0.375
2	0.438	−0.359
3	0.440	−0.360

Exact solution: $x_1 = \frac{11}{25} = 0.44$, $x_2 = -\frac{9}{25} = -0.36$

50. Compute the first three Gauss-Seidel iterations for the system in question 6, using 0 as the initial value for each variable. Then find the exact solution and compare.

ANSWER:

n	x_1	x_2	x_3
0	0	0	0
1	0	-0.125	-0.0875
2	-0.00104	-0.136	-0.0865
3	0.00105	-0.136	-0.0863

Exact solution: $x_1 = -\frac{1}{973} \approx 0.00103$, $x_2 = -\frac{132}{973} \approx -0.136$, $x_3 = -\frac{12}{139} \approx -0.0863$

51. Determine if the system is diagonally dominant. If not, then if possible rewrite the system so that it is diagonally dominant.

$$\begin{aligned} 2x_1 + 16x_2 - x_3 &= 3 \\ x_1 - 2x_2 + 10x_3 &= 1 \\ -8x_1 + 2x_2 + 2x_3 &= -3 \end{aligned}$$

ANSWER: Not diagonally dominant; interchange rows to obtain the diagonally dominant system

$$\begin{aligned} -8x_1 + 2x_2 + 2x_3 &= -3 \\ 2x_1 + 16x_2 - x_3 &= 3 \\ x_1 - 2x_2 + 10x_3 &= 1 \end{aligned}$$

.

52. Compute the first four Jacobi iterations for the system as written, with the initial value of each variable set equal to 0. Then rewrite the system so that it is diagonally dominant, set the value of each variable to 0, and again compute four Jacobi iterations. Finally, find the exact solution and compare.

$$\begin{aligned} x_1 - 6x_2 &= 1 \\ 5x_1 + 2x_2 &= -2 \end{aligned}$$

ANSWER:

Jacobi iteration:

n	x_1	x_2
0	0	0
1	1	-1
2	-5	-3.5
3	-20	11.5
4	70	49

Diagonally dominant system:

n	x_1	x_2
0	0	0
1	-0.4	-0.167
2	-0.333	-0.233
3	-0.307	-0.222
4	-0.311	-0.218

Exact solution: $x_1 = -\frac{5}{16} = -0.3125$, $x_2 = -\frac{7}{32} = -0.21875$

53. Compute the first four Jacobi iterations for the system as written, with the initial value of each variable set equal to 0. Then rewrite the system so that it is diagonally dominant, set the value of each variable to 0, and again compute four Jacobi iterations. Finally, find the exact solution and compare.

Chapter 1 - Systems of Linear Equations

$$\begin{aligned} x_1 + 6x_2 - x_3 &= 2 \\ x_1 - 4x_2 + 7x_3 &= 1 \\ -8x_1 + x_2 + 3x_3 &= -2 \end{aligned}$$

ANSWER:

Jacobi iteration:

n	x_1	x_2	x_3
0	0	0	0
1	2	−0.25	−0.667
2	2.83	−0.917	4.75
3	12.3	8.77	7.19
4	−43.4	15.4	29.1

Diagonally dominant system:

n	x_1	x_2	x_3
0	0	0	0
1	0.25	0.333	0.143
2	0.345	0.315	0.298
3	0.401	0.325	0.274
4	0.393	0.312	0.272

Exact solution: $x_1 = \frac{7}{18} \approx 0.3889,\ x_2 = \frac{107}{342} \approx 0.3129\ , x_3 = \frac{91}{342} \approx 0.2661$

54. Compute the first four Gauss-Seidel iterations for the system in question 10, with the initial value of each variable set equal to 0. Then rewrite the system so that it is diagonally dominant, set the value of each variable to 0, and again compute four Gauss-Seidel iterations.

ANSWER:

Gauss-Seidel iteration:

n	x_1	x_2
0	0	0
1	1	−3.5
2	−20	49
3	295	−738
4	−4430	11074

Diagonally dominant system:

n	x_1	x_2
0	0	0
1	−0.4	−0.233
2	−0.307	−0.218
3	−0.313	−0.219
4	−0.312	−0.219

55. Compute the first four Gauss-Seidel iterations for the system in question 11, with the initial value of each variable set equal to ^{0}. Then rewrite the system so that it is diagonally dominant, set the value of each variable to 0, and again compute four Gauss-Seidel iterations.

ANSWER:

Gauss-Seidel iteration:

n	x_1	x_2	x_3
0	0	0	0
1	2	0.25	4.58
2	5.08	9.04	9.88
3	−42.4	6.44	−116
4	−152	−241	−327

Diagonally dominant system:

n	x_1	x_2	x_3
0	0	0	0
1	0.25	0.292	0.274
2	0.389	0.314	0.267
3	0.389	0.313	0.266
4	0.389	0.313	0.266

56. The values for the first few Jacobi iterations for a linear system are given. Find the values for the next iteration.

n	x_1	x_2
0	0	0
1	4	2
2	8	5
3	?	?

ANSWER: $x_1=14$, $x_2=8$

57. The values for the first few Gauss-Seidel iterations for a linear system are given. Find the values for the next iteration.

n	x_1	x_2
0	0	0
1	−3	1
2	4	4
3	?	?

ANSWER: $x_1=25$, $x_2=13$