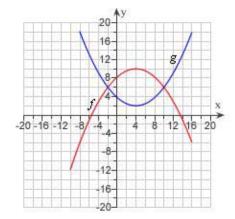
1. Find the domain of the function.

$$f(x) = \frac{5}{4x - 1}$$

2. Determine whether f is even, odd, or neither.

$$f(x) = \frac{8x^2}{x^4 + 1}$$

- 3. The graphs of f(x) and g(x) are given.
 - a) For what values of x is f(x) = g(x)?
 - b) Find the values of f(-2) and g(4).



4. It makes sense that the larger the area of a region, the larger the number of species that inhabit the region. Many ecologists have modeled the species-area relation with a power function and, in particular, the number of species S of bats living in caves in central Mexico has been related to the surface area A measured in m^2 of the caves by the equation

$$S = 0.7A^{0.3}$$

- (a) The cave called mission impossible near puebla, mexico, has suface area of $A = 90 \,\mathrm{m}^2$. How many species of bats would expect to find in that cave?
- (b) If you discover that 5 species of bats live in cave estimate the area of the cave.

5. Express the function in the form of $f \circ g$.

$$v(t) = \sec(t^4) \tan(t^4)$$

6. The position of a car is given by the values in the following table.

t(seconds)	0	1	2	3	4	5
s (feet)	0	16	35	71	112	179

Estimate the instantaneous velocity when t = 2 by averaging the velocities for the periods [1, 2] and [2, 3].

7. Consider the following function.

$$f(x) = \begin{cases} 3 - x & x < -1 \\ x & -1 \le x < 3 \\ (x - 3)^2 & x \ge 3 \end{cases}$$

Determine the values of a for which $\lim_{x \to a} f(x)$ exists.

8. Find the limit.

$$\lim_{x \to 0^+} \tan^{-1} \left(\frac{2}{x} \right)$$

9. Evaluate the limit.

$$\lim_{x \to 0} \frac{(6+x)^{-1} - 6^{-1}}{x}$$

10. Find the limit.

$$\lim_{x \to \frac{10}{x}} \tan^{-1} \left(\frac{5}{x} \right)$$

11. Evaluate the limit.

$$\lim_{x \to 3} \left(\frac{x^3 - 5}{x^2 - 6} \right)$$

12. Evaluate the limit.

$$\lim_{x \to 9} \frac{3 - \sqrt{x}}{x - 9}$$

13. Evaluate the limit.

$$\lim_{x \to 0} \frac{3 - \sqrt{3 - x^2}}{x}$$

- 14. Find a number δ such that if $|x-2| < \delta$, then $|4x-8| < \varepsilon$, where $\varepsilon = 0.1$.
- 15. Find the point at which the given function is discontinuous.

$$f(x) = \begin{cases} \frac{1}{x-7}, & x \neq 7 \\ 7, & x = 7 \end{cases}$$

- 16. Write an equation that expresses the fact that a function f is continuous at the number 4.
- 17. Find a function g that agrees with f for $x \ne 25$ and is continuous on \Re .

$$f(x) = \frac{5 - \sqrt{x}}{25 - x}$$

- 18. Let $f(x) = x^2 18x + 75$ and $g(x) = \sqrt{x+7}$. Find $(f \circ g)(74)(g \circ g)(74)$.
- 19. Find the limit $\lim_{x\to 0^+} \frac{9+\sqrt{x}}{\sqrt{x+16}}$.
- 20. Find the numbers, if any, where the function $f(x) = \frac{x-3}{x^2-9}$ is discontinuous.

Answer Key

$$1. \left\{ x \middle| x \neq \frac{1}{4} \right\}$$

- 2. even
- 3. a) -2, 10

b)
$$f(-2) = 6$$
, $g(4) = 2$

- 4. a) 3 species
 - b) $702 \,\mathrm{m}^2$
- 5. $f(t) = \sec(t)\tan(t)$

$$g(t) = t^4$$

- 6. 27.5 ft/s
- 7. $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$
- 8. $\frac{\pi}{2}$
- 9. $-\frac{1}{36}$
- 10. 0
- 11. $\frac{22}{3}$
- 12. -1/6
- 13. ∝
- 14. $\delta = 0.025$
- 15. 7

16.
$$\lim_{x \to 4} f(x) = f(4)$$

17.
$$g(x) = \frac{1}{5 + \sqrt{x}}$$

19.
$$\frac{9}{4}$$

20.
$$\pm 3$$

1. Find the domain of the function.

$$f(x) = \sqrt{49 - x^2}$$

2. A spherical balloon with radius r inches has volume

$$\frac{4}{3}\pi r^3$$
.

Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of r+1 inches.

3. It makes sense that the larger the area of a region, the larger the number of species that inhabit the region. Many ecologists have modeled the species-area relation with a power function and, in particular, the number of species S of bats living in caves in central Mexico has been related to the surface area A measured in m^2 of the caves by the equation

$$S = 0.7A^{03}$$

- (a) The cave called mission impossible near puebla, mexico, has suface area of $A = 90 \,\text{m}^2$. How many species of bats would expect to find in that cave?
- (b) If you discover that 5 species of bats live in cave estimate the area of the cave.
- 4. A spherical balloon with radius r inches has volume

$$4\frac{\pi r^3}{3}$$

Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of r+3 inches.

- 5. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 45 cm/s. Express the radius r of this circle as a function of the time t (in seconds) and find $A \circ r$, if A is the area of this circle as a function of the radius.
- 6. The position of a car is given by the values in the following table.

t (seconds)	0	1	2	3	4	5
s(feet)	0	16	35	71	112	179

Estimate the instantaneous velocity when t = 2 by averaging the velocities for the periods [1, 2] and [2, 3].

7. Evaluate the function

$$f(x) = 7 \left(\frac{\sqrt{x} - \sqrt{2}}{x - 2} \right)$$

at the given numbers (correct to six decimal places). Use the results to guess the value of the limit $\lim_{x\to 2} f(x)$.

х	f(x)
1.6	
1.8	
1.9	
1.99	
1.999	
2.4	
2.2	
2.1	
2.01	
2.001	
Limit	

8. By graphing the function

$$f(x) = \frac{(\cos x - \cos 5x)}{x^2}$$

and zooming in toward the point where the graph crosses the y-axis, estimate the value of $\lim_{x\to 0} f(x)$.

9. Consider the following function.

$$f(x) = \begin{cases} 3 - x & x < -1 \\ x & -1 \le x < 3 \\ (x - 3)^2 & x \ge 3 \end{cases}$$

Determine the values of a for which $\lim_{x \to a} f(x)$ exists.

- 10. How close to 2 do we have to take x so that 5x + 3 is within a distance of 0.01 from 13?
- 11. Find the limit.

$$\lim_{x\to 0^+} \tan^{-1}\left(\frac{2}{x}\right)$$

12. Evaluate the limit.

$$\lim_{x \to \infty} \frac{10x^2 - 3x + 1}{7x^2 + 3x - 3}$$

13. Find the limit.

$$\lim_{x \to \frac{10}{x}} \tan^{-1} \left(\frac{5}{x} \right)$$

14. Evaluate the limit.

$$\lim_{x \to 9} \frac{3 - \sqrt{x}}{x - 9}$$

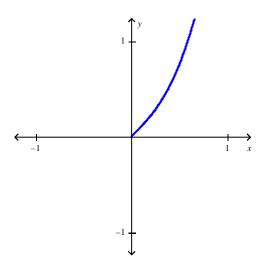
15. Find the limit if $g(x) = x^4$.

$$\lim_{x\to 2} \frac{g(x) - g(2)}{x - 2}$$

16. Use a graph to find a number δ such that $|\sqrt{4x+1}-3| < 0.1$ whenever $|x-2| < \delta$.

© 2016 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

- 17. If f and g are continuous functions with f(7) = 10 and $\lim_{x \to 7} [2f(x) g(x)] = 7$, find g(7).
- 18. The following figure shows a portion of the graph of a function f defined on the interval [-1, 1]. Sketch the complete graph of f if it is known f is odd.

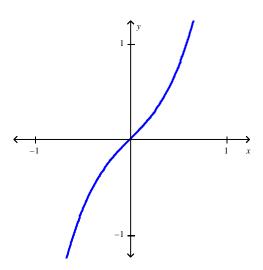


- 19. Let $f(x) = x^2 18x + 75$ and $g(x) = \sqrt{x+7}$. Find $(f \circ g)(74)(g \circ g)(74)$.
- 20. Find the limit $\lim_{x \to 3} \frac{x^2 + x 12}{x^2 9}$, if it exists.

Answer Key

- 1. [-7, 7]
- 2. $\frac{4}{3}\pi(3r^2+3r+1)$
- 3. a) 3 species
 - b) $702 \,\mathrm{m}^2$
- 4. $12\pi(r^2+3r+3)$
- 5. r(t) = 45t, $2025\pi t^2$
- 6. 27.5 ft/s
- 7. 2.612794, 2.540047, 2.506608, 2.477975, 2.475183, 2.362146, 2.415915, 2.444688, 2.471788, 2.474564, *Limit*: 2.474874
- 8. 12
- 9. $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$
- 10. |x-2| < 0.002
- 11. $\frac{\pi}{2}$
- 12. $\frac{10}{7}$
- 13. 0
- 14. -1/6
- 15. 32
- 16. $\delta \le 0.15$
- 17. 13

18.



- 19. –6
- 20. $\frac{7}{6}$

Select the correct answer for each question.

_____ 1. If
$$f(x) = x^2 - x + 6$$
, evaluate the difference quotient $\frac{f(a+h) - f(a)}{h}$.

a.
$$2a + h - 6$$

b.
$$2a - 6$$

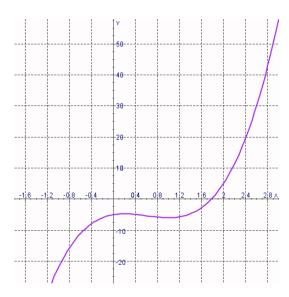
c.
$$2a - h - 6$$

2. Find the domain of the function
$$f(x) = \frac{x}{-2\sin x - 3}$$
.

b.
$$\left[-\frac{3}{2}, \infty\right]$$

c.
$$[-3, -2]$$

3. The graph of the function f is given. State the value of f(-0.4).



a.
$$f(-0.4) = 8$$

b.
$$f(-0.4) = -10$$

c.
$$f(-0.4) = -8$$

d.
$$f(-0.4) = 0$$

e.
$$f(-0.4) = 10$$

4. Scientists have discovered that a linear relationship exists between the amount of flobberworm mucus secretions and the air temperature. When the temperature is 65°F, the flobberworms each secrete 16 grams of mucus a day; when the temperature is 95°F, they each secrete 22 grams of mucus a day. Find a function M(t) that gives the amount of mucus secreted on a given day, where t is the temperature of that day in degrees Fahrenheit.

a.
$$M(t) = 0.2t + 16$$

b.
$$M(t) = 5t + 16$$

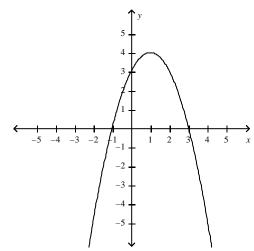
c.
$$M(t) = 5t + 3$$

d.
$$M(t) = 0.2t + 3$$

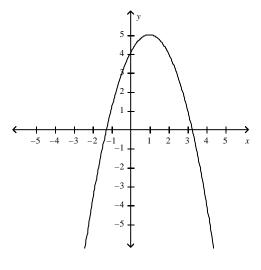
5. Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions and then applying the appropriate transformations.

$$y = 4 + 2x - x^2$$

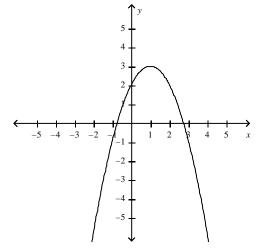
a.



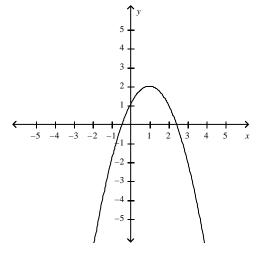
c.



b.

