1. To test whether the law of demand holds using calculus, you should:
A) take the partial derivative of quantity demanded Q^D with respect to P and conclude that the law of demand holds if this derivative is positive at the market price.
B) take the partial derivative of quantity demanded Q^D with respect to P and conclude that the law of demand holds if this derivative is negative at the market price.
C) take the derivative of P with respect to quantity demanded Q^D and conclude that the law of demand holds if this derivative is positive at the market price.
D) take the derivative of P with respect to quantity demanded Q^D and conclude that the law of demand holds if this derivative is negative at the market price.

2. Suppose that the market demand curve for sunflowers is a function of the price of sunflowers, the price of roses, and income. If the partial derivative of quantity demanded of sunflowers with respect to the price of roses is negative, sunflowers and roses are:
A) substitutes.
B) complements.
C) normal goods.
D) inferior goods.

3. Suppose that the market demand curve for sunflowers is a function of the price of sunflowers, the price of roses, and income. If the partial derivative of quantity demanded of sunflowers with respect to income is negative:
A) sunflowers are normal goods.
B) roses are normal goods.
C) sunflowers are inferior goods.
D) roses are inferior goods.

4. Suppose that the market demand curve for cauliflower is a function of the price of cauliflower, the price of broccoli, and income. If the partial derivative of quantity demanded of cauliflower with respect to the price of broccoli is positive, cauliflower and broccoli are:
A) substitutes.
B) complements.
C) normal goods.
D) inferior goods.

5. Suppose that watermelon, with price P_W, and barbecue sauce are related goods. The expanded demand curve for barbecue sauce, then, is $Q^D = 860 - 4P - 6P_W$. Suppose that P_W is \$5 per watermelon. Use calculus to determine whether watermelon is complementary or a substitute for barbecue sauce.

6. Suppose that the market demand curve for cauliflower is a function of the price of cauliflower, the price of broccoli, and income. If the partial derivative of quantity demanded of cauliflower with respect to income is positive:

A) neither cauliflower nor broccoli is a normal good.
B) cauliflower and broccoli are both normal goods.
C) broccoli is a normal good.
D) cauliflower is a normal good.

7. Suppose that the extended market demand curve for maple syrup can be expressed as $Q^D = 1{,}200 - 0.5P - 9P_p + 0.01I$, where P_p is the price of pancake batter and I is income.
a. Use calculus to argue whether pancake batter is a substitute or complement to maple syrup.
b. Use calculus to argue whether maple syrup is a normal or an inferior good.

8. Suppose that the market demand and supply curves for chocolate ice cream are represented by the following equations:

$$Q^D = 10{,}000 - 50P$$
$$Q^S = -200 + 40P$$

where Q^D is the quantity demanded, Q^S is the quantity supplied, and P is the price.
a. Show that the law of demand holds using calculus.
b. Show that the law of supply holds using calculus.

9. Suppose that the market demand and supply curves for granola bars are represented by the following equations:

$$Q^D = 7{,}000 - 120P$$
$$Q^S = -50 + 20P$$

where Q^D is the quantity demanded, Q^S is the quantity supplied, and P is the price.
a. Show that the law of demand holds using calculus.
b. Show that the law of supply holds using calculus.

10. Suppose that the extended supply curve for children's books can be expressed as $Q^S = 8{,}000 + 10P - 2P_p$, where P_p is the price of colored paper. Using calculus, determine whether quantity supplied of children's books increases or decreases as the price of colored paper increases.

11. To test whether the law of supply holds using calculus, you should:

A) take the partial derivative of quantity supplied Q^S with respect to P and conclude that the law of supply holds if this derivative is positive at the market price.
B) take the partial derivative of quantity supplied Q^S with respect to P and conclude that the law of supply holds if this derivative is negative at the market price.
C) take the derivative of P with respect to quantity supplied Q^S and conclude that the law of supply holds if this derivative is positive at the market price.
D) take the derivative of P with respect to quantity supplied Q^S and conclude that the law of supply holds if this derivative is negative at the market price.

