MODERN PORTFOLIO THEORY AND INVESTMENT ANALYSIS
8TH EDITION

The following exam questions are organized according to the text's sections. Within each section, questions follow the order of the text's chapters and are organized by multiple choice, true-false with discussion, problems, and essays. The correct answers to the multiple choice questions are marked with a "**".

PART 1: INTRODUCTION

Part 1: Essays

1. Given a typical set of indifference curves and a budget constraint for a 1-period (2-date) consumption model, where will the optimum consumption pair (for date 1 and date 2) be found on the graph and why is it optimal?

2. List and discuss the characteristics of various types of financial securities.

3. List and discuss the characteristics of various types of financial markets.

PART 2; SECTION 1: MEAN-VARIANCE PORTFOLIO THEORY

Part 2; Section 1: Multiple Choice

1. Diversification among assets improves the opportunities faced by all risk-averse investors
   a. irrespective of the correlation coefficients
   b. only if correlations are not larger than 0
   c. only if the assets have similar variances
   d. for assets with relatively large variances
** e. none of the above

2. Which statement is true?
   a. An efficient portfolio always provides the highest expected rate of return.
** b. An efficient portfolio has less risk than any other asset or portfolio with comparable expected return and more return than any other asset or portfolio with comparable risk.
   c. Neither one of the above statements is true.

3. With a riskless asset and risky assets, the efficient portfolio opportunity set is a straight line. The preceding statement
** a. is true.
   b. is false.
   c. could be true or false, depending on the correlations of the risky assets.

4. Consider the following data for portfolios A and B, which are both on the efficient frontier:

$$\bar{R}_A = 11\%;\ \bar{R}_B = 14\%;\ \sigma_A = 10\%;\ \sigma_B = 17\%$$

If you want to earn 12% by investing in A and B, what portion of your money must you invest in A?

a. 1/5
** b. 2/3
c. 1/3
d. 4/5

5. The separation theorem
** a. says that you can determine the optimum portfolio of risky assets for an investor without having to know anything about the investor.
b. implies that all investors hold the same portfolio of the riskless asset and risky assets.
c. holds even if the lending and borrowing rates are different, provided that both rates are riskless.
d. allows the construction of an efficient portfolio by separating efficient assets from inefficient assets.

Part 2; Section 1: True-False With Discussion

1. Recently, a large pension-fund manager stated that no useful information about the fund's appropriate mix of stocks and risky bonds could be obtained from portfolio theory, since the correlation between the returns on stocks and bonds is essentially zero. Accepting the manager's estimate of the correlation, discuss the correctness of the statement.

2. Discuss whether the following statement is true or false:
The separation theorem tells you how to separate an investor from his or her money.

3. Discuss whether the following statement is true or false:
One function of a capital market is to separate consumption decisions from decisions of investment in physical production facilities.

4. Discuss whether the following statement is true or false:
Diversification does not pay if two assets are positively correlated with each other.

Part 2; Section 1: Problems

1. Consider the following data for securities A, B, and C:
$\bar{R}_A = 20\%;\ \bar{R}_B = 10\%;\ \bar{R}_C = 8\%;\ \sigma_A = 4\%;\ \sigma_B = 2\%;\ \sigma_C = 2\%;\ \rho_{AB} = 0.4;\ \rho_{AC} = 0.2;$
$\rho_{BC} = -1.0$
a. What is the expected return and standard deviation of a portfolio constructed by placing 60% of your money in A and 40% in B?
b. What is the expected return on the portfolio constructed from among the above three securities that has the smallest possible risk?
c. If an investor had to place 100% of his or her money in only one of the above three securities,
1. which security would a risk-neutral investor pick?
2. what can you say about the preference ordering of the three securities for a risk-averse investor?

2. Assume different riskless lending and borrowing rates and the availability of all risky

assets. Draw the efficient frontier.

3. You are in a world where there are only two assets: gold and stocks. You are interested in investing your money in one or both of the assets. Consequently, you collect the following data on the assets' returns over the past six years:

	Gold	Stock Market
average return	8%	20%
standard deviation	25%	22%

Your estimate of the assets' correlation is –0.4.

a. If you were constrained to pick only one of the two assets, which one would you choose?
b. What is the average return and standard deviation of a portfolio composed of equal proportions of gold and stocks?
c. What is the average return and standard deviation of the portfolio composed of gold and stocks that has the lowest risk?
d. You now learn that GPEC (a cartel of gold-producing countries) is going to vary the amount of gold produced depending on stock prices in the U.S. by producing less gold when the stock market is up and more gold when the stock market is down. What effect will this have on portfolios composed of gold and stock? Explain.

