5 MOS Field-Effect Transistors (MOSFETs)

[15 minutes] 5.1
[10 points]

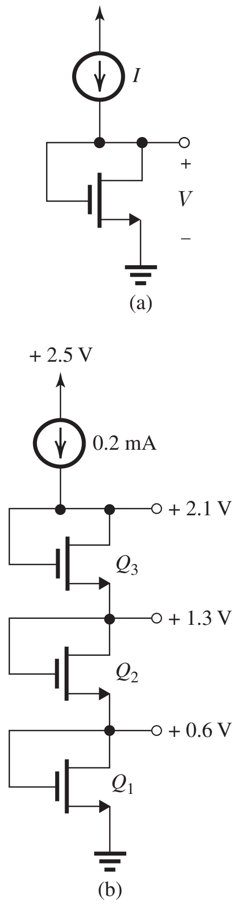

Figure 5.1.1

[4 points] (a) For the circuit shown in Fig. 5.1.1(a), show (neglecting λ) that

$$V = V_t + \sqrt{\frac{2I}{k_n'(W/L)}}$$

(b) The MOSFETs in the circuit of Fig. 5.1.1(b) have $V_t = 0.4\,\text{V}$, $k_n' = 0.4\,\text{mA/V}^2$, and $\lambda = 0$. Find $(W/L)_1$, $(W/L)_2$, and $(W/L)_3$ to obtain the reference voltages shown. [6 points]

5.2 **[18 minutes]** **[10 points]**

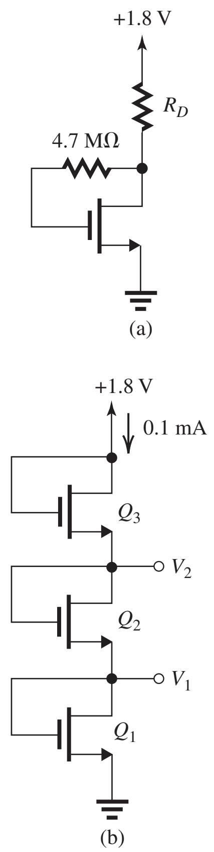

Figure 5.2.1

The MOSFETs in the circuits of Fig. 5.2.1 have $V_t = 0.4\,\text{V}$, $k_n' = 0.4\,\text{mA/V}^2$, $\lambda = 0$, and $L = 0.4\ \mu\text{m}$.

[4 points] (a) For the circuit in Fig. 5.2.1(a), find the values of W and R_D to operate the MOSFET at $I_D = 0.2\,\text{mA}$ and $V_D = 0.6\,\text{V}$.

[6 points] (b) For the circuit in Fig. 5.2.1(b), find W_1, W_2, and W_3 to obtain $V_1 = 0.5\,\text{V}$ and $V_2 = 1.1\,\text{V}$.

[18 minutes] 5.3
[10 points]

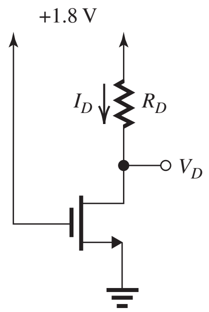

Figure 5.3.1

The MOSFET in Fig. 5.3.1 has $V_t = 0.5\,\text{V}$, $k'_n = 0.4\,\text{mA/V}^2$, $V_A = 10\,\text{V}$, and $W/L = 10$. Find the value of R_D that results in

i. $V_D = 0.1\,\text{V}$

ii. $V_D = 1.5\,\text{V}$

In each case, find I_D and the incremental drain-to-source resistance of the MOSFET.

[25 minutes] 5.4
[15 points]

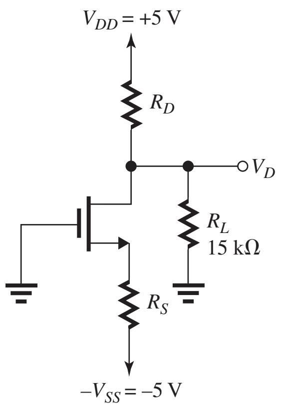

Figure 5.4.1

The MOSFET in the circuit of Fig. 5.4.1 has $V_t = 1\,\text{V}$ and $k_n = 2\,\text{mA/V}^2$, and the Early effect can be neglected.