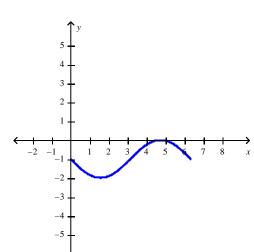


d.

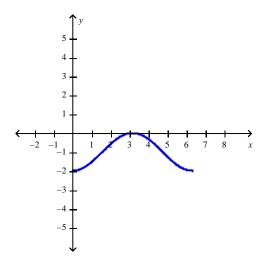


6. Sketch the graph of $y = -1 - \cos x$ over one period.

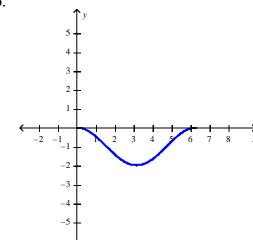
a.



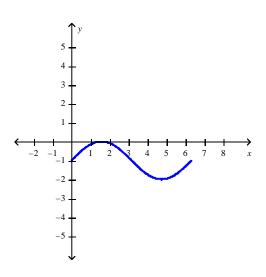
c.



b.



d.



7. The graph of the function $f(x) = x^2 - 11x + 7$ has been stretched horizontally by a factor of 2. Find the function for the transformed graph.

a.
$$g(x) = \frac{x^2 - 11x + 7}{2}$$

b.
$$g(x) = 2x^2 - 22x + 14$$

c.
$$g(x) = \frac{x^2 - 22x + 28}{4}$$

d.
$$g(x) = 4x^2 - 22x + 7$$

8. A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after t minutes. When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute. The monitor estimates this value by calculating the slope of a secant line. Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with t = 38 and t = 42.

t (mins)	36 38		40	42	44
Heartbeats	2570	2720	2840	3020	3070

a. 74

9. If a ball is thrown into the air with a velocity of 58 ft/s, its height (in feet) after *t* seconds is given by

$$H = 58t - 9t^2$$
.

Find the velocity when t = 9.

a.
$$-101 \text{ ft/s}$$

b.
$$-104 \text{ ft/s}$$

c.
$$-106$$
 ft/s

d.
$$-103$$
 ft/s

e.
$$-99$$
 ft/s

© 2016 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

b. 80

c. 85

10. The position of a car is given by the values in the following table.

t (seconds)	0	1	2	3	4
s (meters)	0	21.9	25.8	69.2	92.2

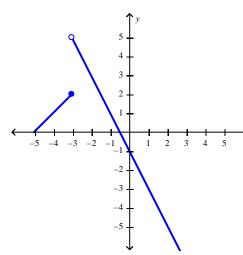
Find the average velocity for the time period beginning when t = 2 and lasting 2 seconds.

- a. 46.1 ft/s
- b. 31.9 ft/s
- c. 33.5 ft/s
- d. 33.8 ft/s
- e. 33.2 ft/s
- 11. Suppose the distance s (in feet) covered by a car moving along a straight road after t sec is given by the function $s = f(t) = 3t^2 + 13t$. Calculate the (instantaneous) velocity of the car when t = 35.
 - a. 223 ft/sec
 - b. 16 ft/sec
 - c. 560 ft/sec
 - d. 4130 ft/sec

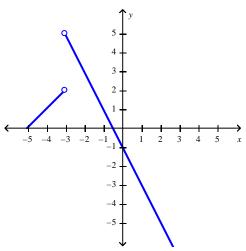
12. Sketch the graph of the function f and evaluate $\lim_{x \to -3^+} f(x)$.

$$f(x) = \begin{cases} x + 5, & \text{if } x \le -3 \\ -2x - 1, & \text{if } x > -3 \end{cases}$$

a.



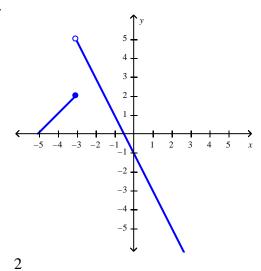
c.



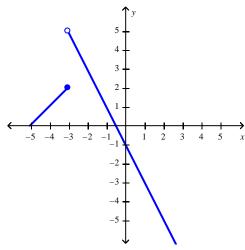
Does not exist

Does not exist

b.



d.



5

____ 13. Evaluate the limit.

$$\lim_{x \to 1} (x+5)^3 \left(x^2 - 6\right)$$

- a. -448
- b. -1070
- c. -1090
- d. -1080
- e. 320

____ 14. If $6x - 1 \le f(x) \le x^2 - 1$, find $\lim_{x \to 6} f(x)$.

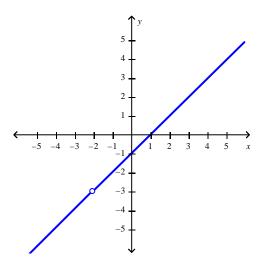
- a. 1
- b. 35
- c. 0
- d. -6
- e. -35

15. Find the limit.

$$\lim_{x \to 2} \frac{x^2 + 2x - 12}{x - 2}$$

- a. 6
- b. 6
- c. 10
- d. 12
- e. 1

16. Use the graph of $f(x) = \frac{x^2 + x - 2}{x + 2}$ to guess at the limit $\lim_{x \to -2} \frac{x^2 + x - 2}{x + 2}$, if it exists.



- a. -2
- b. -1
- c. Does not exist
- d. -3

17. Let
$$F(x) = \frac{x-5}{|x-5|}$$
. Find the following limits.

$$\lim_{x \to 5^+} F(x), \lim_{x \to 5^-} F(x)$$

- a. both 1
- b. 2 and 1
- c. 2 and -1
- d. 1 and -1
- e. both-1

18. Which of the given functions is discontinuous?

a.
$$f(x) = \begin{cases} \frac{1}{x - 11}, & x \neq 11 \end{cases}$$

a.
$$f(x) = \begin{cases} \frac{1}{x - 11}, & x \neq 11 \\ 9, & x = 11 \end{cases}$$
b.
$$f(x) = \begin{cases} \frac{1}{x - 2}, & x \geq 11 \\ \frac{1}{9}, & x < 11 \end{cases}$$

_____ 19. Find the numbers, if any, where the function
$$f(x) = \begin{cases} 3x - 2 & \text{if } x \le 1 \\ 0 & \text{if } x > 1 \end{cases}$$
 is discontinuous.

____ 20. Find the interval(s) where
$$f(x) = \sqrt{x^2 - 2x + 3}$$
 is continuous.

Answer Key

- 1. E
- 2. A
- 3. C
- 4. D
- 5. C
- 6. C
- 7. C
- 8. F
- 9. B
- 10. E
- 11. A
- 12. D
- 13. D
- 14. B
- 15. A
- 16. C
- 17. D
- 18. A
- 19. C
- 20. C

Select the correct answer for each question.

1. Which of the following graphs is neither even nor odd?

a.
$$f(x) = \frac{4x^2}{x^4 + 1}$$

b.
$$f(x) = 8x^3 + 10x^2 + 1$$

c.
$$f(x) = x^3 - 9x$$

2. Find the domain of the function.

$$f(x) = \frac{7x + 1}{x^2}$$

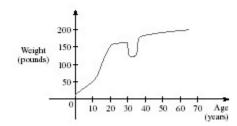
a.
$$(-\infty, 0)$$

b.
$$\left(-\infty, -\frac{1}{7}\right) \cup \left(-\frac{1}{7}, \infty\right)$$

c.
$$\left(-\infty, \frac{1}{7}\right) \cup \left(\frac{1}{7}, \infty\right)$$

d.
$$(-\infty, 0) \cup (0, \infty)$$

3. The graph shown gives the weight of a certain person as a function of age. Find the age at which the person started an exercise program.



4. The relationship between the Fahrenheit and Celsius temperature scales is given by the linear function.

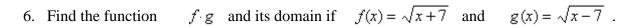
$$F = \frac{9}{5}C + 32$$

What is the *F*-intercept and what does it represent?

- a. $\frac{9}{5}$, Fahrenheit temperature corresponding to $0 \,^{\circ} C$
- b. $\frac{9}{5}$, Celsius temperature corresponding to 32° C
- c. 32, Celsius temperature corresponding to $0 \,{}^{\circ} F$
- d. 0, Fahrenheit temperature corresponding to $32^{\circ}C$
- e. 32, Fahrenheit temperature corresponding to $0 \,{}^{\circ}C$
- ___ 5. Use the table to evaluate the expression $(f \circ g)(6)$.

х	1	2	3	4	5	6
f(x)	3	2	1	0	1	2
g(x)	6	5	2	3	4	6

- a. 5
- b. 2
- c
- d 1
- e. 6



a.
$$\sqrt{x^2 - 49}$$

$$D = [7, \infty)$$

b.
$$\sqrt{x^2 - 49}$$

$$D = [-7, \infty)$$

c.
$$\sqrt{x^2 + 49}$$

$$D = [-7, \infty)$$

d.
$$\sqrt{x^2 + 49}$$

$$D = [7, \infty)$$

7. The displacement (in feet) of a certain particle moving in a straight line is given by $\varepsilon = \frac{t^3}{2}$

where t is measured in seconds. Find the average velocity over the interval [1, 1.19].

Round your answer to three decimal places.

 8.	If a rock is thrown upward on the planet Mars with a velocity of 12 m/s , its height in meters t
	seconds later is given by

$$y = 12t - 1.92t^2.$$

Find the average velocity over the time interval [2, 3].

a.
$$-0.6 \text{ m/s}$$

9. The point
$$P(16, 4)$$
 lies on the curve $y = \sqrt{x}$. If is the point $Q(x, \sqrt{x})$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the value $x = 3.89$.

a.
$$m_{PQ} = -0.044439$$

b.
$$m_{PQ} = -0.167439$$

c.
$$m_{PQ} = -0.307439$$

d.
$$m_{PQ} = 0.377439$$

e.
$$m_{PQ} = 0.167439$$

10. Find the value of
$$\lim_{x\to 0^+} f(x)$$
.

$$f(x) = \frac{1}{1+6^{1/x}}.$$

b.
$$-0.7$$

c.
$$-0.7$$

____ 11. If $1 \le f(x) \le x^2 + 6x + 6$, for all x, find $\lim_{x \to -1} f(x)$.

a.
$$-\frac{1}{8}$$

- b. 1
- c. $-\frac{1}{16}$
- d. 8
- e. does not exist

____ 12. Find the limit.

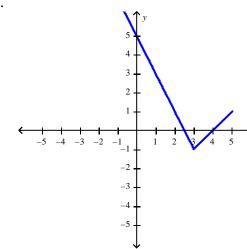
$$\lim_{t \to \infty} \frac{t^2 + 3}{t^3 + t^2 - 7}$$

- a. 00
- b. -3
- c. 0
- d. 3
- e. 7

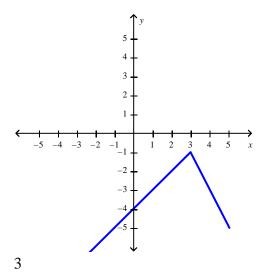
13. Sketch the graph of the function f and evaluate $\lim_{x \to 3} f(x)$.

$$f(x) = \begin{cases} x - 4, & \text{if } x \le 3\\ -2x + 5, & \text{if } x > 3 \end{cases}$$

a.



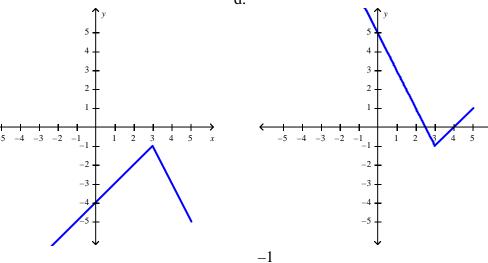
c.



3

b.

d.



