Name: ______________________ Class: ______________ Date: ________

Chapter 02: Numeration Systems

1. Is the old Greek numeration system (α = 1, β = 2, γ = 3, etc.) a place-value system? Explain.

ANSWER: No. A place-value system requires that the placement of the symbol have meaning, which is not true of the Greek system.

2. Most present-day societies use the Hindu-Arabic numeration system.

a. True

b. False

ANSWER: a

3. How many tens are in 7654? How many whole tens are in 7654?

ANSWER: 765.4; 765

4. How many hundreds are in 23? How many whole hundreds?

ANSWER: 0.23; 0

5. How many tenths are in 1.03? How many whole tenths?

ANSWER: 10.3; 10

6. How many ones are in 4352.678? How many whole ones?

ANSWER: 4352.678; 4352

7. In base ten, 3421 is exactly _____ ones, exactly _____ tens, exactly _____ hundreds, and exactly _____ thousands; also, 3421 is exactly _____ tenths and exactly _____ hundredths.

ANSWER: 3421; 342.1; 34.21; 3.421; 3421; 34210; 342100

8. In base ten, 215.687 is exactly _____ ones, exactly _____ tens, exactly _____ hundreds, and exactly _____ thousands; also, 3421 is exactly _____ tenths and exactly _____ hundredths.

ANSWER: 215.687; 21.5687; 2.15687; 0.215687; 34210; 342100

9. (Roman numerals) IX = $\_\_\_\_\__{\text{ten}}$ and XI = $\_\_\_\_\__{\text{ten}}$.

ANSWER: 9; 11

10. 34,597 has 345 whole thousands in it.

a. True

b. False

ANSWER: b

11. 34.597 has 345 whole tenths in it.

a. True

b. False

ANSWER: a

12. 56 has 560 tenths in it.

a. True

b. False

ANSWER: a

13. 23 has 230 hundredths in it.

a. True

b. False

ANSWER: b

14. 45 has 4500 hundredths in it.

a. True

b. False

ANSWER: a

15. 632.1 has 632.1 ones in it.

a. True

b. False

ANSWER: a

16. A soap factory packs 100 bars of soap in each box for shipment. If the factory makes 15,287 bars of soap, how many *full* boxes will they have for shipment? Explain.

ANSWER: They will have 152 boxes because there are 152 hundreds in 15,287.

17. How many $10 bills could one get for $10 million?

a. 10,000,000

b. 1,000,000

c. 100,000

d. 10,000

e. 1000

ANSWER: b

18. How many $100 bills could one get for a billion dollars?

a. 100,000,000

b. 10,000,000

c. 1,000,000

d. 100,000

e. 10,000

ANSWER: b

19. How many $100 bills would make $45 billion?

ANSWER: 450,000,000

Chapter 02: Numeration Systems

20. Judy says, "Well, hundredths are smaller than tenths. So 0.36 is smaller than 0.4." Comment on Judy's reasoning.

ANSWER: Although Judy does choose the smaller number correctly, her reasoning is risky. If the numbers were 0.56 and 0.4, using just her reasoning would give an incorrect choice for the smaller number.

21. Tyler thinks that 0.36 is bigger than 0.4 because 36 is bigger than 4. Comment on Tyler's reasoning.

ANSWER: Tyler is reasoning as though the numbers were whole numbers. Tyler does not recognize that 4 tenths will be bigger than 3 tenths and only 6 hundredths.

22. Four children are talking about different ways to think about the number 196.83. Following are their answers. For each answer, tell whether it is correct or incorrect. If it is incorrect, please explain.

Kim's answer: 196.83 could be thought about as 190 tens and 683 hundredths.
A) Is Kim's answer correct or incorrect?
B) If Kim's answer is incorrect, please explain the error.

Laurie's answer: 196.83 could be thought about as 1 hundred, 9 tens, and 6.83 hundredths.
A) Is Laurie's answer correct or incorrect?
B) If Laurie's answer is incorrect, please explain the error.