[6 points] (a) Find the values of R_S and R_D that result in the MOSFET operating with an overdrive voltage of 0.5 V and a drain voltage of 1.5 V. What is the resulting I_D value?

[4 points] (b) If R_L is reduced from 15 kΩ to 10 kΩ, what does V_D become?

(c) If R_L is disconnected, what does V_D become? [2 points]

(d) With R_L disconnected, what is the largest R_D that can be used while the MOSFET is remaining in saturation? [3 points]

5.5 **[20 minutes]**
[20 points]

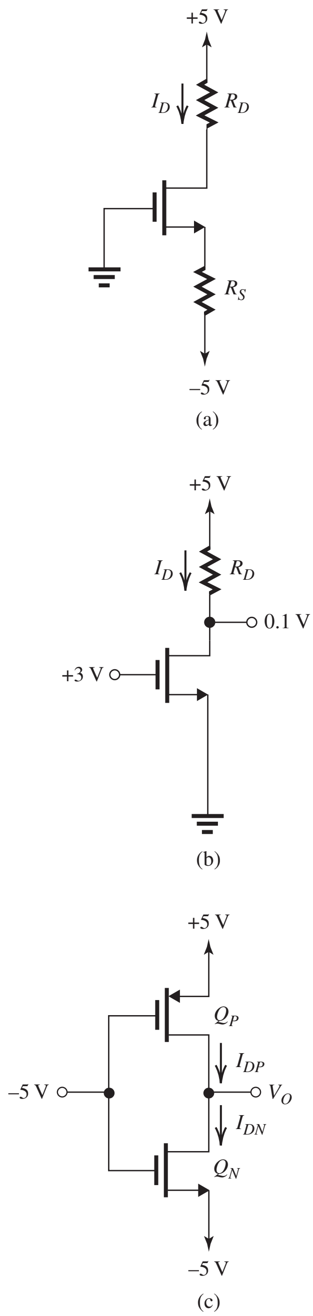

Figure 5.5.1

The MOSFETs in the circuits of Fig. 5.5.1 have $k = 1\,\text{mA/V}^2$, $|V_t| = 1\,\text{V}$, and $\lambda = 0$.

[6 points] (a) For the circuit in Fig. 5.5.1(a), find the values of R_D and R_S that result in the MOSFET operating at the edge of the saturation region with $I_D = 0.1$ mA.

[4 points] (b) For the circuit in Fig. 5.5.1(b), find I_D and R_D.

[10 points] (c) For the circuit in Fig. 5.5.1(c), find I_{DN}, I_{DP}, and V_O. Also, find the drain-to-source incremental resistance of each of Q_N and Q_P.

[25 minutes] [20 points] 5.6

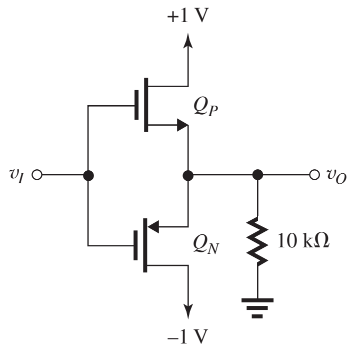

Figure 5.6.1

The transistors in the circuit of Fig. 5.6.1 have $k_n = k_p = 2\ \text{mA/V}^2$ and $V_{tn} = -V_{tp} = 0.4$ V. Find v_O for each of the following cases:

(a) $v_I = 0$ V [3 points]
(b) $v_I = +1$ V [5 points]
(c) $v_I = -1$ V [3 points]
(d) $v_I = +2$ V [6 points]
(e) $v_I = -2$ V [3 points]

5.7 [15 minutes] [15 points]