4. Assume that the average variance of return for an individual security is 50 and that the average covariance is 10. What are the expected variances of portfolios composed of 5, 10, 20, 50, and 100 securities?

5. You are evaluating two risky investments, A and B, that have the following distributions:

probability	return on A	return on B
0.6	20%	30%
0.4	10%	10%

a. What are the expected returns and standard deviations for A and B?
b. Suppose that an investor must pick either A or B to hold in some combination with the riskless asset (R_F = 8%). Which risky asset should the investor choose?

6. The stock returns for firm A and firm B have the following characteristics:

firm	expected return	standard deviation
A	10%	8%
B	12%	20%

The correlation between the two stocks is 1.0.

a. If there are no restrictions on short sales or borrowing, what are the portfolio weights, expected return and standard deviation on the portfolio of these two assets with the lowest risk (minimum variance)?
b. Susan is an officer of firm A. Under a company stock purchase plan, she currently holds $200,000 worth of A's stock, and this represents her total assets. This stock cannot be sold. Susan can purchase additional amounts of stock A or stock B, and she can sell stock B short. It is illegal for her to sell stock A short. How can Susan eliminate the risk in her holding? Be specific (give numbers).

7. Consider the following data for assets A and B:
$\bar{R}_A = 10\%;\ \bar{R}_B = 14\%;\ \sigma_A = 4\%;\ \sigma_B = 6\%;\ \rho_{AB} = 0.5$

Assuming that only the riskless asset and assets A and B are available,

a. if the risk-free rate is 6%, what is the optimal portfolio of risky assets?
b. what riskless rate would cause the investor to hold none of asset A?

8. Assume riskless lending and borrowing and zero correlation between returns on commodity funds and stock funds. Derive algebraically what the minimum return must be on commodity funds for those funds to be held in positive proportions.

9. Assume that the returns of assets A and B depend on the state of the market and that the return of asset C depends on the state of the weather. Assume that the state of the market is independent of the state of the weather. Given the following

state of market	probability	R_A	R_B	state of weather	probability	R_C
good	0.2	18%	5%	good	0.3	12%
average	0.6	12%	10%	average	0.6	15%
poor	0.2	6%	5%	poor	0.1	18%

a. What is the mean return of asset C?
b. What is the standard deviation of asset C?
c. What is the covariance between each pair of assets?
d. Which of the following two portfolios dominates the other: portfolio 1 consisting of 1/2 invested in A and 1/2 invested in B, or portfolio 2 consisting of ½ invested in B and 1/2 invested in C?
e. Find a portfolio that dominates portfolios 1 and 2 in part d.
f. Set up the first order conditions to solve for the optimum portfolio of assets A, B and C if the riskless rate is 8%.

10. Consider the following data for assets A, B, and C:
$\bar{R}_A = 10\%;\ \bar{R}_B = 8\%;\ \bar{R}_C = 6\%;\ \sigma_A = 9\%;\ \sigma_B = 8\%;\ \sigma_C = 5\%;\ \rho_{AB} = \rho_{AC} = \rho_{BC} = 0.5$

Assume that a riskless asset exists and set up the first order conditions to solve for the optimum portfolio of assets A, B and C.

Part 2; Section 1: Essays

1. Under what condition will adding a security with a high standard deviation decrease the risk of a portfolio?

PART 2; SECTION 2: SIMPLIFYING THE PORTFOLIO SELECTION PROCESS

Part 2; Section 2: Multiple Choice

1. If the returns on different assets are uncorrelated

** a. an increase in the number of assets in a portfolio may bring the standard deviation of the portfolio close to zero.
b. there will be little gain from diversification.
c. diversification will result in risk averaging but not in risk reduction.
d. the expected return on a portfolio of such assets should be zero.

2. Using the Sharpe single-index model with a random portfolio of U.S. common stocks, as one increases the number of stocks in the portfolio, the total risk of the portfolio will
a. approach zero.
** b. approach the portfolio's systematic risk.
c. approach the portfolio's non-systematic risk.
d. not be affected.

Part 2; Section 2: True-False With Discussion

1. Discuss whether the following statement is true or false:
One can always construct a multi-index model that explains more of the returns on a security than a single-index model does.

2. Discuss whether the following statement is true or false:
A multi-index model will predict returns better than a single-index model.