Mick's answer: 196.83 could be thought about as 1 hundred and 9683 hundredths.
A) Is Mick's answer correct or incorrect?
B) If Mick's answer is incorrect, please explain the error.

Nola's answer: 196.83 could be thought about as 18 tens, 15 ones, 17 tenths, and 13 hundredths.
A) Is Nola's answer correct or incorrect?
B) If Nola's answer is incorrect, please explain the error.

ANSWER: Kim's answer: A) Incorrect; B) 19 tens and 683 hundredths
Laurie's answer: A) Incorrect; B) 1hundred, 9 tens, and 683 hundredths, or …
Mick's answer: A) Correct
Nola's answer: A) Correct

23. For whole numbers, any two-digit numeral in base five represents a smaller number than the *same* two-digit numeral in base twenty.

a. True
b. False

ANSWER: a

24. In base b, there are $b - 1$ different digits.

a. True
b. False

ANSWER: b

25. These are the digits that are needed for a base *seven* place-value system: 0, 1, 2, 3, 4, 5, 6, 7.

a. True

b. False

ANSWER: b

26. In base b, $3 + 2b^3 + b$ would be written _____.

ANSWER: $2013b$

27. A place-value, base-twenty system would require _____ digits.

ANSWER: 20

28. 524_{eight} = _____$_{ten}$.

ANSWER: 340

29. 287_{ten} = _____$_{four}$.

ANSWER: 10133

30. 1012_{five} = _____ in base ten.

ANSWER: 132_{ten}

31. 32_{ten} = _____ in base four.

ANSWER: 200_{four}

32. 2.31_{four} = _____ as a mixed number in base ten.

ANSWER: $2\frac{13}{16}_{ten}$

33. $6\frac{2}{3}$ in base ten = _____ in base three.

ANSWER: $20\frac{2}{10}_{three}$, or 20.2_{three}

34. 1_{ten} = _____$_{twelve}$.

ANSWER: 1

35. 214.3_{five} = _____ in base ten.

ANSWER: 59.6_{ten}, or $59\frac{3}{5}$ ten

36. 29_{ten} = _____ in base three.

ANSWER: 1002_{three}

37. 7_{ten} = ______ in base nine.
ANSWER: 7_{nine}

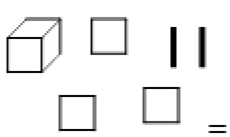
38. 203.6_{ten} = $\_\_\_\_\_\__{five}$.
ANSWER: 1303.3

39. 2003_{five} = $\_\_\_\_\_\__{ten}$.
ANSWER: 253

40. 200.3_{five} = $\_\_\_\_\_\__{ten}$.
ANSWER: 50.6_{ten}, or $50\frac{3}{5}$ ten (If you used item 39, you might ask how the two answers are related.)

41. Write 49_{ten} in base seven.
ANSWER: 100_{seven}

42. Do the "translations" in parts A–D. Show your work.
A) 3102_{five} = $\_\_\_\_\_\__{ten}$
B) 310.2_{five} = $\_\_\_\_\_\__{ten}$
C) 203.6_{ten} = $\_\_\_\_\_\__{five}$
D) (base-<u>six</u> pieces with small block as the unit)



= $\_\_\_\_\_\__{ten}$

ANSWER:
A) 402
B) 80.4, or $80\frac{2}{5}$
C) 1303.3_{five}
D) 336

43. You are living and working on a planet that uses only base five. How many $5 bills can you get for $\$1234.20_{five}$? Write your answer in base five because you are living on the planet. Write enough (numbers, words, etc.) to make your thinking clear.
ANSWER: 123. 123. As in base ten, $1230_{five} = 123 \times 10_{five}$ = 123 fives. More symbolically,

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$$1234.20 = (1 \times five^3) + (2 \times five^2) + (3 \times five) + \text{ something less than five}$$
$$\text{or} \quad = ((1 \times five^2) \times five) + ((2 \times five) \times five) + (3 \times five) + (small)$$
$$= ((1 \times five^2) + (2 \times five) + 3) \times five + (small)$$
$$= 123_{five} \times five + (small)$$

So, in base five, 1234.20_five$ is: 123 fives + something small.