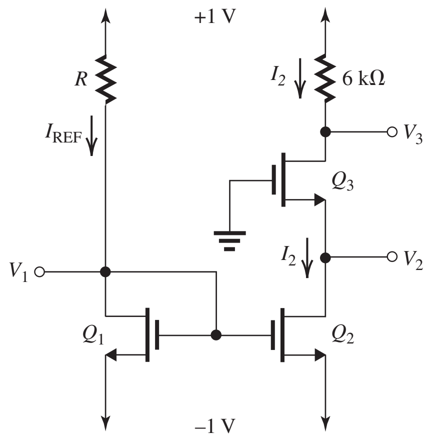

Figure 5.7.1

The MOSFETs in the circuit of Fig. 5.7.1 have $\mu_n C_{ox} = 400\ \mu\text{A/V}^2$, $V_t = 0.4$ V, $\lambda = 0$, $L = 0.4\ \mu$m, $W_1 = 2\ \mu$m, and $W_2 = W_3 = 10\ \mu$m. Find R to obtain a reference current I_{REF} of 40 μA. Also, find the values of V_1, I_2, V_2, and V_3.

5 MOS Field-Effect Transistors (MOSFETs)

5.1

(a)

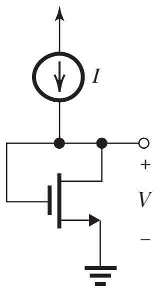

Figure 5.1.1(a)

The MOSFET is operating in saturation; thus, for $\lambda = 0$,

$$I_D = \frac{1}{2}k_n'(W/L)(V_{GS} - V_t)^2$$

Here,

$$I_D = I, \qquad V_{GS} = V$$

Thus,

$$I = \frac{1}{2}k_n'(W/L)(V - V_t)^2$$

$$\Rightarrow V = V_t + \sqrt{\frac{2I}{k_n'(W/L)}} \qquad \text{Q.E.D}$$

(b)

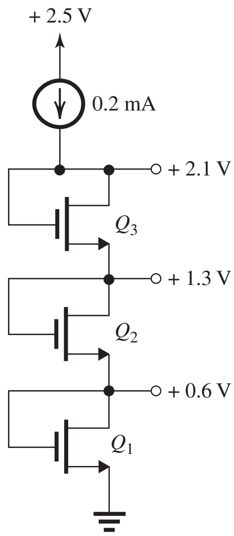

Figure 5.1.1(b)

For Q_1,

$$0.6 = 0.4 + \sqrt{\frac{2 \times 0.2}{0.4(W/L)_1}}$$

$$\Rightarrow (W/L)_1 = 25$$

For Q_2,

$$1.3 - 0.6 = 0.4 + \sqrt{\frac{2 \times 0.2}{0.4(W/L)_2}}$$

$$\Rightarrow (W/L)_2 = 11.11$$

For Q_3,

$$2.1 - 1.3 = 0.4 + \sqrt{\frac{2 \times 0.2}{0.4(W/L)_3}}$$

$$\Rightarrow (W/L)_3 = 6.25$$

5.2

(a)

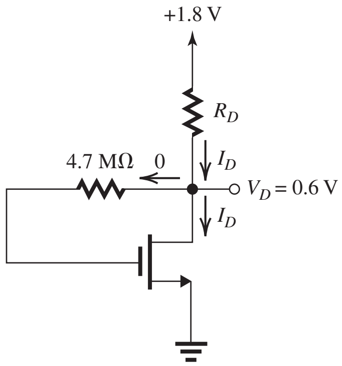

Figure 5.2.2

$$V_{GS} = V_G = V_D = 0.6 \text{ V}$$

The MOSFET is operating in saturation, $\lambda = 0$,

$$I_D = \tfrac{1}{2} k_n'(W/L)(V_{GS} - V_t)^2$$

$$0.2 = \tfrac{1}{2} \times 0.4(W/L)(0.6 - 0.4)^2$$

$$\Rightarrow W/L = 25$$

For $L = 0.4$ μm,

$$W = 10 \text{ μm}$$

From Fig. 5.2.2, we see that

$$I_D = \frac{1.8 - V_D}{R_D}$$

$$0.2 = \frac{1.8 - 0.6}{R_D}$$

$$\Rightarrow R_D = 6 \text{ k}\Omega$$

(b)

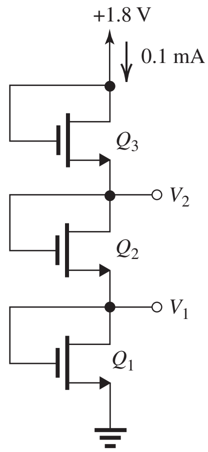

Figure 5.2.1(b)