12. In the standard model, we expect the partial derivative of quantity supplied with respect to input price to be:

A) positive.
B) negative.
C) either positive or negative.
D) The correct answer is uncertain without more information.

13. Suppose the demand for fabric softener can be described as $Q^D = 1{,}000 - P + 0.01I$, where Q^D is the quantity of fabric softener demanded, P is the price of fabric softener, and I is income. Suppose that income is initially 1,000, but it falls to 800. The *new* equation for the demand for fabric softener is:

A) $800 - P + 0.01I$, and demand has shifted to the left.
B) $800 - P + 0.01I$, and demand has shifted to the right.
C) $1{,}080 - P$ and, demand has shifted to the left.
D) $1{,}080 - P$ and, demand has shifted to the right.

14. Suppose the demand for fabric softener can be described as $Q^D = 800 - P - P_D$, where Q^D is the quantity of fabric softener demanded, P is the price of fabric softener, and P_D is the price of laundry detergent. Suppose that the price of detergent is initially 10 but increases to 15. The *new* equation for the demand of fabric softener is:

A) $785 - P$, and demand has shifted to the left.
B) $785 - P$, and demand has shifted to the right.
C) $815 - P$, and demand has shifted to the left.
D) $815 - P$, and demand has shifted to the right.

15. Suppose that the inverse demand curve for energy drinks can be expressed as $P = 10 - Q^{0.5}$. The price elasticity of demand at a quantity of 25 is:

A) –0.5.
B) 0.5.
C) –2.
D) 2.

16. For an elastic demand function, the derivative of the revenue function with respect to price is:

A) positive.
B) negative.
C) zero.
D) infinite.

17. Suppose the demand for baby shoes in a small town is described by the following equation:

$$Q = 50 - 2P$$

where Q is the quantity of baby shoes demanded and P is the average price of a pair of baby shoes.
a. What is the price elasticity of demand for baby shoes when the price is \$15? Use calculus to show the answer.
b. What is the price elasticity of demand for baby shoes when the price is \$10? Use calculus to show the answer.

18. Suppose that the inverse demand curve for a well-known sports car can be expressed as $P = 100{,}000 - 2Q^2$, where price is in dollars and quantity is in numbers of cars.
a. What is the price elasticity of demand at a quantity of 100?
b. Is the demand for these sports cars elastic or inelastic?

19. Suppose that the inverse demand curve for a new laptop computer can be expressed as $P = 2{,}000 - 25Q^{0.5}$, where price is in dollars and quantity is in number of laptops.
a. What is the price elasticity of demand at a quantity of 625?
b. Is the demand for these laptops elastic or inelastic?

20. Suppose that the demand curve for a new product can be expressed as $Q^D = 900 - 3P$. At what price and quantity is total expenditure maximized?

21. Suppose that the demand curve for scissors can be expressed as $Q^D = 600 - 0.5P$. At what price and quantity is total expenditure maximized?

Answer Key

1. B
2. B
3. C
4. A
5. Since the partial derivative of quantity demanded of barbecue sauce with respect to the price of watermelon $\frac{\partial Q^D}{\partial P_W} = 0 - 0 - 6(1)P_W^{1-1} = -6P_W^0 = -6 < 0$, watermelon is a complement to barbecue sauce.
6. D
7. a. Since $\frac{\partial Q^D}{\partial P_p} = -9 < 0$, pancake batter is a substitute to maple syrup.
   b. Since $\frac{\partial Q^D}{\partial I} = 0.01 > 0$, maple syrup is a normal good.
8. a. Using the derivative of a constant and power rules of derivatives, $\frac{\partial Q^D}{\partial P} = 0 - 50(1)P^{1-1} = -50P^0 = -50$. Since this is less than zero, the law of demand holds.
   b. Using the derivative of a constant and power rules of derivatives, $\frac{\partial Q^S}{\partial P} = 0 + 40(1)P^{1-1} = 40P^0 = 40 > 0$. Since this is more than zero, the law of supply holds.
9. a. Using the derivative of a constant and power rules of derivatives, $\frac{\partial Q^D}{\partial P} = 0 - 120(1)P^{1-1} = -120P^0 = -120$. Since this is less than zero, the law of demand holds.
   b. Using the derivative of a constant and power rules of derivatives, $\frac{\partial Q^S}{\partial P} = 0 + 20(1)P^{1-1} = 20P^0 = 20 > 0$. Since this is more than zero, the law of supply holds.
10. Since $\frac{\partial Q^S}{\partial P_p} = -2 < 0$, we know that the quantity supplied of children's books decreases as the price of colored paper increases, which is the expected relationship given that colored paper is a likely input to production of children's books.
11. A
12. B
13. C
14. A
15. C
16. B
17. a. At \$15, quantity is 50 – 2(15) = 20. The derivative of a constant and power rules of derivatives show that $\frac{\partial Q^D}{\partial P} = 0 - 2(1)P^{1-1} = -2P^0 = -2$.