-1

14. Find the limit.

$$\lim_{x \to 2} \sqrt{\frac{4x^2 + 1}{3x - 2}}$$

- a. 0
- b. $-\frac{4}{3}$
- c. $\frac{\sqrt{17}}{2}$
- d. $\frac{4}{3}$
- e. does not exist

____ 15. If f and g are continuous functions with f(9) = 6 and $\lim_{x \to 9} [2f(x) - g(x)] = 9$, find g(9).

a.
$$g(9) = 21$$

b.
$$g(9) = 15$$

c.
$$g(9) = 12$$

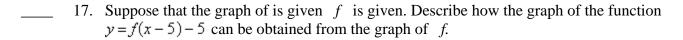
d.
$$g(9) = 24$$

e.
$$g(9) = 3$$

16. Let

$$f(x) = \begin{cases} x - 4 & \text{if } x \le 5\\ kx^2 - 24x + 46 & \text{if } x > 5 \end{cases}$$

Find the value of k that will make f continuous on $(-\infty, \infty)$.



- a. Shift the graph 5 units to the left and 5 units down.
- b. Shift the graph 5 units to the left and 5 units up.
- c. Shift the graph 5 units to the right and 5 units up.
- d. Shift the graph 5 units to the right and 5 units down.
- e. None of these
- 18. The position of a car is given by the values in the following table.

t (seconds)	0	1	2	3	4
s (meters)	0	21.9	25.8	69.2	92.2

Find the average velocity for the time period beginning when t = 2 and lasting 2 seconds.

- a. 46.1 ft/s
- b. 31.9 ft/s
- c. 33.5 ft/s
- d. 33.8 ft/s
- e. 33.2 ft/s
- 19. Let $F(x) = \frac{x-5}{|x-5|}$. Find the following limits.

$$\lim_{x \to 5^+} F(x), \lim_{x \to 5^-} F(x)$$

- a. both 1
- b. 2 and 1
- c. 2 and -1
- d. 1 and -1
- e. both-1

- ____ 20. Define the function $f(x) = \frac{6x^3 + x}{4x}$ at 0 so as to make it continuous at 0.
 - a. $f(0) = \frac{3}{2}$
 - b. $f(0) = \frac{7}{4}$
 - c. f(0) = 0
 - d. $f(0) = \frac{1}{4}$

Answer Key

- 1. B
- 2. D
- 3. D
- 4. E
- 5. B
- 6. A
- 7. F
- 8. C
- 9. E
- 10. A
- 11. B
- 12. C
- 13. B
- 14. C
- 15. E
- 16. D
- 17. D
- 18. E
- 19. D
- 20. D

- 1. An open rectangular box with volume 2 m^3 has a square base. Express the surface area of the box as a function S(x) of the length x of a side of the base.
- 2. Find a, such that the function $f(x) = 4x + \sqrt{a x^2}$ has the domain (-4, 4).

Select the correct answer.

a.
$$a = -16$$

b.
$$a = \sqrt{4}$$

c.
$$a = -\sqrt{4}$$

d.
$$a = 16$$

e.
$$a = 4$$

3. Find the domain.

$$g(u) = \sqrt{u} - \sqrt{3-u}$$

4. Find the range of the function.

$$y = 2 + \cos x$$

Select the correct answer.

5. The relationship between the Fahrenheit and Celsius temperature scales is given by the linear function.

$$F = \frac{9}{5}C + 32$$

What is the *F*-intercept and what does it represent?

6. The monthly cost of driving a car depends on the number of miles driven. Julia found that in October it cost her \$200 to drive 300 mi and in July it cost her \$350 to drive 600 mi. Express the monthly cost *C* as a function of the distance driven *d* assuming that a linear relationship gives a suitable model.

- 7. Suppose that the graph of is given f is given. Describe how the graph of the function y = f(x-5) 5 can be obtained from the graph of f.
- 8. Find the function $f \cdot g$ and its domain if $f(x) = \sqrt{x+7}$ and $g(x) = \sqrt{x-7}$.
- 9. A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after t minutes. When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute. The monitor estimates this value by calculating the slope of a secant line. Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with t = 38 and t = 42.

t (mins)	36	38	40	42	44
Heartbeats	2570	2720	2840	3020	3070

Select the correct answer.

- a. 74
- b. 80
- c. 85
- d. 70
- e. 76
- f. 75
- 10. Estimate the value of the following limit by graphing the function $f(x) = \frac{(5 \sin x)}{(\sin \pi x)}$.

$$\lim_{x \to 0} \frac{5 \sin x}{\sin \pi x}$$

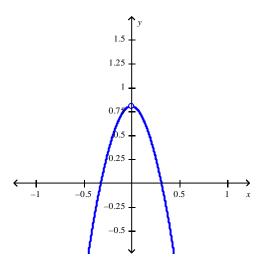
Round your answer correct to two decimal places.

11. Evaluate the limit.

$$\lim_{x \to 1} (x+5)^3 \left(x^2 - 6\right)$$

12. Find the limit $\lim_{x\to 0} \frac{\sqrt{x+6} - \sqrt{6}}{x}$, if it exists.

- 13. Is there a number a such that $\lim_{x \to -3} \frac{3x^2 + ax + a + 3}{x^2 + x 6}$ exists? If so, find the value of a and the value of the limit.
- 14. Use the graph of $f(x) = \frac{\sin 4x}{\tan 5x}$ to guess at the limit $\lim_{x \to 0} \frac{\sin 4x}{\tan 5x}$, if it exists.



15. Let $F(x) = \frac{x-5}{|x-5|}$. Find the following limits.

$$\lim_{x \to 5^+} F(x), \lim_{x \to 5^-} F(x)$$

16. You are given $\lim_{x \to a} f(x) = L$ and a tolerance ε . Find a number δ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

$$\lim_{x \to 3} 4x = 12; \quad \varepsilon = 0.01$$

17. If f and g are continuous functions with f(9) = 6 and $\lim_{x \to 9} [2f(x) - g(x)] = 9$, find g(9).

18. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx + 5 & \text{for } x \le 2\\ cx^2 - 5 & \text{for } x > 2 \end{cases}$$

Select the correct answer.

a.
$$c = 1$$

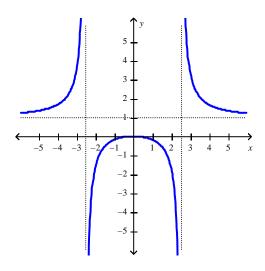
b.
$$c = 5$$

c.
$$c = -2$$

d.
$$c = -5$$

e.
$$c = 2$$

19. Use the graph to determine where the function is discontinuous.



20. Find an expression for the function y = f(x) whose graph is the bottom half of the parabola $x + (6 - y)^2 = 0$.

Answer Key

1.
$$S(x) = x^2 + \frac{8}{x}$$

- 2. D
- 3. [0, 3]
- 4. E
- 5. 32, Fahrenheit temperature corresponding to $0 \,{}^{\circ}C$
- 6. C = 0.5d + 50
- 7. Shift the graph 5 units to the right and 5 units down.

8.
$$\sqrt{x^2 - 49}$$

$$D = [7, \infty)$$

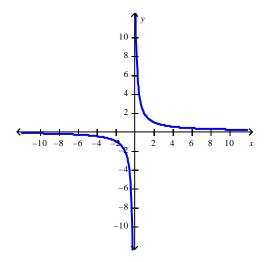
- 9. F
- 10. 1.59
- 11. -1080

12.
$$\frac{\sqrt{6}}{12}$$

- 13. a = 15, limit equals 0.6
- 14. 0.8
- 15. 1 and -1
- 16. 0.0025
- 17. g(9) = 3
- 18. B
- 19. At ±2.5
- 20. $y = 6 \sqrt{-x}$

- 1. If the point (7,3) is on the graph of an even function, what other point must also be on the graph? Select the correct answer.
 - a. (-7, -3)
 - b. (7,-3) c. (0,0)

 - d. (-7, 3)
 - e. None of these
 - 2. Find an expression for the function y = f(x) whose graph is the bottom half of the parabola $x + \left(6 - y\right)^2 = 0.$
 - 3. Determine whether the function whose graph is given is even, odd, or neither.



- 4. Find the domain of the function $f(x) = \frac{x}{-2\sin x 3}$.
 - a. $(-\infty, \infty)$
 - b. $\left[-\frac{3}{2}, \infty\right]$
 - c. [-3, -2]
 - d. [2, 3]

5. Find a, such that the function
$$f(x) = 4x + \sqrt{a - x^2}$$
 has the domain $(-4, 4)$.

a.
$$a = -16$$

b.
$$a = \sqrt{4}$$

c.
$$a = -\sqrt{4}$$

d.
$$a = 16$$

e.
$$\alpha = 4$$

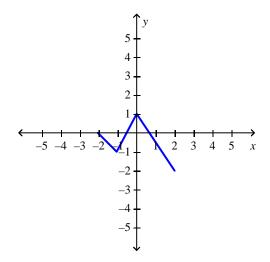
6. Determine whether
$$f$$
 is even, odd, or neither.

$$f(x) = \frac{4x^2}{x^4 + 5}$$

$$y = 2 + \cos x$$

$$y = 4 + 2x - x^2$$

9. The graph of the function
$$f$$
 follows. Choose the graph of $y = \frac{1}{2}f(x-1)$.



_	10.	If a rock is thrown upward on the planet Mars with a velocity of 12 m/s , its height in meters t
		seconds later is given by

$$y = 12t - 1.92t^2$$
.

Find the average velocity over the time interval [2, 3].

- a. -0.6 m/s
- b. 4.4 m/s
- c. 2.4 m/s
- d. 3.4 m/s
- e. 1.4 m/s
- 11. Use the graph of the function to state the value of $\lim_{x\to 0} f(x)$, if it exists.

$$f(x) = \frac{1}{1 + 4^{1/x}}$$

12. Find the limit.

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 - 25}}{2x - 6}$$

13. Find the limit
$$\lim_{x\to 5} \frac{x+1}{x^2-4x+2}$$
.

14. Evaluate
$$\lim_{h \to 0} \frac{\cot\left(\frac{\pi}{4} + h\right) - 1}{h}$$
.

15. Find the limit.

$$\lim_{x \to 2} \frac{x^2 + 2x - 12}{x - 2}$$

16. Which of the given functions is discontinuous?

17. How would you define f(7) in order to make f continuous at 7?

$$f(x) = \frac{x^2 - 2x - 3}{x - 7}$$

a.
$$f(3) = -8$$

b.
$$f(7) = 12$$

c. $f(7) = 0$

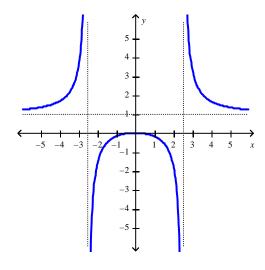
c.
$$f(7) = 0$$

d.
$$f(7) = -12$$

18. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx + 5 & \text{for } x \le 2\\ cx^2 - 5 & \text{for } x > 2 \end{cases}$$

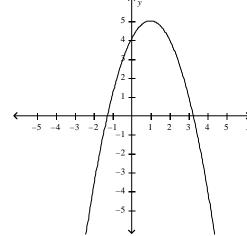
19. Use the graph to determine where the function is discontinuous.



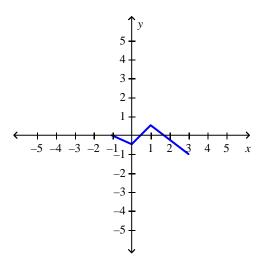
20. Find the numbers, if any, where the function $f(x) = \begin{cases} 3x - 2 & \text{if } x \le 1 \\ 0 & \text{if } x > 1 \end{cases}$ is discontinuous.

Answer Key

- 1. D
- 2. $y = 6 \sqrt{-x}$
- 3. Odd
- 4. A
- 5. D
- 6. C
- 7. [1, 3]
- 8.



9.