For Q_1,

$$I_D = \tfrac{1}{2} k_n'(W/L)_1 (V_1 - V_t)^2$$

$$0.1 = \tfrac{1}{2} \times 0.4(W/L)_1 (0.5 - 0.4)^2$$

$$\Rightarrow (W/L)_1 = 50$$

For $L = 0.4$ μm,

$$W_1 = 20 \text{ μm}$$

For Q_2,

$$0.1 = \tfrac{1}{2} \times 0.4(W/L)_2 (V_2 - V_1 - V_t)^2$$

$$0.5 = (W/L)_2 (1.1 - 0.5 - 0.4)^2$$

$$\Rightarrow (W/L)_2 = 12.5$$

For $L = 0.4$ μm,

$$W_2 = 5 \text{ μm}$$

For Q_3,

$$0.1 = \tfrac{1}{2} \times 0.4(W/L)_3 (1.8 - V_2 - V_t)^2$$

$$= 0.2(W/L)_3 (1.8 - 1.1 - 0.4)^2$$

$$\Rightarrow (W/L)_3 = 5.56$$

For $L = 0.4$,

$$W_3 = 2.22 \text{ μm}$$

5.3

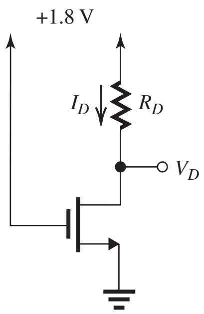

Figure 5.3.1

(i) For $V_D = 0.1$ V, $V_{GD} = 1.8 - 0.1 = 1.7$ V, which is greater than $V_t = 0.5$ V; thus, the MOSFET is operating in the triode region. Neglecting the Early effect, we have

$$I_D = k_n'(W/L)\left[(V_{GS} - V_t)V_{DS} - \tfrac{1}{2}V_{DS}^2\right]$$
$$= 0.4 \times 10\left[(1.8 - 0.5) \times 0.1 - \tfrac{1}{2} \times 0.1^2\right]$$
$$= 0.5 \text{ mA}$$

From the circuit,

$$I_D = \frac{1.8 - V_D}{R_D}$$
$$0.5 = \frac{1.8 - 0.1}{R_D}$$
$$\Rightarrow R_D = 3.4 \text{ k}\Omega$$
$$r_{DS} = \frac{1}{k_n'(W/L)(V_{GS} - V_t)}$$
$$= 0.192 \text{ k}\Omega = 192\ \Omega$$

(ii) For $V_D = 1.5$ V and $V_{GS} = 1.8$ V, $V_{OV} = 1.8 - 0.5 = 1.3$ V.

Thus, $V_{DS} > V_{OV}$, which implies operation in the saturation mode and

$$I_D = \tfrac{1}{2}k_n'(W/L)(V_{GS} - V_t)^2\left(1 + \frac{V_{DS}}{V_A}\right)$$
$$= \tfrac{1}{2} \times 0.4 \times 10(1.8 - 0.5)^2\left(1 + \frac{1.5}{10}\right)$$
$$= 3.89 \text{ mA}$$

Since

$$I_D = \frac{1.8 - V_D}{R_D}$$
$$3.89 = \frac{1.8 - 1.5}{R_D}$$
$$\Rightarrow R_D = 77\ \Omega$$

The incremental drain-to-source resistance is

$$r_o = \frac{V_A}{I_D} = \frac{10}{3.89} = 2.57 \text{ k}\Omega$$

5.4

(a)

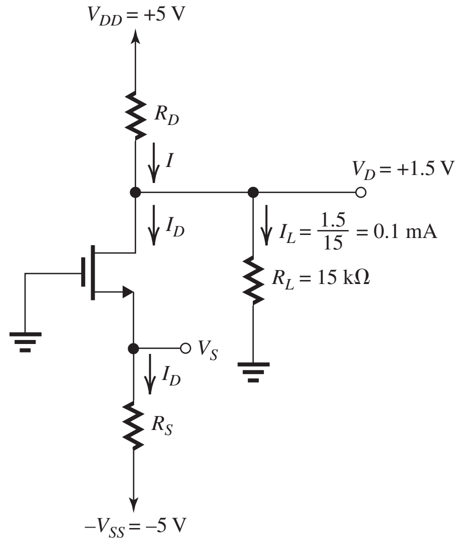

Figure 5.4.2

With $V_D = 1.5$ V and $V_G = 0$ V, the transistor operates in saturation, thus

$$I_D = \tfrac{1}{2}k_n V_{OV}^2$$
$$= \tfrac{1}{2} \times 2 \times 0.5^2$$
$$= 0.25 \text{ mA}$$