Part 2; Section 2: Problems

1. Consider the following data for assets A and B:
$\bar{R}_A = 10\%$; $\bar{R}_B = 19\%$; $\sigma_A = 3\%$; $\sigma_B = 5\%$; $\beta_A = 0.6$; $\beta_B = 1.4$; $\rho_{AB} = 0.4$.

a. Calculate the expected return, variance, and beta of a portfolio constructed by investing 1/3 of your funds in asset A and 2/3 in asset B.
b. If only the riskless asset and assets A and B are available, find the optimum risky-asset portfolio if the risk-free rate is 8%.

2. Consider the following data for assets A, B, and C
$\bar{R}_A = 12\%$; $\bar{R}_B = 8\%$; $\bar{R}_C = 6\%$; $\beta_A = 1.1$; $\beta_B = 0.8$; $\beta_C = 0.9$; $\sigma^2_{eA} = 10$; $\sigma^2_{eB} = 15$; $\sigma^2_{eC} = 5$.

Assume the variance of the market portfolio is 20 and that a riskless asset exists. Set up the first-order conditions for the optimum risky-asset portfolio.

3. Consider the following historical data for the returns on assets A and B and the market portfolio:

period	asset A	asset B	market portfolio
1	10%	6%	4%
2	-3%	6%	1%
3	5%	2%	5%
4	2%	4%	2%
5	1%	2%	1%

a. What is the covariance between asset A and asset B?

b. If the beta of asset B is 0.5, what is the systematic return and non-systematic return for asset B in each period?

4. The annual returns of Wonder Widgets, Inc. and the S&P 500 Composite Index over the last ten years were as follows:

year	Wonder Widgets	S&P 500
1	-15%	-8.5%
2	10%	4.0%
3	12%	14.0%
4	20%	15.0%
5	-20%	-14.0%
6	-15%	-26.0%
7	25%	37.0%
8	30%	24.0%
9	-10%	-7.0%
10	3%	6.5%

Find the following for Wonder Widgets:

a. beta (β_W, slope of regression line)
b. alpha (α_W, intercept of regression line)
c. unsystematic variance ($\sigma^2_W - \beta^2_W\sigma^2_m$)
d. correlation coefficient (ρ)

5. You are the pension fund manager for a major university with $100 million in an index fund that invests in the S&P 500 stocks. (The fund holds all the stocks in the index in proportion to their market values.) Due to recent pressure from student groups, the regents have decided to divest themselves of the stocks of firms that invest in South Africa. You estimate that this will eliminate 100 of the 500 stocks in your portfolio. You have been asked to evaluate the effect of the divestiture decision. You estimate that the correlation between acceptable and eliminated stocks is 0.6. You also have the following data:

	acceptable stocks	eliminated stocks
number of firms	400	100
total market value	$3 billion	$2 billion
average beta	1.0	1.25
standard deviation	25%	30%

a. What will the effect of the divestment be on the beta of your portfolio? (Report the beta before and after the divestment.)
b. How will divestment affect the standard deviation of your portfolio? (Report the standard deviation before and after the divestment.)
c. Assume that the standard deviation of the overall market is 20%. What is the effect of divestment on the proportion of your portfolio's risk that is unsystematic? (Report the proportion before and after the divestment.)

6. A security analyst works for a large institution that uses the single-index model as part of its portfolio-management scheme. The security analyst believes the following values are relevant for the four stocks she follows

$\bar{R}_A = 14\%;\ \bar{R}_B = 12\%;\ \bar{R}_C = 8\%;\ \bar{R}_D = 11\%;\ \beta_A = 2.0;\ \beta_B = 1.5;\ \beta_C = 1.0;\ \beta_D = 1.0;\ \sigma^2_{eA} = 15;$

$\sigma_{eB}^2 = 7.5$; $\sigma_{eC}^2 = 9$; $\sigma_{eD}^2 = 10$.

The institution assumes that the risk-free rate is 6%, and short selling is not allowed. The institution accepts the Sharpe single-index model and uses the procedure described by Elton, Gruber and Padberg (EGP) to determine the optimum risky-asset portfolio for the institution to hold. The procedure is to compute

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2} \times \left[(\text{ranking criterion for asset } i) - C^*\right]$$

where the ranking criterion is as described by EGP and where C^* depends on *all* risky assets the institution holds. The institution's management has determined that $C^* = 3$.

a. Which stocks that the analyst follows will be held in the institution's optimum portfolio?
b. If the sum of the Z_i's for all the institution's stocks in the optimum portfolio is equal to 4, what fraction of the institution's optimum portfolio will each of the stocks that the analyst follows represent?
c. Why should σ_{ei}^2 (diversifiable risk) enter into the optimal solution?