- 10. C
- 11. does not exist
- 12. $\frac{1}{2}$
- 13. $\frac{6}{7}$

15. 6

16.
$$f(x) = \begin{cases} \frac{1}{x - 11}, & x \neq 11 \\ 9, & x = 11 \end{cases}$$

18.
$$c = 5$$

1. If
$$f(x) = x^2 - x + 6$$
, evaluate the difference quotient $\frac{f(a+h) - f(a)}{h}$.

2. If
$$f(x) = 4x^2 + 2$$
, find and simplify $\frac{f(1+h) - f(1)}{h}$, where $h \ne 0$.

3. Find the domain of the function
$$f(x) = \frac{x}{-2\sin x - 3}$$
.

4. Determine whether f is even, odd, or neither.

$$f(x) = \frac{4x^2}{x^4 + 5}$$

5. The monthly cost of driving a car depends on the number of miles driven. Julia found that in October it cost her \$200 to drive 300 mi and in July it cost her \$350 to drive 600 mi. Express the monthly cost *C* as a function of the distance driven *d* assuming that a linear relationship gives a suitable model.

a.
$$C = -50d + 0.5$$

b.
$$C = 50d - 0.5$$

c.
$$C = 0.5d + 50$$

d.
$$C = 2d + 50$$

e.
$$C = 0.5d - 50$$

6. What is
$$\sqrt[10]{x}$$
, given that $H = f \circ g \circ h$ and $H(x) = \sqrt[10]{\sqrt{x} - 3}$?

7. Find the function
$$f \cdot g$$
 and its domain if $f(x) = \sqrt{x+7}$ and $g(x) = \sqrt{x-7}$.

8. Find the function
$$f \circ g$$
 and its domain if $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{x}{x+3}$.

 9.	Suppose the distance s (in feet) covered by a car moving along a straight road after t sec is given
	by the function $s = f(t) = 3t^2 + 13t$. Calculate the (instantaneous) velocity of the car when $t = 35$.

Select the correct answer.

- a. 223 ft/sec
- b. 16 ft/sec
- c. 560 ft/sec
- d. 4130 ft/sec

10. If
$$\lim_{x \to 2^+} f(x) = 7.9$$
, then if $\lim_{x \to 2} f(x)$ exists, to what value does it converge?

Select the correct answer.

- a. 6.9
- b. 8.9
- c. 9.9
- d. 7.9
- e. 5.9
- 11. Find the value of the limit.

$$\lim_{x \to 0} 3 \frac{\tan 4x - 4x}{x^3}$$

12. Find the limit.

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 - 25}}{2x - 6}$$

13. If
$$1 \le f(x) \le x^2 + 6x + 6$$
, for all x , find $\lim_{x \to -1} f(x)$.

14. Find the limit
$$\lim_{x \to 0} \frac{\sqrt{x+6} - \sqrt{6}}{x}$$
, if it exists.

15. Is there a number a such that $\lim_{x \to -3} \frac{3x^2 + ax + a + 3}{x^2 + x - 6}$ exists? If so, find the value of a and the value of the limit.

Select the correct answer.

- a. a = 15, limit equals -0.6
- b. a = 15, limit equals 0.6
- c. a = -15, limit equals 0.6
- d. a = -15, limit equals -0.6
- e. a = 15, limit equals 18
- 16. Find the limit.

$$\lim_{x\to 2} \sqrt{\frac{4x^2+1}{3x-2}}$$

17. Use continuity to evaluate the limit.

$$\lim_{x \to 3\pi} \sin(x + 4\sin x)$$

- 18. Which of the given functions is discontinuous?
- 19. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx + 5 & \text{for } x \le 2\\ cx^2 - 5 & \text{for } x > 2 \end{cases}$$

20. Find the interval(s) where $f(x) = \sqrt{x^2 - 2x + 3}$ is continuous.

Answer Key

- 1. none of these
- 2. 8+4h
- 3. (-∞, ∞)
- 4. even
- 5. C
- 6. f(x)
- 7. $\sqrt{x^2 49}$

$$D = [7, \infty)$$

8.
$$-\frac{3}{x}$$
,

$$D = (-\infty, -3) \cup (-3, 0) \cup (0, \infty)$$

- 9. A
- 10. D
- 11. 64
- 12. $\frac{1}{2}$
- 13. 1
- 14. $\frac{\sqrt{6}}{12}$
- 15. B
- 16. $\frac{\sqrt{17}}{2}$
- 17. 0

17. 0

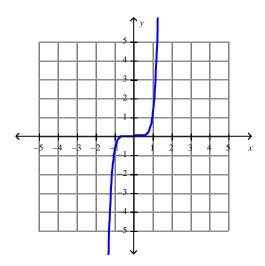
18.
$$f(x) = \begin{cases} \frac{1}{x - 11}, & x \neq 11 \\ 9, & x = 11 \end{cases}$$

- 19. c = 5
- 20. (-∞, ∞)

1. Find the domain of the function.

$$f(x) = \frac{7x + 1}{x^2}$$

2. What is the equation of this graph?



3. Determine whether f is even, odd, or neither.

$$f(x) = \frac{4x^2}{x^4 + 5}$$

4. Classify the function as a Polynomial function, a Rational function, an algebraic function, or other.

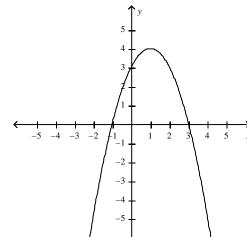
$$f(x) = -8x^{-7} - x^{-5} - 7$$

5. Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions and then applying the appropriate transformations.

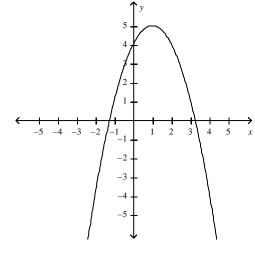
$$y = 4 + 2x - x^2$$

Select the correct answer.

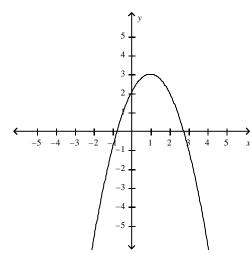
a.



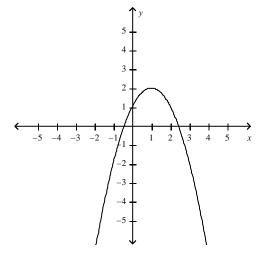
c.



b.



d.



6. Suppose that the graph of is given f is given. Describe how the graph of the function y = f(x-5) - 5 can be obtained from the graph of f.

7. If a ball is thrown into the air with a velocity of 58 ft/s, its height (in feet) after *t* seconds is given by

$$H = 58t - 9t^2$$
.

Find the velocity when t = 9.

- 8. The point P(16, 4) lies on the curve $y = \sqrt{x}$. If is the point $Q(x, \sqrt{x})$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the value x = 3.89.
- 9. Estimate the value of the following limit by graphing the function $f(x) = \frac{(5 \sin x)}{(\sin \pi x)}$.

$$\lim_{x \to 0} \frac{5 \sin x}{\sin \pi x}$$

Round your answer correct to two decimal places.

10. Use the graph of the function to state the value of $\lim_{x\to 0} f(x)$, if it exists.

$$f(x) = \frac{x^2 + x}{2\sqrt{x^3 + x^2}}$$

11. Sketch the graph of the function f and evaluate $\lim_{x \to -3^+} f(x)$.

$$f(x) = \begin{cases} x+5, & \text{if } x \le -3\\ -2x-1, & \text{if } x > -3 \end{cases}$$

12. If $\lim_{x \to 2^+} f(x) = 7.9$, then if $\lim_{x \to 2} f(x)$ exists, to what value does it converge?

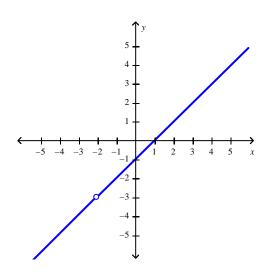
_____ 13. Find the limit
$$\lim_{x\to 0} \frac{\sqrt{x+6} - \sqrt{6}}{x}$$
, if it exists.

- a. Does not exist
- b. $\frac{\sqrt{6}}{12}$
- c. $\frac{\sqrt{6}}{2}$
- d. $\frac{\sqrt{6}}{6}$

14. Evaluate
$$\lim_{h \to 0} \frac{\cot\left(\frac{\pi}{4} + h\right) - 1}{h}$$
.

15. If
$$6x - 1 \le f(x) \le x^2 - 1$$
, find $\lim_{x \to 6} f(x)$.

16. Use the graph of
$$f(x) = \frac{x^2 + x - 2}{x + 2}$$
 to guess at the limit $\lim_{x \to -2} \frac{x^2 + x - 2}{x + 2}$, if it exists.



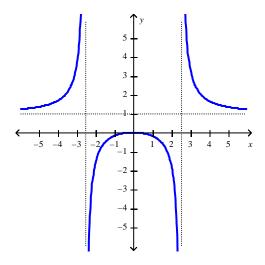
17. A machinist is required to manufacture a circular metal disk with area 1000 cm^2 . If the machinist is allowed an error tolerance of $\pm 15 \text{ cm}^2$ in the area of the disk, how close to the ideal radius must the machinist control the radius?

Round your answer to the nearest hundred thousandth. Select the correct answer.

- a. $\delta \le 0.13131 \, \text{cm}$
- b. $\delta \le 0.13281 \, \text{cm}$
- c. $\delta \le 0.13231 \, \text{cm}$
- d. $\delta \leq 0.13331 \, \text{cm}$
- e. $\delta \le 0.13431 \, \text{cm}$
- 18. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx + 5 & \text{for } x \le 2\\ cx^2 - 5 & \text{for } x > 2 \end{cases}$$

19. Use the graph to determine where the function is discontinuous.



20. Find the numbers, if any, where the function $f(x) = \begin{cases} 3x - 2 & \text{if } x \le 1 \\ 0 & \text{if } x > 1 \end{cases}$ is discontinuous.

- a. -2
- b. 2
- c. 1
- d. 0

Answer Key

1.
$$(-\infty, 0) \cup (0, \infty)$$

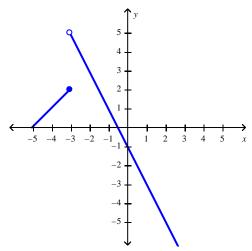
2.
$$y = x^7$$

6. Shift the graph 5 units to the right and 5 units down.

7.
$$-104$$
 ft/s

8.
$$m_{PQ} = 0.167439$$

11.



5

12. D

13. B

14. 2

15. 35

16. Does not exist

17. D

18. c = 5

19. At ±2.5

20. C

1. Differentiate.

$$y = \frac{\sin x}{6 + \cos x}$$

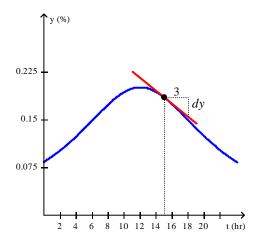
2. Find the limit.

$$\lim_{\theta \to 0} 4 \frac{\sin(\sin 4\theta)}{\sec 4\theta}$$

3. Differentiate.

$$y = \frac{\sin x}{3 + \cos x}$$

4. The graph shows the percentage of households in a certain city watching television during a 24-hr period on a weekday (t = 0 corresponds to 6 a.m.). By computing the slope of the respective tangent line, estimate the rate of change of the percentage of households watching television at a-12 p.m. Note that dy = 0.03



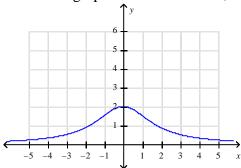
- 5. Suppose the total cost in maunufacturing x units of a certain product is C(x) dollars.
 - **a.** What does C'(x) measure? Give units.
 - **b.** What can you say about the sign of C'?
 - **c.** Given that C'(3000) = 11, estimate the additional cost in producing the 3001st unit of the product.

6. The level of nitrogen dioxide present on a certain June day in downtown Megapolis is approximated by

$$A(t) = 0.03t^3(t-7)^4 + 64.8$$
 $0 \le t \le 7$

where A(t) is measured in pollutant standard index and t is measured in hours with t = 0 corresponding to 7 a.m. What is the average level of nitrogen dioxide in the atmosphere from 1 a.m. to 2 p.m. on that day? Round to three decimal places.