Refer to Fig. 5.4.2 above. A node equation at the drain provides

$$I = I_D + I_L = 0.25 + 0.1 = 0.35 \text{ mA}$$

We can now find R_D from

$$R_D = \frac{V_{DD} - V_D}{I} = \frac{5 - 1.5}{0.35} = 10 \text{ k}\Omega$$

To find R_S, we first determine V_S from

$$V_S = -V_{GS} = -(V_t + V_{OV}) = -(1+0.5) = -1.5\text{ V}$$

Then, we determine R_S from

$$R_S = \frac{V_S - V_{SS}}{I_D} = \frac{-1.5-(-5)}{0.25} = 14\text{ k}\Omega$$

(b) Refer to Figure 5.4.3 below. We assume that with R_L reduced to 10 kΩ, the MOSFET remains in saturation. Thus, I_D remains unchanged at 0.25 mA. Figure 5.4.3 shows the circuit before and after Thévenin theorem is applied to simplify the drain circuit. Now,

$$V_D = 2.5 - 0.25 \times 5 = 1.25\text{ V}$$

Thus, $V_D > V_G$, and saturation-mode operation is confirmed. The drain voltage becomes

$$V_D = 1.25\text{ V}$$

(c)

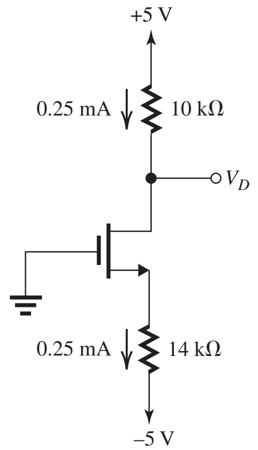

Figure 5.4.4

If R_L is disconnected, the circuit becomes as in Fig. 5.4.4. Assuming that the transistor remains in saturation, then $I_D = 0.25$ mA and

$$V_D = 5 - 0.25 \times 10 = 2.5\text{ V}$$

which is greater than V_G, thus confirming saturation-mode operation. The drain voltage is now

$$V_D = +2.5\text{ V}$$

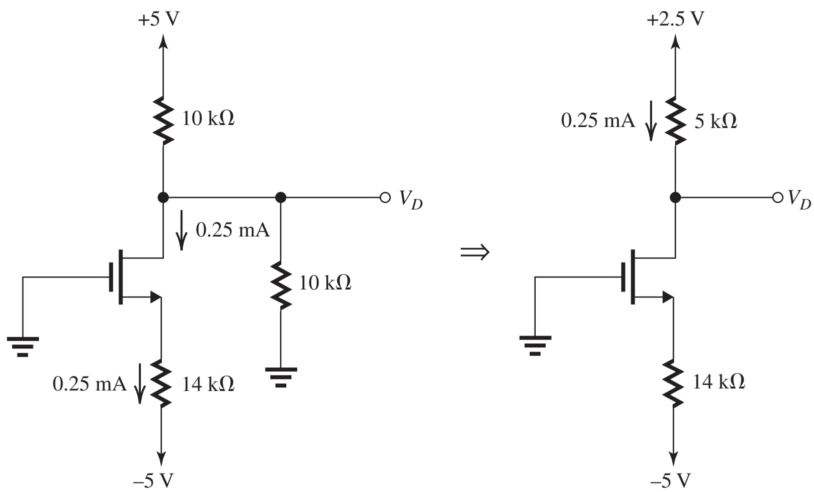

Figure 5.4.3

(d)