44. In base five, the two whole numbers immediately BEFORE 2001_{five} are $\_\_\_\_\__{five}$ and $\_\_\_\_\__{five}$.
ANSWER: 1444; 2000 (either order)

45. If you are counting in base five, what would be the next six numerals AFTER 2314_{five}?
ANSWER: 2320, 2321, 2322, 2323, 2324, 2330

46. If you have been counting in base five, what would have been the five numerals BEFORE 2314_{five}?
ANSWER: 2304, 2310, 2311, 2312, 2313

47. Write how many fingers you have in base five. In base two. In base ten.
ANSWER: 20_{five}; 1010_{two}; 10_{ten}

48. Which is LARGER? 2_{four} or 2_{five}? Explain.
ANSWER: When a numeral has more than one digit, it will vary in value if written in different bases because the place values will differ. $21_{four} = 9_{ten}$ and $21_{five} = 11_{ten}$. Here, without converting, you can argue that the two groups of five will be more that the two groups of four.

49. Consider: $x = 81765_{fifteen}$ and $y = 81765_{thirteen}$. Which of x and y is GREATER? Explain.
ANSWER: x is greater because each digit other than the ones place represents more in base fifteen than in base thirteen.

50. Consider: $x = 74213_{sixteen}$ and $y = 74213_{fourteen}$. Which is GREATER, x or y (or are they equal)? Explain.
ANSWER: x is greater because each digit other than the ones place represents more.

51. Consider $x = 0.3147_{eight}$ and $y = 0.3147_{nine}$. Which of x and y is GREATER? Explain. (Be careful.)
ANSWER: x is greater. Consider only $x = 0.3_{eight}$ and $y = 0.3_{nine}$. $0.3_{eight} = 3/8$ in base ten, and $0.3_{nine} = 3/9$ in base ten. 3/8 is greater than 3/9. Extending this reasoning, x is greater.

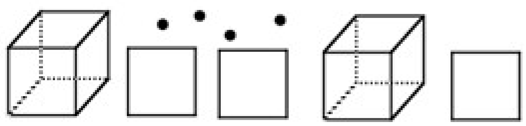
52. Write an algebraic expression for 204_b.
ANSWER: $2b^2 + 4$ (or, $2b^2 + 0b + 4$)

53. If a base-*eight* flat represents 1, the numeral _____ would give the numerical value of the small cube. (You may give your answer either in base eight or in base ten—just make clear which.)

ANSWER: 0.01_{eight}, or 1/64 in base ten

54. Write the base-ten numeral for the following base-*eight* pieces.

Base-*eight* pieces, with the small cube (a dot here) as the unit:

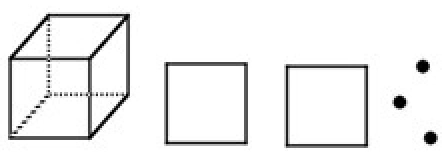


= ________________ten

ANSWER: 1220_{ten}

55. Sketch the base blocks that show 1203_{seven}, and give the English *words* for the base-*ten* value of each different-sized piece.