7. Sketch the graph of the derivative f' of the function f whose graph is given.



- 8. Let $f(x) = x |x^3|$.
 - **a.** Sketch the graph of f.
 - **b.** For what values of x is f differentiable?
 - **c.** Find a formula for f'(x).
- 9. Suppose that f and g are functions that are differentiable at x = 1 and that f(1) = 1, f'(1) = -3, g(1) = 2, and g'(1) = 5. Find h'(1).

$$h(x) = \frac{xf(x)}{x + g(x)}$$

10. Find the derivative of the function.

$$f(x) = -x^2 + x + 2$$

11. Identify the "inside function" u = f(x) and the "outside function" y = g(u). Then find dy/dx using the Chain Rule.

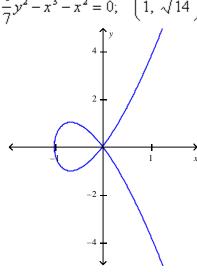
$$y = \sqrt{x^2 - 2}$$

12. Find the derivative of the function.

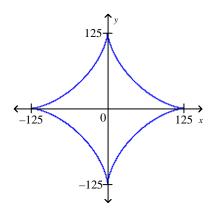
$$f(x) = x \sin^8 x$$

13. Find an equation of the tangent line to the given curve at the indicated point.

$$\frac{1}{7}y^2 - x^3 - x^2 = 0; \quad \left(1, \sqrt{14}\right)$$

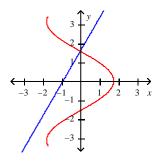


14. The curve with the equation $x^{2/3} + y^{2/3} = 25$ is called an asteroid. Find an equation of the tangent to the curve at the point $(48\sqrt{6}, 1)$.



15. Two curves are said to be **orthogonal** if their tangent lines are perpendicular at each point of intersection of the curves. Show that the curves of the given equations are orthogonal.

$$y - \frac{7}{4}x = \frac{\pi}{2}$$
, $x = \frac{7}{4} \cos y$

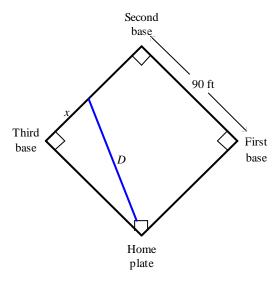


16. s(t) is the position of a body moving along a coordinate line; s(t) is measured in feet and t in seconds, where $t \ge 0$. Find the position, velocity, and speed of the body at the indicated time.

$$s(t) = \frac{4t}{t^2 + 1}; \qquad t = 3$$

- 17. In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the polluted area is circular, determine how fast the area is increasing when the radius of the circle is 20 ft and is increasing at the rate of $\frac{1}{6}$ ft/sec. Round to the nearest tenth if necessary.
- 18. The volume of a right circular cone of radius r and height h is $V = \frac{\pi}{3}r^2h$. Suppose that the radius and height of the cone are changing with respect to time t.
 - **a.** Find a relationship between $\frac{dV}{dt}$, $\frac{dr}{dt}$, and $\frac{dh}{dt}$.
 - **b.** At a certain instant of time, the radius and height of the cone are 12 in. and 13 in. and are increasing at the rate of 0.2 in./sec and 0.5 in./sec, respectively. How fast is the volume of the cone increasing?
- 19. In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the polluted area is circular, determine how fast the area is increasing when the radius of the circle is 20 ft and is increasing at the rate of $\frac{1}{6}$ ft/sec. Round to the nearest tenth if necessary.

20. The sides of a square baseball diamond are 90 ft long. When a player who is between the second and third base is 30 ft from second base and heading toward third base at a speed of 24 ft/sec, how fast is the distance between the player and home plate changing? Round to two decimal places.

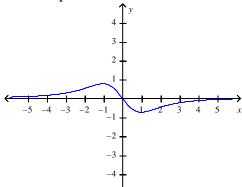


Answer Key

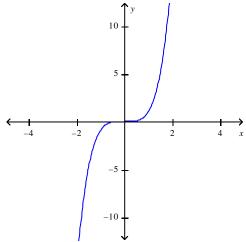
$$1. \quad \frac{dy}{dx} = \frac{6\cos x + 1}{\left(6 + \cos x\right)^2}$$

$$3. \quad \frac{dy}{dx} = \frac{3\cos x + 1}{\left(3 + \cos x\right)^2}$$

- 4. Falling at 1%/hr
- 5. **a.** C'(x), measured in dollars per unit, gives the instantaneous rate of changes of the total manufacturing cost C when x units of a certain product are produced.
 - **b.** Positive
 - **c.** \$11
- 6. 153.037 pollutant standard index



- 7.
- 8. **a.**



- **b.** $x \in (-\infty, \infty)$
- **c.** $f'(x) = \begin{cases} -4x^3 & \text{if } x < 0 \\ 4x^3 & \text{if } x \ge 0 \end{cases}$

9.
$$-\frac{4}{3}$$

10.
$$-2x + 1$$

11.
$$u = x^2 - 2$$
$$y = \sqrt{u}$$
$$dy \qquad x$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2 - 2}}$$

12.
$$\sin^8 x + 8x \cos x \sin^7 x$$

13.
$$y = \frac{5\sqrt{14}}{4}x - \frac{\sqrt{14}}{4}$$

14.
$$y = -\frac{\sqrt{6}}{12}x + 25$$

15. The curves intersect at
$$\left(0, \frac{\pi}{2}\right)$$
.

For
$$y - \frac{7}{4}x = \frac{\pi}{2}$$
, $m = \frac{7}{4}$.

For
$$x = \frac{7}{4} \cos y$$
, $m = -\frac{4}{7} \csc y$; at $\left(0, \frac{\pi}{2}\right)$, $m = -\frac{4}{7}$.

16.
$$\frac{6}{5}$$
 ft, $-\frac{8}{25}$ ft/sec, $\frac{8}{25}$ ft/sec

17.
$$20.9 \text{ ft}^2/\text{sec}$$

18. **a.**
$$\frac{dV}{dt} = \frac{\pi}{3} \left(r^2 \frac{dh}{dt} + 2rh \frac{dr}{dt} \right)$$

b.
$$44.8\pi \, \text{in.}^3/\text{sec}$$

1. The position function of a particle is given by

$$s = t^3 - 10.5t^2 - 2t$$
, $t \le 0$

When does the particle reach a velocity of 22 m/s?

- 2. Find an equation of the tangent line to the graph of $f(x) = 2x^2 7$ at the point (3, 11).
- 3. Find $\frac{dy}{dx}$ by implicit differentiation.

$$8\sqrt{x} + \sqrt{y} = 8$$

4. s(t) is the position of a body moving along a coordinate line; s(t) is measured in feet and t in seconds, where $t \ge 0$. Find the position, velocity, and speed of the body at the indicated time.

$$s(t) = \frac{4t}{t^2 + 1}; \qquad t = 3$$

- 5. The circumference of a sphere was measured to be 86 cm with a possible error of 0.8 cm. Use differentials to estimate the maximum error in the calculated volume.
- 6. If a cylindrical tank holds 10000 gallons of water, which can be drained from the bottom of the tank in an hour, then Torricelli's Law gives the volume of water remaining in the tank after *t* minutes as

$$V(t) = 10000 \left(1 - \frac{1}{60} t \right)^2, 0 \le t \le 60$$

Find the rate at which the water is flowing out of the tank (the instantaneous rate of change of V with respect to t) as a function of t.

- 7. Suppose the total cost in maunufacturing x units of a certain product is C(x) dollars.
 - **a.** What does C'(x) measure? Give units.
 - **b.** What can you say about the sign of C'?
 - **c.** Given that C'(3000) = 11, estimate the additional cost in producing the 3001st unit of the product.
- 8. Find the derivative of the function.

$$f(x) = -x^2 + x + 2$$

9. s(t) is the position of a body moving along a coordinate line; s(t) is measured in feet and t in seconds, where $t \ge 0$. Find the position, velocity, and speed of the body at the indicated time.

$$s(t) = \frac{3t}{t^2 + 1}; \qquad t = 2$$

10. s(t) is the position of a body moving along a coordinate line, where $t \ge 0$, and s(t) is measured in feet and t in seconds.

$$s(t) = -3 + 2t - t^2$$

- **a.** Determine the time(s) and the position(s) when the body is stationary.
- **b.** When is the body moving in the positive direction? In the negative direction?
- c. Sketch a schematic showing the position of the body at any time t.
- 11. Find the equation of the tangent to the curve at the given point.

$$y = \sqrt{16 + 4\sin x}$$
, (0,4)

12. Find the rate of change of y with respect to x at the given values of x and y.

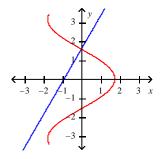
$$2xy^2 - 5x^2y + 192 = 0;$$
 $x = 4, y = 4$

13. Find an equation of the tangent line to the curve

$$xe^{y} + x + 2y = 2$$
 at $(1, 0)$.

14. Two curves are said to be **orthogonal** if their tangent lines are perpendicular at each point of intersection of the curves. Show that the curves of the given equations are orthogonal.

$$y - \frac{7}{4}x = \frac{\pi}{2}, \quad x = \frac{7}{4} \cos y$$



15. A spherical balloon is being inflated. Find the rate of increase of the surface area $S = 4\pi r^2$ with respect to the radius r when r = 1 ft.

16. s(t) is the position of a body moving along a coordinate line; s(t) is measured in feet and t in seconds, where $t \ge 0$. Find the position, velocity, and speed of the body at the indicated time.

$$s(t) = t^{10}e^{-t};$$
 $t = 1$

17. Find the differential of the function at the indicated number.

$$f(x) = e^{7x} + \ln(x+8); x = 0$$

18. Two chemicals react to form another chemical. Suppose that the amount of chemical formed in time *t* (in hours) is given by

$$x(t) = \frac{11\left[1 - \left(\frac{2}{3}\right)^{3t}\right]}{1 - \frac{1}{4}\left(\frac{2}{3}\right)^{3t}}$$

where x(t) is measured in pounds.

- a. Find the rate at which the chemical is formed when t = 4. Round to two decimal places.
- b. How many pounds of the chemical are formed eventually?

19. The volume of a right circular cone of radius r and height h is $V = \frac{\pi}{3}r^2h$. Suppose that the radius and height of the cone are changing with respect to time t.

- **a.** Find a relationship between $\frac{dV}{dt}$, $\frac{dr}{dt}$, and $\frac{dh}{dt}$.
- **b.** At a certain instant of time, the radius and height of the cone are 12 in. and 13 in. and are increasing at the rate of 0.2 in./sec and 0.5 in./sec, respectively. How fast is the volume of the cone increasing?
- 20. In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the polluted area is circular, determine how fast the area is increasing when the radius of the circle is 20 ft and is increasing at the rate of $\frac{1}{6}$ ft/sec. Round to the nearest tenth if necessary.

Answer Key

2.
$$y = 12x - 25$$

$$3. -\frac{8\sqrt{y}}{\sqrt{x}}$$

4.
$$\frac{6}{5}$$
 ft, $-\frac{8}{25}$ ft/sec, $\frac{8}{25}$ ft/sec

6.
$$V'(t) = \frac{-1000}{3} + \frac{50t}{9}$$

- 7. **a.** C'(x), measured in dollars per unit, gives the instantaneous rate of changes of the total manufacturing cost C when x units of a certain product are produced.
 - **b.** Positive

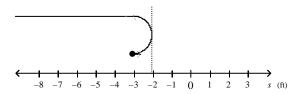
8.
$$-2x + 1$$

9.
$$\frac{6}{5}$$
 ft, $-\frac{9}{25}$ ft/sec, $\frac{9}{25}$ ft/sec

10. **a.**
$$s(1) = -2$$

b. Positive when
$$0 < t < 1$$
, negative when $t > 1$

c.



11.
$$y = \frac{1}{2}x + 4$$

13.
$$y = -\frac{2}{3}x + \frac{2}{3}$$

14. The curves intersect at
$$\left(0, \frac{\pi}{2}\right)$$
.

For
$$y - \frac{7}{4}x = \frac{\pi}{2}$$
, $m = \frac{7}{4}$.

For
$$x = \frac{7}{4} \cos y$$
, $m = -\frac{4}{7} \csc y$; at $\left(0, \frac{\pi}{2}\right)$, $m = -\frac{4}{7}$.