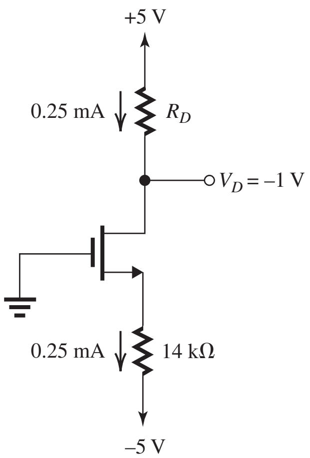

Figure 5.4.5

With R_L disconnected and the MOSFET operating at the edge of saturation, $V_{DG} = -V_t$, thus

$$V_D = -1 \text{ V}$$

and the current remains unchanged,

$$I_D = 0.25 \text{ mA}$$

From the circuit in Fig. 5.4.5, we can write

$$R_D = \frac{5 - V_D}{I_D} = \frac{5 - (-1)}{0.25}$$
$$= 24 \text{ k}\Omega$$

5.5
(a)

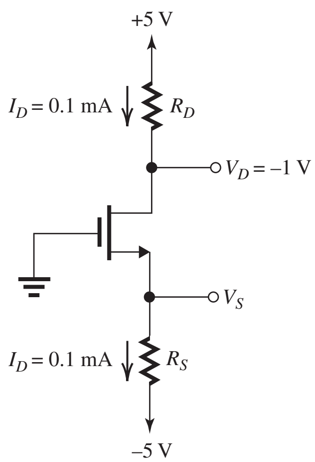

Figure 5.5.2

For operation at the edge of saturation,

$$V_{GD} = V_t = 1 \text{ V}$$

Thus,

$$V_D = -1 \text{ V}$$

The circuit is shown in Fig. 5.5.2. To obtain $I_D =$ 0.1 mA, V_{OV} can be found from

$$I_D = \tfrac{1}{2} k_n V_{OV}^2$$
$$0.1 = \tfrac{1}{2} \times 1 \times V_{OV}^2$$
$$\Rightarrow V_{OV} = 0.447 \text{ V}$$

Thus,

$$V_{GS} = V_t + V_{OV} = 1 + 0.447 = 1.447 \text{ V}$$

and

$$V_S = -V_{GS} = -1.447 \text{ V}$$

The value of R_S can be found from

$$R_S = \frac{V_S - (-5)}{I_D} = \frac{-1.447 + 5}{0.1}$$
$$= 35.5 \text{ k}\Omega$$

The value of R_D can be found from

$$R_D = \frac{+5 - V_D}{I_D} = \frac{5 - (-1)}{0.1} = 60 \text{ k}\Omega$$

(b)

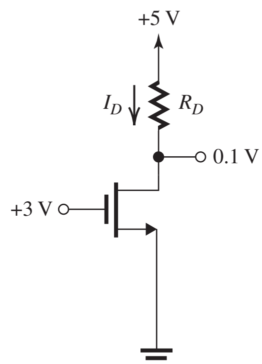

Figure 5.5.1(b)

Here, $V_{GD} = 3 - 0.1 = 2.9$ V, which is greater than V_t. Thus, the transistor is operating in the triode region with

$$I_D = k_n\left[(V_{GS} - V_t)V_{DS} - \tfrac{1}{2}V_{DS}^2\right]$$
$$= 1 \times \left[(3-1) \times 0.1 - \tfrac{1}{2} \times 0.1^2\right]$$
$$= 0.195 \text{ mA}$$

From the circuit,

$$I_D = \frac{5 - 0.1}{R_D} = 0.195 \text{ mA}$$
$$\Rightarrow R_D = 25.1 \text{ k}\Omega$$

(c)

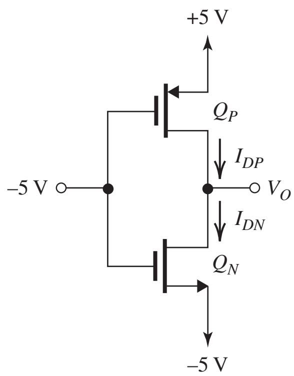

Figure 5.5.1(c)