ANSWER: With the large dot representing a small cube as the unit



(small cube = 1; no longs)

large cube = $7^3 = 343$, or three hundred forty-three
flat = $7^2 = 49$, or forty-nine
small "cube" = 1, or one

56. Write the base *b* numeral for $2b^4 + b^2 + 3b + 1$.

ANSWER: 20131_b

57. Write out 32004_m in the algebraic form of the last item.

ANSWER: $3m^4 + 2m^3 + 4$, or $3m^4 + 2m^3 + 0m^2 + 0m + 4$

58. The best coins to use in thinking about the first three whole-number place values in base five would be the penny, the nickel, and the quarter.

a. True
b. False

ANSWER: a

59. The best coins to use in thinking about the first three whole-number place values in base ten would be the penny, the dime, and the half-dollar.

a. True
b. False

ANSWER: b

60. If $10000_{ten} + 10_b = 10023_{ten}$, what is base *b*?

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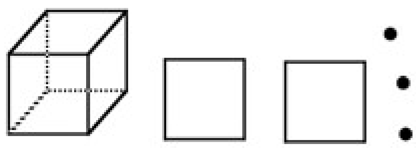
ANSWER: $b = 23$. (The given equation gives $10_b = 10023_{ten} - 10000_{ten}$, or $10_b = 23_{ten}$ —i.e., $b = 23$.)

61. Define your unit and sketch base blocks to represent 32.67_{eight}.

ANSWER: Using the flat = 1, three large cubes, two flats, six longs, seven small cubes.
Note: Answers will vary depending on the unit used.

62. Define your unit, sketch the base blocks that show 1203_{nine}, and give the English *words* for the base-*ten* value of each piece of wood.

ANSWER:



large cube = seven hundred twenty-nine;
flat = eighty-one
small cube = one

63. 53_{six} is the same number as which of these base-ten numerals?

a. 18
b. 183
c. 12
d. 85
e. 33

ANSWER: e

64. In base ten, 111_{five} would be written:

a. 421.
b. 155.
c. 31.
d. 21.
e. 555.

ANSWER: c

65. The base *b* numeral 321_b means:

a. $3 \cdot b^2 + 2 \cdot b^1 + 1$.
b. $3 \cdot b^3 + 2 \cdot b^2 + 1 \cdot b^1$.
c. $6b$.
d. $3 \cdot b + 2 \cdot b + 1$.

ANSWER: a

66. In base five, 32_{ten} would be written:

a. 152_{five}.

b. 112_{five}.

c. 62_{five}.

d. 17_{five}.

ANSWER: b

67. The base two numeral 100_{two} equals the base ten numeral:

a. 1100100.

b. 1011100.

c. 8.

d. 4.

ANSWER: d

68. In base ten, 32_{four} would be written:

a. 400.

b. 200.

c. 122.

d. 14.

e. 8.

ANSWER: d

69. The base-four numeral 11.1_{four} could be written in base ten as:

a. $33\frac{1}{4}$.

b. $33\frac{1}{10}$.

c. $11\frac{1}{4}$.

d. $5\frac{1}{4}$.

ANSWER: d

70. The base-ten decimal 18.5 could be written in base six as:

a. 10.5_{six}.

b. 20.3_{six}.

c. 30.3_{six}.

d. 128.5_{six}.

ANSWER: c

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71. The base-ten fraction 1/4 equals which base-eight numeral?

a. 0.2_{eight}

b. 0.14_{eight}

c. 0.02_{eight}

d. 1.4_{eight}

ANSWER: a

72. If $31_b = 28_{ten}$, then b =

a. 4.

b. 5.

c. 7.

d. 9.

e. This is impossible for any whole number *b*.

ANSWER: d

73. In what base does the following counting work:
1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, …

a. base two

b. base four

c. base five

d. base six

ANSWER: c

74. What is the base-ten fraction representation for 1.21_{four}?

a. $1\frac{9}{16}$

b. $1\frac{3}{4}$

c. $1\frac{21}{100}$

d. $1\frac{3}{5}$

ANSWER: a

75. Write an addition equation for (# fingers) + (# toes) = (answer) in some base other than base ten.

ANSWER: Samples:
base five: 20 + 20 = 40

base three: 101 + 101 = 202
base eight: 12 + 12 = 24

76. Three-fourths in base ten is what in base two?

ANSWER:

$$\left(\frac{3}{4}\right)_{ten} = \left(\frac{1}{2} + \frac{1}{4}\right)_{ten} = 0.11_{two}$$