15.
$$8\pi$$

16.
$$\frac{1}{e}$$
 ft, $\frac{9}{e}$ ft/sec, $\frac{9}{e}$ ft/sec

17.
$$\frac{57}{8}dx$$

18. a. 0.08 lb/hr, b. 11 lbs

19. **a.**
$$\frac{dV}{dt} = \frac{\pi}{3} \left(r^2 \frac{dh}{dt} + 2rh \frac{dr}{dt} \right)$$

b.
$$44.8\pi \, \text{in.}^3/\text{sec}$$
 20. $20.9 \, \text{ft}^2/\text{sec}$

Select the correct answer for each question.

- 1. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.0017 cm thick to a hemispherical dome with diameter 70 m.
 - a. 4.165π
 - b. 2.52π
 - c. 4.11π
 - d. 3.82π
 - e. 2.28π
 - 2. Determine the values of x for which the given linear approximation is accurate to within 0.07 at a = 0.

 $\tan x \approx x$

a.
$$-0.19 < x < 0.28$$

b.
$$-0.57 < x < 0.57$$

c.
$$0.06 < x < 0.68$$

d.
$$-1.04 < x < 1.55$$

e.
$$-0.71 < x < 0.48$$

3. Find the differential of the function at the indicated number.

$$f(x) = 13\sin x + 4\cos x; \quad x = \frac{\pi}{4}$$

a.
$$\frac{9\sqrt{2}}{2} dx$$

b.
$$-\frac{17\sqrt{2}}{2} dx$$

c.
$$\frac{17\sqrt{2}}{2} dx$$

d.
$$-\frac{9\sqrt{2}}{2} dx$$

4. Find the linearization L(x) of the function at a.

$$f(x) = x^{2/3}$$
; $a = 64$

a.
$$\frac{8}{3}x + \frac{464}{3}$$

b.
$$\frac{8}{3}x + \frac{512}{3}$$

c.
$$\frac{8}{3}x - \frac{512}{3}$$

d.
$$\frac{8}{3}x - \frac{464}{3}$$

5. The slope of the tangent line to the graph of the exponential function $y = 6^x$ at the point (0, 1) is

 $\lim_{x \to 0} \frac{6^x - 1}{x}$. Estimate the slope to three decimal places.

6. If $g(x) = \sqrt{2-3x}$, use the definition of derivative to find g'(x).

a.
$$g'(x) = -\frac{1}{2}(2-3x)^{-1/2}$$

b.
$$g'(x) = -(2-3x)^{-1/2}$$

c.
$$g'(x) = -\frac{3}{2}(2-3x)^{1/2}$$

d.
$$g'(x) = -\frac{3}{2}(2-3x)^{-1/2}$$

e. None of these

7. Suppose that F(x) = f(g(x)) and g(14) = 2, g'(14) = 4, f'(14) = 15, and f'(2) = 13.

Find F'(14).

- a. 140
- b. 20
- c. 24
- d. 52
- e. 17

8. Find f' in terms of g'.

$$f(x) = \left[g(x) \right]^4$$

- a. f'(x) = 4g(x)
- b. $f'(x) = 4[g(x)]^3 g'(x)$
- c. $f'(x) = 4 [g'(x)]^3$
- d. f'(x) = 4[gx][xg'+g]
- e. f'(x) = 4g'(x)

9. Find the point(s) on the graph of f where the tangent line is horizontal.

$$f(x) = x^2 e^{-x}$$

- a. $(0,0), \left(2, \frac{2^2}{e^2}\right)$
- b. $\left(1, \frac{1}{e}\right)$
- c. (0,0)d. $\left[2, \frac{2^2}{e^2}\right]$

10. Find the derivative of the function.

$$f(x) = (4x + 9)^9$$

- a. $36(4x+9)^8$ b. $9(4x+9)^8$
- c. $9x(4x+9)^8$
- d. $36x(4x+9)^8$

11. Find an equation of the tangent line to the curve $120(x^2 + y^2)^2 = 2312(x^2 - y^2)$ at the point (4,1).

- a. y = -1.11x + 17
- b. y = -1.11x + 3.43
- c. y = 1.11x + 5.43
- d. y = -1.11x + 5.43
- e. None of these

12. The mass of the part of a metal rod that lies between its left end and a point x meters to the right is

$$S = 4x^2$$
.

Find the linear density when x is 3 m.

- a. 20
- b. 24
- c. 18
- d. 12
- e. 4

13.	In an adiabatic process (one in which no heat transfer takes place), the pressure P and volume V of
	an ideal gas such as oxygen satisfy the equation

$$P^5V^7=C,$$

where C is a constant. Suppose that at a certain instant of time, the volume of the gas is 2L, the pressure is 100 kPa, and the pressure is decreasing at the rate of 5 kPa/sec. Find the rate at which the volume is changing.

- a. 14 L/sec
- b. *C* 14 L/sec
- c. $C \frac{1}{14}$ L/sec
- d. $\frac{1}{14}$ L/sec
- 14. The quantity Q of charge in coulombs C that has passed through a point in a wire up to time t (measured in seconds) is given by

$$Q(t) = t^3 - 3t^2 + 4t + 3.$$

Find the current when t = 1s.

- a. 24
- b. 15
- c. 18
- d. 26
- e. 1

15.	If f is the focal length of a convex lens and an object is placed at a distance v from the lens, then its
	image will be at a distance u from the lens, where f , v , and u are related by the lens equation

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}.$$

Find the rate of change of v with respect to u.

a.
$$\frac{dv}{du} = -\frac{f}{\left(u - f\right)^2}$$

b.
$$\frac{dv}{du} = -\frac{f^2}{u - f}$$

c.
$$\frac{dv}{du} = \frac{2f^2}{\left(u - f\right)^2}$$

$$\frac{\mathrm{d}.}{du} = \frac{f^2}{\left(u - f\right)^2}$$

e.
$$\frac{dv}{du} = -\frac{f^2}{\left(u - f\right)^2}$$

___ 16. Find the instantaneous rate of change of the function
$$f(x) = \sqrt{3x}$$
 when $x = 3$.

- a. <u>1</u> 3
- b. 3
- c. 9
- d. $\frac{1}{2}$

- a. 3.9 m
- b. 2.9 m
- c. 4.4 m
- d. 3.4 m
- e. 2.4 m

_____ 18. Find equations of the tangent lines to the curve $y = \frac{x-10}{x+10}$ that are parallel to the line x-y=10.

a.
$$x - y = -4.5$$

b.
$$x - y = -2$$

c.
$$x-y=-12.5$$

d.
$$x-y=-19.75$$

e.
$$x - y = -15$$

____ 19. If $f(x) = 6\cos x + \sin^2 x$, find f'(x) and f''(x).

a.
$$f''(x) = -6\cos(2x) + 2\cos(x)$$

b.
$$f'(x) = -6\sin(x) + \sin(2x)$$

c.
$$f'(x) = -6\sin(2x) + \sin(x)$$

d.
$$f''(x) = -6\cos(x) + 2\cos(2x)$$

e.
$$f''(x) = -2\cos(2x) + 6\cos(x)$$

_____ 20. Find equations of the tangent lines to the curve $y = \frac{x-8}{x+8}$ that are parallel to the line x-y=8.

a.
$$x - y = -18.5$$

b.
$$x - y = -4.5$$

c.
$$x - y = -1.5$$

d.
$$x - y = -12.5$$

e.
$$x - y = -15$$

Answer Key

- 1. A
- 2. B
- 3. A
- 4. D
- 5. D
- 6. D
- 7. D
- 8. B
- 9. A
- 10. A
- 11. D
- 12. B
- 13. D
- 14. E
- 15. E
- 16. D
- 17. D
- 18. B, D
- 19. B, D
- 20. A, C

Select the correct answer for each question.

1. Find the differential of the function at the indicated number.

$$f(x) = 13\sin x + 4\cos x; \quad x = \frac{\pi}{4}$$

a.
$$\frac{9\sqrt{2}}{2} dx$$

b.
$$-\frac{17\sqrt{2}}{2} dx$$

c.
$$\frac{17\sqrt{2}}{2} dx$$

d.
$$-\frac{9\sqrt{2}}{2} dx$$

2. The cost (in dollars) of producing x units of a certain commodity is

$$C(x) = 4,280 + 13x + 0.03x^2$$
.

Find the average rate of change with respect to x when the production level is changed from x = 102 to x = 122.

3. If $g(x) = \sqrt{2-3x}$, use the definition of derivative to find g'(x).

a.
$$g'(x) = -\frac{1}{2}(2-3x)^{-1/2}$$

b.
$$g'(x) = -(2-3x)^{-1/2}$$

c.
$$g'(x) = -\frac{3}{2}(2-3x)^{1/2}$$

d.
$$g'(x) = -\frac{3}{2}(2-3x)^{-1/2}$$

- e. None of these
- 4. Differentiate.

$$K(x) = \left(3x^5 + 1\right)\left(x^6 - 4x\right)$$

a.
$$15x^4(x^6-4x)+(3x^5+1)(6x^5-4)$$

b.
$$(x^6 - 4x) + (3x^5 + 1)$$

c.
$$15x^4(6x^5-4)+(3x^5+1)(x^6-4x)$$

d.
$$15x^4(6x^5) + (3x^5)(x^6 - 4x)$$

e.
$$(3x^5+1)(x^6-4x)+15x^4(6x^5-4)+1$$

5. Find f' in terms of g'.

$$f(x) = x^7 g(x)$$

a.
$$f'(x) = 7x^6 f'(x) + x^7 g'(x)$$

b.
$$f'(x) = 7x^6g'(x)$$

c.
$$f'(x) = 7x^6 g(x) + 7x^7 g'(x)$$

d.
$$f'(x) = 7x^6g(x) + x^7g'(x)$$

e.
$$f'(x) = 7x^6 + g'(x)$$

___ 6. Find the derivative of the function.

$$f(x) = 0.2x^{-17}$$

a.
$$-\frac{0.34}{x^{0.7}}$$

b.
$$-\frac{0.34}{x^{23}}$$

c.
$$-0.34x^{27}$$

d.
$$-0.34x^{0.7}$$

_ 7. Find the derivative of the function.

$$f(x) = \frac{2\sqrt{x}}{x^2 + 9}$$

a.
$$\frac{-3x^2 + 9}{\sqrt{x}(x^2 + 9)^2}$$

b.
$$\frac{1}{2x\sqrt{x}\left(x^2+9\right)}$$

c.
$$\frac{1}{2x\sqrt{x}}$$

$$\frac{d.}{\sqrt{x}\left(x^2+9\right)}$$

8. Find the derivative of the function.

$$f(x) = \left(x^2 + 1\right) \left(\frac{9x - 1}{7x + 1}\right)$$

a.
$$\frac{63x^3 + 135x^2 + 49x + 9}{7(7x+1)^2}$$

b.
$$\frac{63x^3 + 135x^2 + 49x + 9}{7(7x+1)}$$

c.
$$\frac{126x^3 + 20x^2 - 2x + 16}{(7x+1)^2}$$

d.
$$\frac{126x^3 + 20x^2 - 2x + 16}{(7x+1)}$$

9. If f is a differentiable function, find an expression for the derivative of $y = x^3 f(x)$.

a.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^2 f(x) + x^3 f'(x)$$

b.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^3 f(x) + x^2 f'(x)$$

c.
$$\frac{d}{dx}\left(x^3f(x)\right) = 2x^2f(x) - x^3f'(x)$$

d.
$$\frac{d}{dx}\left(x^3f(x)\right) = 3x^2f(x) - x^3f'(x)$$

e.
$$\frac{d}{dx}\left(x^3f(x)\right) = 3x^3f(x) - x^2f'(x)$$

10. Find the points on the curve $y = 2x^3 + 3x^2 - 36x + 19$ where the tangent is horizontal.

a.
$$(-3, 100), (2, -25)$$

d.
$$(-4,71)$$
, $(2,-37)$

11. If $f(t) = \sqrt{9t+1}$, find f''(5).

$$d. -0.22$$

12. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$y\sin 3x = x\cos 3y, \left(\frac{\pi}{3}, \frac{\pi}{6}\right)$$

a.
$$y = \frac{x}{2}$$

b.
$$y = 2x - \frac{3\pi}{3}$$

c.
$$y = -\frac{x}{2} + \frac{\pi}{2}$$

d.
$$y = \frac{x}{6}$$

e.
$$y = \frac{x}{3} + \frac{\pi}{6}$$

13. Calculate v'.

$$xy^3 + x^3y = x + 3y$$

a.
$$y' = \frac{1 - y^3 - 3x^2y}{3xy^2 + x^3 - 3}$$

b.
$$y' = \frac{1 - y^3 - 2x^3}{3xy^2 + x^2 - 3}$$

c.
$$y' = \frac{-y^4 - 3xy}{4xy^3 + x^2}$$

d.
$$y' = \frac{xy^2 + 2x - 3}{x^2y^2(3x - 1)}$$

- e. none of these
- 14. Find the derivative of the function.