Since $V_{GSN} = 0$ V, Q_N will be cut off and

$$I_{DN} = 0$$

Since $V_{SGP} = 10$ V, Q_P can conduct. However, since the drain current has no path to ground,

$$I_{DP} = 0$$

and Q_P will be operating in the triode region with $V_{SD} = 0$, thus

$$V_O = +5 \text{ V}$$

Since Q_N is off, its incremental resistance will be infinite. Since Q_P is operating in the triode region, its incremental resistance will be

$$r_{SD} = \frac{1}{k(V_{SG} - |V_t|)}$$
$$= \frac{1}{1 \times (10-1)} = 0.111 \text{ k}\Omega$$
$$= 111 \text{ }\Omega$$

5.6

(a)

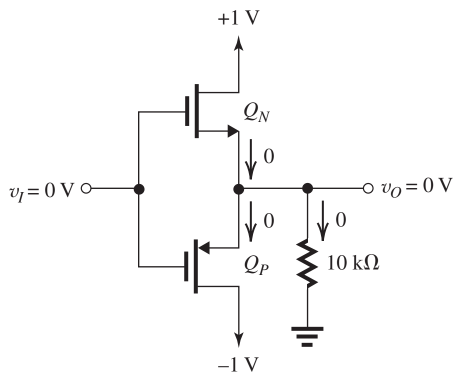

Figure 5.6.2

From Fig. 5.6.2, we see that when $v_I = 0$ V, both transistors are cut off and

$$v_O = 0 \text{ V}$$

(b)

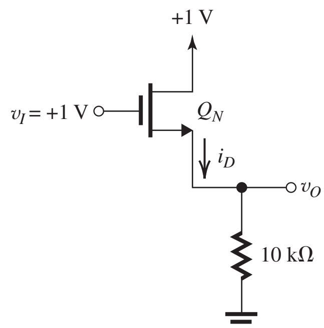

Figure 5.6.3

With $v_I = +1$ V, Q_P cannot conduct and can be eliminated, reducing the circuit to that shown in Fig. 5.6.3. Since $v_{GDN} = 0$ V, Q_N will be operating in saturation with

$$i_D = \tfrac{1}{2}k_n(v_{GS} - V_{tn})^2$$

But

$$i_D = \frac{v_O}{10\,\text{k}\Omega} = 0.1v_O, \qquad \text{mA}$$

and

$$v_{GS} = v_I - v_O = 1 - v_O$$

Thus,

$$\begin{aligned} 0.1v_O &= \tfrac{1}{2} \times 2(1 - v_O - 0.4)^2 \\ &= (0.6 - v_O)^2 \\ &= 0.36 - 1.2v_O + v_O^2 \end{aligned}$$

which can be rearranged in the form

$$v_O^2 - 1.3v_O + 0.36 = 0$$

This quadratic has two solutions: 0.775 V, which is physically meaningless since v_{GS} becomes $1 - 0.775 = 0.225$ V, which is less than V_{tn}; and 0.4 V, which is possible. Thus,

$$v_O = 0.4\text{ V}$$

(c)

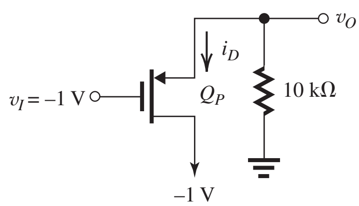

Figure 5.6.4

With $v_I = -1$ V, Q_N cannot conduct and can be eliminated, reducing the circuit to that shown in Fig. 5.6.4. Since $v_{GDP} = 0$ V, Q_P will be operating in saturation. We recognize this situation to be the complement of that in (b) above. Thus, we do not need to do the analysis again and we can simply write

$$v_O = -0.4\text{ V}$$

(d)

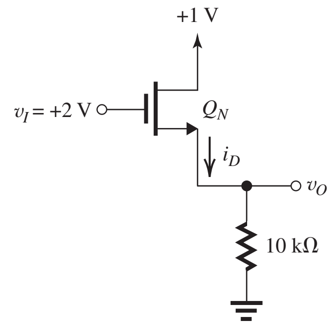

Figure 5.6.5

With $v_I = +2$ V, Q_P cannot conduct and can be removed, thus reducing the circuit to that shown in Fig. 5.6.5. Since $v_{GDN} = 1$ V, which is greater than V_{tn}, Q_N will be operating in the triode region. Thus,

$$i_D = k_n\left[(v_{GS} - V_{tn})v_{DS} - \tfrac{1}{2}v_{DS}^2\right]$$

Here,

$$\begin{aligned} i_D &= \frac{v_O}{10\,\text{k}\Omega} = 0.1v_O, \qquad \text{mA} \\ v_{GS} &= 2 - v_O \\ v_{DS} &= 1 - v_O \end{aligned}$$