77. Seven-eighths in base ten is what in base two?

ANSWER:

$$\left(\frac{7}{8}\right)_{ten} = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)_{ten} = 0.111_{two}$$

78. Suppose you visited an alien planet and the creatures wrote numbers in order as follows: "obi, fin, mus, obi-na, obi-obi, obi-fin, obi-mus…" These creatures must be using which base?

a. base ten
b. base three
c. base four
d. base nine
e. base twelve

ANSWER: c

79. A) Add $24_{five} + 33_{five}$ in base five. (The numbers are already written in base five, so no conversions are necessary.)
B) How would you illustrate this with the base-five blocks using drawings and showing the intermediate steps?

ANSWER: A) 112_{five}
B) With the small cube = 1, the first drawing shows two longs, four small cubes and three longs, three small cubes. Next, five of the seven small cubes are traded for a long, giving six longs and two remaining small cubes. Next, five longs are traded for a flat, giving one flat, one remaining long, and the two remaining small cubes, or 112_{five}.

80.

A) 0.5_{ten} = ______$_{eight}$

B) $312.2_{four} + 22.3_{four}$ = ______$_{four}$

C) 84_{ten} = ______$_{three}$

D) 33.3_{six} = ______$_{ten}$

ANSWER: A) 0.4_{eight}

B) 1001.1_{four}
C) 10010_{three}
D) 21.5

81. Determine the possible value(s) for base *b*:

$$\begin{array}{r} 321_b \\ -\ \underline{234_b} \\ 43_b \end{array}$$

ANSWER: b = six ($11_b - 4_b = 3_b$, or $3_b + 4_b = 11_b$)

82. Below is a partially completed addition, written in connection with base five pieces. *At the time of the work below*, what base pieces would be displayed if the small piece were the unit? (Drawings or word descriptions are okay.)
Finish the numerical calculation. (You do not have to draw the base pieces for the rest of the work.)

$$\begin{array}{r} 1\phantom{\ 1\ 4_{five}} \\ 2\ 1\ 4_{five} \\ +\ \underline{3\ 3}_{five} \\ 2 \end{array}$$

ANSWER: There would be two flats, five longs, and two small cubes at the time of the work. (The trade of the five longs for a flat is not reflected in the work yet.) Final sum: 302

83.

$$\begin{array}{r} 241_{six} \\ +\ \underline{135}_{six} \end{array}$$

ANSWER: 420_{six}

84.

$$\begin{array}{r} 127_{nine} \\ -\ \underline{58}_{nine} \end{array}$$

ANSWER: 58_{nine}

85.

4.4_{five}

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$+ \underline{3.3}_{five}$

ANSWER: 13.2_{five}

86.

0.24_{seven}

$- \underline{0.15}_{seven}$

ANSWER: 0.06_{seven}

87.

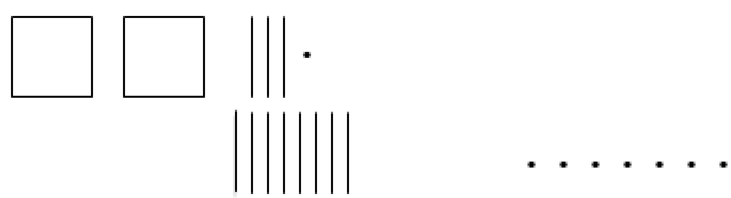
21_{six}

$+ \underline{35.2}_{six}$

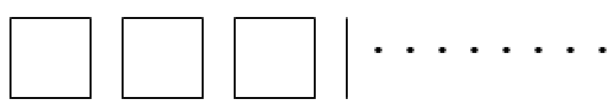
ANSWER: 100.2_{six}

88. Use drawings of multibase blocks to illustrate $231_{ten} + 87_{ten}$.

ANSWER: Answer using a small square/block as the unit:



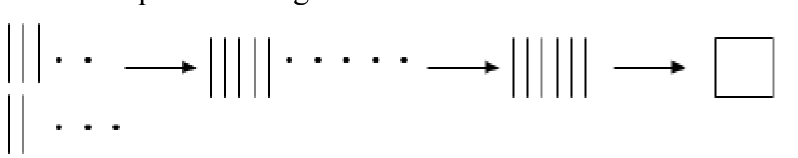
Place together and then trade ten longs for a flat:



Using small squares (dots here) as the unit: Put the ones together to form 8 ones; put the tens (longs) together to form 11 tens; trade 10 tens for a 100 (flat). One would now have 3 hundreds (flats), 1 ten (long), and 8 ones. The answer is 318.

89. Use drawings of multibase blocks to illustrate $32_{five} + 23_{five}$.

ANSWER: The small square is being used as the unit.



The five small squares can be traded for a five (long) leaving 0 ones. There are now six longs. Five would be traded for a flat of 25, leaving 1 five (long). The answer is therefore 110_{five}.

90. In what base does the following addition NOT work: 13 + 13 = 26?

a. base six
b. base seven
c. base eight
d. base ten
e. It works in all of these bases.

ANSWER: a

91. Two-thirds in base ten is what in base twelve?

a. 0.8
b. 0.4
c. 0.2
d. 0.12
e. 0.3

ANSWER: a

92. A) Subtract the following in base five. Show all your work.

$$\begin{array}{r} 2\,2\,1_{\text{five}} \\ -\ \ 4\,2_{\text{five}} \\ \hline \end{array}$$

B) Use your work in part A to explain how the way we regroup in base-five subtraction is similar to the way that we regroup in base-ten subtraction.

ANSWER:

$$\begin{array}{r} 1\ 11 \\ \not{2}\,\not{2}\,{}^{1}1 \\ -\ \ 4\,2 \\ \hline 1\,2\,4 \end{array}$$

We first consider the ones. Regrouping may be necessary to subtract, as in A, where we regrouped to make 6 ones, and again when we regrouped to make 6 fives.

93. 221.2_{three} = ______$_{ten}$.

ANSWER: 13.1_{ten}

94. The blocks are base four.
A) What base-four numeral is illustrated with the long as the unit?
B) Give the English *words* for the base-*ten* value.
C) Suppose the small block is the unit. Give the English *words* for the base-*ten* value.

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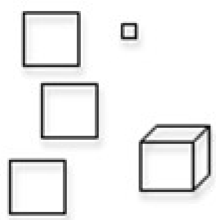
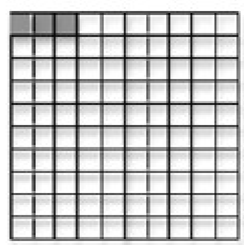


ANSWER: A) 130.1 base four
B) twenty-eight and one-fourth
C) One hundred thirteen; if the small block is the unit, the flat is 16 and the cube is 64. 64 + (3)16 + 1 = 113.

95. A teacher challenged her class to find out how many ways the number 345.23 could be thought about. Macy said 345.23 could be thought of as 340 ones and 52.3 tenths. Is Macy correct or incorrect? Explain how you know.

ANSWER: Macy is correct because 345.23 is 340 ones and 5.23 ones. Five-ones is 50 tenths. Three-hundredths is 0.3 tenths. (See diagram. What part of the whole is each row? What part of the row is the shaded part?) Therefore, 5.23 is 52.3 tenths.



96. Write the "basimal" (expanded) place values and then the usual base-ten fraction or mixed number for 20.11_{three}.

ANSWER:

$$20.11_{three} = (3 \times 2) + (0 \times 1) + (\frac{1}{3} \times 1) + (\frac{1}{9} \times 1) = 6\frac{4}{9}$$