$$y = 3\cos^{-1}\left(\sin^{-1}t\right)$$

a.
$$y' = -\frac{3}{\sqrt{\left(1 - t^2\right) \left(1 - \left(\sin^{-1}(t)\right)^2\right)}}$$

b.
$$y' = -\frac{3}{\sqrt{(1-t^2)(1-\sin^{-1}(t))}}$$

c.
$$y' = -\frac{3}{\sqrt{(1-t^2)}}$$

d.
$$y' = -\frac{3}{\sqrt{1 - \left(\sin^{-1}(t)\right)^2}}$$

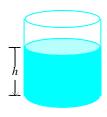
d.
$$y' = -\frac{3}{\sqrt{1 - \left(\sin^{-1}(t)\right)^2}}$$

e. $y' = -\frac{3}{\sqrt{\left(1 + t^2\right)\left(1 + \left(\sin^{-1}(t)\right)^2\right)}}$

15. Water flows from a tank of constant cross-sectional area 50 ft² through an orifice of constant cross-sectional area $\frac{1}{4}$ ft² located at the bottom of the tank. Initially, the height of the water in the tank was 20 ft, and t sec later it was given by the equation

$$2\sqrt{h} + \frac{1}{25}t - 2\sqrt{20} = 0 \qquad 0 \le t \le 50\sqrt{20}$$

How fast was the height of the water decreasing when its height was 2 ft?



a.
$$100\sqrt{5} - 50\sqrt{2}$$
 ft/sec
b. $100\sqrt{5} - 50\sqrt{2}$ ft/sec

b.
$$100\sqrt{5} - 50\sqrt{2}$$
 ft/sec

c.
$$\frac{2}{25}$$
 ft/sec

d.
$$\frac{\sqrt{2}}{25}$$
 ft/sec

16. The mass of part of a wire is $x(1+\sqrt{x})$ kilograms, where x is measured in meters from one end of the wire. Find the linear density of the wire when x = 36m.

- a. 6kg/m
- b. 4 kg/m
- c. 9kg/m
- d. 1.5 kg/m
- e. None of these

 17.	A plane flying horizontally at an altitude of 1 mi and a speed of 550 mi/h passes directly over a
	radar station. Find the rate at which the distance from the plane to the station is increasing when it
	is 2 mi away from the station.

- a. ≈ 476 mi/h
- b. ≈ 670 mi/h
- c. ≈ 455 mi/h
- d. ≈ 570 mi/h
- e. ≈ 495 mi/h
- 18. Two sides of a triangle are 2 m and 3 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\frac{\pi}{3}$.
 - a. $1.145 \,\mathrm{m}^2/\mathrm{s}$
 - b. $-0.955 \,\mathrm{m}^2/\mathrm{s}$
 - c. $0.090 \, \text{m}^2/\text{s}$
 - d. $5.045 \,\mathrm{m}^2/\mathrm{s}$
 - e. $-1.955 \,\mathrm{m}^2/\mathrm{s}$

19. Gravel is being dumped from a conveyor belt at a rate of 34 ft/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 13 ft high? Round the result to the nearest hundredth.



- a. 0.6 ft/min
- b. 0.26 ft/min
- c. 0.14 ft/min
- d. 0.27 ft/min
- e. 1.24 ft/min

20. A point moves along the curve $3y + y^2 - 8x = 2$. When the point is at $\left(-\frac{1}{2}, -1\right)$, its x-coordinate is increasing at the rate of 3 units per second. How fast is its y-coordinate changing at that instant of time?

- a. 24 units/sec
- b. 26 units/sec
- c. -24 units/sec
- d. -22 units/sec

Answer Key

- 1. A
- 2. E
- 3. D
- 4. A
- 5. D
- 6. B
- 7. A
- 8. C
- 9. A
- 10. A
- 11. A
- 12. A
- 13. A
- 14. A
- 15. D
- 16. C
- 17. A
- 18. C
- 19. B
- 19. B 20. A

- 1. Use the linear approximation of the function $f(x) = \sqrt{9-x}$ at a = 0 to approximate the number $\sqrt{9.08}$.
- 2. Compute $\triangle y$ and dy for the given values of x and $dx = \triangle x$.

$$y = x^2$$
, $x = 1$, $\triangle x = 0.5$

3. If the tangent line to y = f(x) at (8, 4) passes through the point (4, -32), find f'(8). Select the correct answer.

a.
$$f'(8) = 29$$

b.
$$f'(8) = 19$$

c.
$$f'(8) = 9$$

d.
$$f'(8) = 34$$

e.
$$f'(8) = -9$$

- 4. If $g(x) = \sqrt{8-7x}$, find the domain of g'(x).
- 5. Differentiate.

$$K(x) = \left(3x^5 + 1\right)\left(x^6 - 4x\right)$$

6. Use the Product Rule to find the derivative of the function. Select the correct answer.

$$f(x) = (4x+5)(x^2 - 8)$$

b.
$$2x + 4$$

c.
$$12x^2 + 10x - 32$$

d. $8x^2 - 40$

d.
$$8x^2 - 40$$

7. Use the Quotient Rule to find the derivative of the function.

$$P(t) = \frac{1-t}{7-8t}$$

8. Find the derivative of the function.

$$f(x) = \left(x^2 + 1\right) \left(\frac{9x - 1}{7x + 1}\right)$$

9. Find f''(x).

$$f(x) = (2x)^5 - (7x)^2 + 5$$

10. Find f' in terms of g'.

$$f(x) = \left[g(x) \right]^4$$

11. Find f' in terms of g'.

$$f(x) = x^5 g(x)$$

Select the correct answer.

a.
$$f'(x) = 5x^4 + g'(x)$$

b.
$$f'(x) = x^5 g(x) + 5x^5 g'(x)$$

c.
$$f'(x) = 5x^4g(x) + x^5g'(x)$$

d.
$$f'(x) = 5x^4 g'(x)$$

e.
$$f'(x) = 5x f'(x) + 5xg'(x)$$

12. Suppose that F(x) = f(g(x)) and g(14) = 2, g'(14) = 5, f'(14) = 15, and f'(2) = 16.

Find
$$F'(14)$$
.

13. Calculate y'.

$$xy^3 + x^3y = x + 3y$$

14. Find the derivative of the function.

$$y = 3\cos^{-1}\left(\sin^{-1}t\right)$$

© 2016 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

15. Find an equation of the tangent line to the curve $120(x^2 + y^2)^2 = 2312(x^2 - y^2)$ at the point (4,1).

Select the correct answer.

a.
$$y = -1.11x + 17$$

b.
$$y = -1.11x + 3.43$$

c.
$$y = 1.11x + 5.43$$

d.
$$y = -1.11x + 5.43$$

- e. None of these
- 16. The mass of the part of a metal rod that lies between its left end and a point x meters to the right is

$$S = 4x^2$$
.

Find the linear density when x is 3 m.

17. In an adiabatic process (one in which no heat transfer takes place), the pressure *P* and volume *V* of an ideal gas such as oxygen satisfy the equation

$$P^5V^7=C$$
.

where C is a constant. Suppose that at a certain instant of time, the volume of the gas is 2L, the pressure is 100 kPa, and the pressure is decreasing at the rate of 5 kPa/sec. Find the rate at which the volume is changing.

- 18. Find the instantaneous rate of change of the function $f(x) = \sqrt{3x}$ when x = 3.
- 19. Let C(t) be the total value of US currency (coins and banknotes) in circulation at time. The table gives values of this function from 1980 to 2000, as of September 30, in billions of dollars. Estimate the value of C(1990).

t	1980	1985	1990	1995	2000
C(t)	129.9	176.3	275.9	405.3	568.6

Answers are in billions of dollars per year. Round your answer to two decimal places.

 20.	A car leaves an intersection traveling west. Its position 4 sec later is 26 ft from the intersection	ction. At
	the same time, another car leaves the same intersection heading north so that its position 4	4 sec later
	is 26 ft from the intersection. If the speeds of the cars at that instant of time are 12 ft/sec a	ınd 10
	ft/sec, respectively, find the rate at which the distance between the two cars is changing.	Round to
	the nearest tenth if necessary.	

Select the correct answer.

- a. 15.6 ft/sec
- b. 3.7 ft/sec
- c. 3.1 ft/sec
- d. 36.8 ft/sec

Answer Key

2.
$$\triangle y = 1.25$$
, $dy = 1$

4.
$$\left(-\infty, \frac{8}{7}\right)$$

5.
$$15x^4(x^6-4x)+(3x^5+1)(6x^5-4)$$

7.
$$\frac{1}{(7-8t)^2}$$

8.
$$\frac{126x^3 + 20x^2 - 2x + 16}{(7x+1)^2}$$

9.
$$640x^3 - 98$$

10.
$$f'(x) = 4[g(x)]^3 g'(x)$$

13.
$$y' = \frac{1 - y^3 - 3x^2y}{3xy^2 + x^3 - 3}$$

14.
$$y' = -\frac{3}{\sqrt{\left(1-t^2\right)\left(1-\left(\sin^{-1}(t)\right)^2\right)}}$$

17.
$$\frac{1}{14}$$
 L/sec

18.
$$\frac{1}{2}$$

1. Find the differential of the function at the indicated number.

Select the correct answer.

$$f(x) = \sqrt{x^2 + 7}; \quad x = 3$$

- a. $\frac{3}{8} dx$ b. $\frac{3}{4} dx$ c. $\frac{3}{2} dx$
- d. $\frac{1}{8} dx$
- 2. The slope of the tangent line to the graph of the exponential function $y = 6^x$ at the point (0, 1) is $\lim_{x\to 0} \frac{6^x - 1}{x}$. Estimate the slope to three decimal places.
- 3. A turkey is removed from the oven when its temperature reaches 175 °F and is placed on a table in a room where the temperature is 70 °F. After 10 minutes the temperature of the turkey is 161 °F and after 20 minutes it is 149 °F. Use a linear approximation to predict the temperature of the turkey after 30 minutes.
- 4. If $g(x) = \sqrt{8-7x}$, find the domain of g'(x)
- 5. Suppose that F(x) = f(g(x)) and g(14) = 2, g'(14) = 4, f'(14) = 15, and f'(2) = 13.

Find F'(14).

6. Plot the graph of the function f in an appropriate viewing window.

$$f(x) = \frac{x^4}{x^4 + 1}$$

7. Find the derivative of the function.

Select the correct answer.

$$f(x) = \frac{2\sqrt{x}}{x^2 + 9}$$

a.
$$\frac{-3x^2+9}{\sqrt{x}\left(x^2+9\right)^2}$$

b.
$$\frac{1}{2x\sqrt{x}\left(x^2+9\right)}$$

c.
$$\frac{1}{2x\sqrt{x}}$$

$$d. \quad \frac{-3x^2 + 9}{\sqrt{x}\left(x^2 + 9\right)}$$

8. Find the derivative of the function.

$$f(x) = \left(x^2 + 1\right) \left(\frac{9x - 1}{7x + 1}\right)$$

- 9. If f is a differentiable function, find an expression for the derivative of $y = x^3 f(x)$.
- 10. Find the derivative of the function.

$$g(v) = \sin v - 8v \csc v$$

11. Find f' in terms of g'.

$$f(x) = \left[g(x) \right]^4$$

12. Find the second derivative of the function.

Select the correct answer.

$$f(x) = x\left(3x^2 - 1\right)^4$$

a.
$$4x(3x^2-1)^3$$

b.
$$72x(3x^2-1)^2(9x^2-1)$$

c.
$$12x(3x^2-1)^2$$

d.
$$(27x^2-1)(3x^2-1)^3$$

13. Use implicit differentiation to find an equation of the tangent line to the curve at the indicated point.

$$y = \sin xy^6; \quad \left(\frac{\pi}{2}, 1\right)$$

14. Find $\frac{d^2y}{dx^2}$ in terms of x and y.

$$x^7 - y^7 = 1$$

15. Calculate y'.

$$xy^3 + x^3y = x + 3y$$

16. If $f(t) = \sqrt{9t+1}$, find f''(4).

17. In an adiabatic process (one in which no heat transfer takes place), the pressure *P* and volume *V* of an ideal gas such as oxygen satisfy the equation

$$P^5V^7=C$$
,

where C is a constant. Suppose that at a certain instant of time, the volume of the gas is 2L, the pressure is 100 kPa, and the pressure is decreasing at the rate of 5 kPa/sec. Find the rate at which the volume is changing.