Thus,

$$\begin{aligned} 0.1v_O &= 2\left[(1.6 - v_O)(1 - v_O) - \tfrac{1}{2}(1 - v_O)^2\right] \\ &= 2\left[1.6 - 2.6v_O + v_O^2 - \tfrac{1}{2} + v_O - \tfrac{1}{2}v_O^2\right] \\ &= v_O^2 - 3.2v_O + 2.2 \\ &\Rightarrow v_O^2 - 3.3v_O + 2.2 = 0 \end{aligned}$$

which has the following solution

$$v_O = \frac{+3.3 \pm \sqrt{3.3^2 - 8.8}}{2} = 0.927\text{ V or } 2.37\text{ V}$$

The second solution is physically meaningless as v_O exceeds v_D. Thus,

$$v_O = 0.927\text{ V}$$

(e)

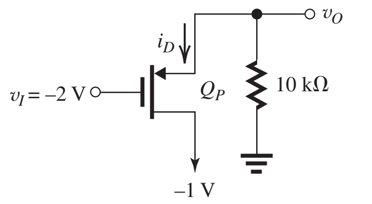

Figure 5.6.6

With $v_I = -2$ V, Q_N cannot conduct and the circuit reduces to that shown in Fig. 5.6.6. We recognize this situation to be the complement of that in (d) above. Thus,

$$v_O = -0.927 \text{ V}$$

5.7

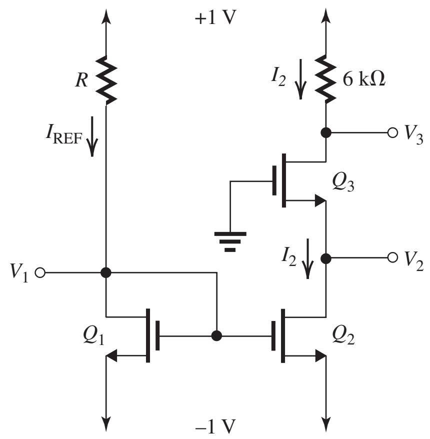

Figure 5.7.1

Consider first the R, Q_1 branch. Transistor Q_1 has $V_{DG} = 0$ and thus is operating in saturation,

$$I_{REF} = I_{D1} = \frac{1}{2}\mu_n C_{ox}(W/L)_1 (V_{GS1} - V_t)^2$$

$$40 = \frac{1}{2} \times 400 \times \frac{2}{0.4}[V_1 - (-1) - 0.4]^2$$

$$\Rightarrow V_1 = -0.4 \text{ V}$$

The value of R can now be determined from

$$R = \frac{1 - V_1}{I_{REF}} = \frac{1 - (-0.4)}{0.04}$$

$$R = 35 \text{ k}\Omega$$

Next, consider Q_2; it has a width W_2 that is five times that of Q_1. Since Q_2 and Q_1 have the same V_{GS}, and assuming that Q_2 is in saturation, the current in Q_2 will be five times that in Q_1:

$$I_2 = 5I_{REF} = 5 \times 40 = 200 \ \mu\text{A}$$

Transistor Q_3 has I_D equal to I_2. Since Q_3 and Q_2 have the same W/L, then, if we assume that Q_3 is in the saturation region, we obtain

$$V_{GS3} = V_{GS2} = V_1 - (-1)$$

$$\Rightarrow V_{GS3} = 0.6 \text{ V}$$

Thus,

$$V_2 = -V_{GS3} = -0.6 \text{ V}$$

which is lower than $V_{G2} = V_1 = -0.4$ V by 0.2 V, which is less than V_{tn}, thus Q_2 is in saturation as assumed.

Finally, V_3 can be found as

$$V_3 = +1 - 6 \times 0.2 = -0.2 \text{ V}$$

Thus, $V_{GD3} = 0.2$ V, which is less than V_t, confirming that Q_3 is operating in saturation, as assumed.