The price elasticity of demand is $E^D = \frac{\partial Q^D}{\partial P}\frac{P}{Q^D} = -2\frac{15}{20} = -1.5$.

b. At a price of \$10, quantity is 50 – 2(10) = 30. The price elasticity of demand at this price is $E^D = \frac{\partial Q^D}{\partial P}\frac{P}{Q^D} = -2\frac{10}{30} \approx -.67$.

18. a. First, for a quantity of 100 cars, $P = 100{,}000 - 2(100)^2 = 80{,}000$, and therefore this sports car costs \$80,000. The formula for the price elasticity of demand, written in terms of calculus, is $E^D = \frac{\partial Q^D}{\partial P}\frac{P}{Q^D} = \frac{1}{\frac{\partial P}{\partial Q^D}}\frac{P}{Q^D}$. With the inverse demand curve equation as given, we can see that $\frac{\partial P}{\partial Q^D} = 0 - 2(2)Q^{2-1} = -4Q$. At a quantity of 100 sports cars, this value is $-4(100) = -400$. Making the appropriate substitutions, we find that $E^D = \frac{1}{-400}\frac{80{,}000}{100} = -2$.

b. Since this is greater than one in absolute value, these sports cars are price elastic.

19. a. For a quantity of 625 laptops, $P = 2{,}000 - 25(625)^{0.5} = 1{,}375$. The price elasticity of demand formula, written in terms of calculus, is $E^D = \frac{\partial Q^D}{\partial P}\frac{P}{Q^D} = \frac{1}{\frac{\partial P}{\partial Q^D}}\frac{P}{Q^D}$.

With the inverse demand curve equation as given, we can see that $\frac{\partial P}{\partial Q^D} = 0 - 25(0.5)Q^{0.5-1} = -12.5Q^{-0.5}$. At a quantity of 625 laptops, this value is $-\frac{12.5}{25} = -0.5$. Making the appropriate substitutions, we find that $E^D = \frac{1}{-0.5}\frac{1{,}375}{625} = -4.4$.

b. Since this is greater than one in absolute value, these laptop computers are price elastic.

20. Total expenditure is maximized when the price elasticity of demand is exactly unit elastic. This is true when:

$$E^D = \frac{\partial Q^D}{\partial P}\frac{P}{Q^D} = -3\frac{P}{Q^D} = -1$$

Rearranging, we can see that this holds when $3P = Q^D$. Substituting into the demand equation, we get:

$$Q^D = 900 - 3P$$
$$3P = 900 - 3P$$
$$6P = 900$$
$$P = 150$$
$$Q^D = 900 - 3(150) = 450$$

21. Total expenditure is maximized when the price elasticity of demand is exactly unit elastic. This is true when:

$$E^D = \frac{\partial Q^D}{\partial P}\frac{P}{Q^D} = -0.5\frac{P}{Q^D} = -1$$

Rearranging, we can see that this holds where $0.5P = Q^D$. Substituting into the demand equation, we get:

$Q^D = 600 - 0.5P$

$0.5P = 600 - 0.5P$

$P = 600$

$Q^D = 600 - 0.5(600) = 300$