18. A plane flying horizontally at an altitude of 1 mi and a speed of 550 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

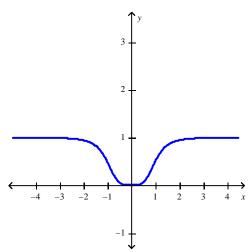
Select the correct answer.

- a. ≈ 476 mi/h
- b. ≈ 670 mi/h
- c. ≈ 455 mi/h
- d. $\approx 570 \, \text{mi/h}$
- e. ≈ 495 mi/h
- 19. Two sides of a triangle are 2 m and 3 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\frac{\pi}{3}$.
- 20. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 1.5 m from the wall, it slides away from the wall at a rate of 0.3 m/s. How long is the ladder?

Answer Key

4.
$$\left(-\infty, \frac{8}{7}\right)$$

6.



8.
$$\frac{126x^3 + 20x^2 - 2x + 16}{(7x+1)^2}$$

9.
$$\frac{d}{dx}\left(x^3f(x)\right) = 3x^2f(x) + x^3f'(x)$$

10.
$$\cos v - 8 \csc v + 8v \csc v \cot v$$

11.
$$f'(x) = 4[g(x)]^3 g'(x)$$

13.
$$y = 1$$

14.
$$\frac{6x^5}{y^6} - \frac{6x^{12}}{y^{13}}$$

15.
$$y' = \frac{1 - y^3 - 3x^2y}{3xy^2 + x^3 - 3}$$

17.
$$\frac{1}{14}$$
 L/sec

© 2016 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

18. A

19. $0.090 \,\mathrm{m}^2/\mathrm{s}$

20. 3.4 m

1. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.0017 cm thick to a hemispherical dome with diameter 70 m.

Select the correct answer.

- a. 4.165π
- b. 2.52π
- c. 4.11π
- d. 3.82π
- e. 2.28π
- 2. A turkey is removed from the oven when its temperature reaches $175^{\circ}F$ and is placed on a table in a room where the temperature is $70^{\circ}F$. After 10 minutes the temperature of the turkey is $160^{\circ}F$ and after 20 minutes it is $150^{\circ}F$. Use a linear approximation to predict the temperature of the turkey after 40 minutes.

Select the correct answer.

- a. 160
- b. 36
- c. 134
- d. 135
- e. 130
- 3. If f is a differentiable function, find an expression for the derivative of $y = x^3 f(x)$.
- 4. Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

$$\frac{d^{89}}{dx^{89}}(\sin x)$$

Select the correct answer.

- a. $-\sin x$
- b. $\sin x$
- $c. \cos x$
- d. $\cos x$
- e. None of these

5. If
$$f(0) = 4$$
, $f'(0) = 3$, $g(0) = 1$ and $g'(0) = -6$, find $(f+g)'(0)$

6. Find
$$\frac{d^2y}{dx^2}$$
 in terms of x and y.

$$x^7 - y^7 = 1$$

7. Calculate y'.

$$xy^3 + x^3y = x + 3y$$

- 8. Find the average rate of change of the area of a circle with respect to its radius *r* as *r* changes from 3 to 8.
- 9. Two cars start moving from the same point. One travels south at 70 mi/h and the other travels west at 20 mi/h. At what rate is the distance between the cars increasing 2 hours later? Round the result to the nearest hundredth.
- 10. A plane flying horizontally at an altitude of 1 mi and a speed of 550 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

d.
$$\approx 570 \, \text{mi/h}$$

11. If a cylindrical tank holds 10000 gallons of water, which can be drained from the bottom of the tank in an hour, then Torricelli's Law gives the volume of water remaining in the tank after *t* minutes as

$$V(t) = 10000 \left(1 - \frac{1}{60} t \right)^2, 0 \le t \le 60$$

Find the rate at which the water is flowing out of the tank (the instantaneous rate of change of V with respect to t) as a function of t.

12. Differentiate the function.

$$f(t) = \frac{1}{3}t^3 - 2t^7 + t$$

- 13. Find an equation of the tangent line to the curve $y = 7 \tan x$ at the point $\left(\frac{\pi}{4}, 7\right)$.
- 14. Find the limit.

$$\lim_{\theta \to 0} 4 \frac{\sin(\sin 4\theta)}{\sec 4\theta}$$

15. Differentiate.

$$y = \frac{\sin x}{3 + \cos x}$$

16. Find the equation of the tangent to the curve at the given point.

$$y = \sqrt{16 + 4 \sin x}$$
, (0,4)

17. Find y' by implicit differentiation.

$$10\cos x \sin y = 16$$

- 18. A spherical balloon is being inflated. Find the rate of increase of the surface area $S = 4\pi r^2$ with respect to the radius r when r = 1 ft.
- 19. If a snowball melts so that its surface area decreases at a rate of $4 \,\mathrm{cm}^2/\mathrm{min}$, find the rate at which the diameter decreases when the diameter is 37 cm.
- 20. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of $2 \text{ cm}^2/\text{min}$. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2 .

Answer Key

- 1. A
- 2. E

3.
$$\frac{d}{dx}\left(x^3f(x)\right) = 3x^2f(x) + x^3f'(x)$$

- 4. D
- 5. -3

6.
$$\frac{6x^5}{y^6} - \frac{6x^{12}}{y^{13}}$$

7.
$$y' = \frac{1 - y^3 - 3x^2y}{3xy^2 + x^3 - 3}$$

- 8. 11π
- 9. 72.80 mi/h
- 10. A

11.
$$V'(t) = \frac{-1000}{3} + \frac{50t}{9}$$

12.
$$f'(t) = t^2 - 14t^6 + 1$$

13.
$$y = 14x + 7\left(1 - \frac{\pi}{2}\right)$$

14 (

15.
$$\frac{dy}{dx} = \frac{3\cos x + 1}{(3 + \cos x)^2}$$

16.
$$y = \frac{1}{2}x + 4$$

- 17. tan(x)tan(y)
- 18. 8π

19.
$$\frac{2}{37\pi}$$

- 1. Find an equation of the tangent line to the curve $y = x^3 6x$ at the point (6, 8).
- 2. A turkey is removed from the oven when its temperature reaches 175 °F and is placed on a table in a room where the temperature is 70 °F. After 10 minutes the temperature of the turkey is 161 °F and after 20 minutes it is 149 °F. Use a linear approximation to predict the temperature of the turkey after 30 minutes.
- 3. The equation of motion is given for a particle, where *s* is in meters and *t* is in seconds. Find the acceleration after 5 seconds.

$$s = t^3 - 3t$$

4. If f is a differentiable function, find an expression for the derivative of $y = x^3 f(x)$.

Select the correct answer.

a.
$$\frac{d}{dx}\left(x^3f(x)\right) = 3x^2f(x) + x^3f'(x)$$

b.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^3 f(x) + x^2 f'(x)$$

c.
$$\frac{d}{dx}\left(x^3f(x)\right) = 2x^2f(x) - x^3f'(x)$$

d.
$$\frac{d}{dx}\left(x^3f(x)\right) = 3x^2f(x) - x^3f'(x)$$

e.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^3 f(x) - x^2 f'(x)$$

5. Differentiate the function.

$$B(y) = cy^{-4}$$

6. Find the points on the curve $y = 2x^3 + 3x^2 - 36x + 19$ where the tangent is horizontal.

7. Find the derivative of the function.

$$f(x) = 2\cos x - 2x - 8$$

8. Calculate y'.

$$y = \frac{e^x}{x^3}$$

Select the correct answer.

a.
$$y' = e^{x} \left(\frac{x-1}{x^5} \right)$$

b.
$$y' = e^x \left(\frac{x+3}{x^4} \right)$$

$$y' = \frac{e^x}{3x}$$

d.
$$y' = e^x \left(\frac{x-3}{x^4} \right)$$

e.
$$y' = e^x \left(\frac{x-3}{3x} \right)$$

9. If
$$y = 2x^2 + 7x$$
 and $\frac{dx}{dt} = 6$, find $\frac{dy}{dt}$ when $x = 4$.

10. Find the tangent line to the ellipse
$$\frac{x^2}{40} + \frac{y^2}{10} = 1$$
 at the point $(2, -\sqrt{3})$.

11. The equation of motion is given for a particle, where *s* is in meters and *t* is in seconds. Find the acceleration after 2.5 seconds.

$$s = \sin 2\pi t$$

12. In an adiabatic process (one in which no heat transfer takes place), the pressure *P* and volume *V* of an ideal gas such as oxygen satisfy the equation

$$P^5V^7=C$$
.

where C is a constant. Suppose that at a certain instant of time, the volume of the gas is 2L, the pressure is 100 kPa, and the pressure is decreasing at the rate of 5 kPa/sec. Find the rate at which the volume is changing.

- 13. A plane flying horizontally at an altitude of 1 mi and a speed of 550 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.
- 14. Two sides of a triangle are 2 m and 3 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\frac{\pi}{3}$.

Select the correct answer.

a.
$$1.145 \,\mathrm{m}^2/\mathrm{s}$$

b.
$$-0.955 \,\mathrm{m}^2/\mathrm{s}$$

c.
$$0.090 \,\mathrm{m}^2/\mathrm{s}$$

d.
$$5.045 \,\mathrm{m}^2/\mathrm{s}$$

e.
$$-1.955 \,\mathrm{m}^2/\mathrm{s}$$

15. A car leaves an intersection traveling west. Its position 4 sec later is 26 ft from the intersection. At the same time, another car leaves the same intersection heading north so that its position 4 sec later is 26 ft from the intersection. If the speeds of the cars at that instant of time are 12 ft/sec and 10 ft/sec, respectively, find the rate at which the distance between the two cars is changing. Round to the nearest tenth if necessary.

16. If
$$h(2) = 16$$
 and $h'(2) = -2$, find $\frac{d}{dx} \left(\frac{h(x)}{x} \right) \Big|_{x=2}$

17. Differentiate.

$$g(x) = 2 \sec x + \tan x$$

18. The position function of a particle is given by

$$s = t^3 - 10.5t^2 - 2t$$
, $t \ge 0$

When does the particle reach a velocity of 22 m/s?

- 19. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 40 ft/s. At what rate is his distance from second base decreasing when he is halfway to first base? Round the result to the nearest hundredth.
- 20. A television camera is positioned 4,600 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Also, the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Let's assume the rocket rises vertically and its speed is 680 ft/s when it has risen 2,600 ft. If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at this moment? Round the result to the nearest thousandth.

Answer Key

- 1. None of these
- 2. 136°F
- 3. 30 m/s²
- 4. A

5.
$$B'(y) = -\frac{4c}{y^5}$$

- 6. (-3,100), (2,-25)
- 7. $-2 \sin x 2$
- 8. D
- 9. None of these

10.
$$y = \frac{\sqrt{3}}{6}x - \frac{4\sqrt{3}}{3}$$

- 11. $0 \, \text{m/s}^2$
- 12. $\frac{1}{14}$ L/sec
- 13. ≈ 476 mi/h
- 14. C
- 15. 15.6 ft/sec
- 16. -5
- 17. $g'(x) = 2 \sec(x) \tan(x) + \sec^2 x$
- 18. 8
- 19. 17.89 ft/s
- 20. 0.112 rad/s